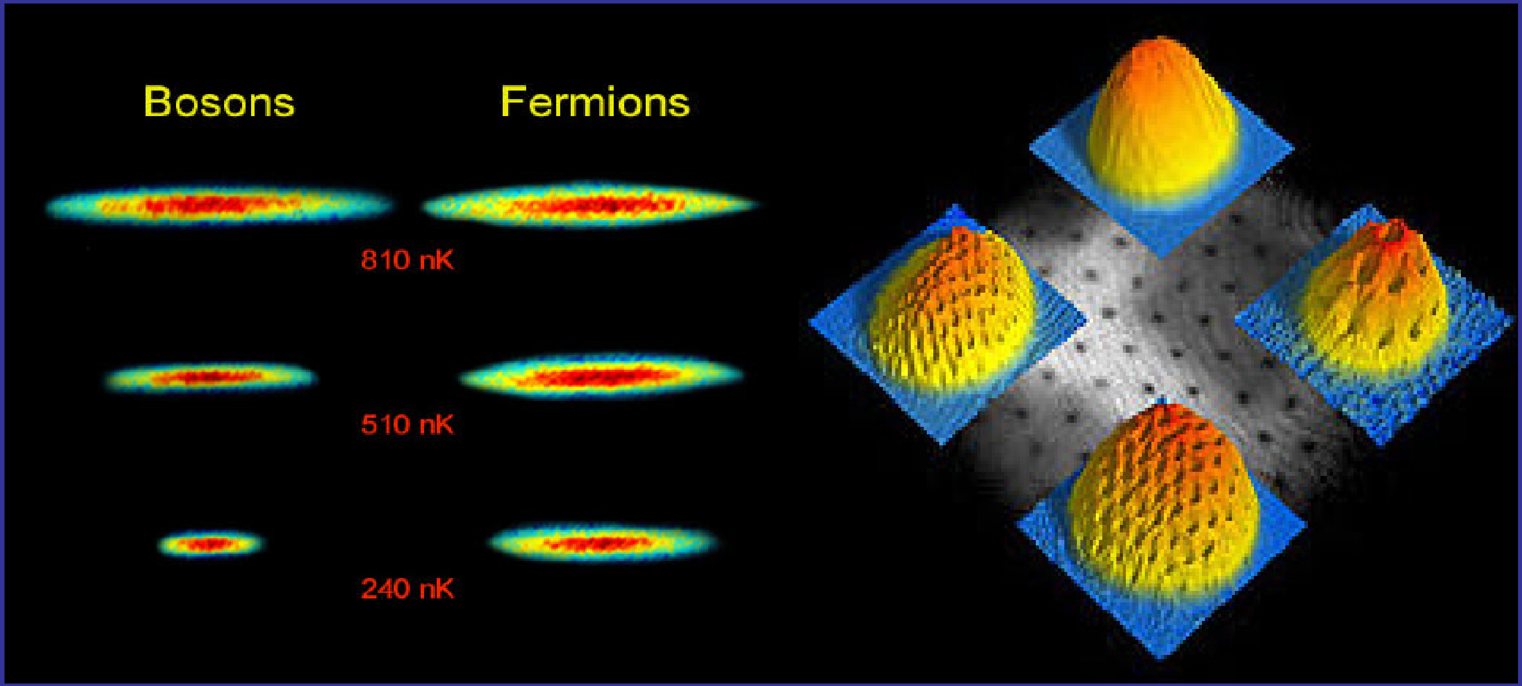
Between bosonic condensate and fermionic superfluid



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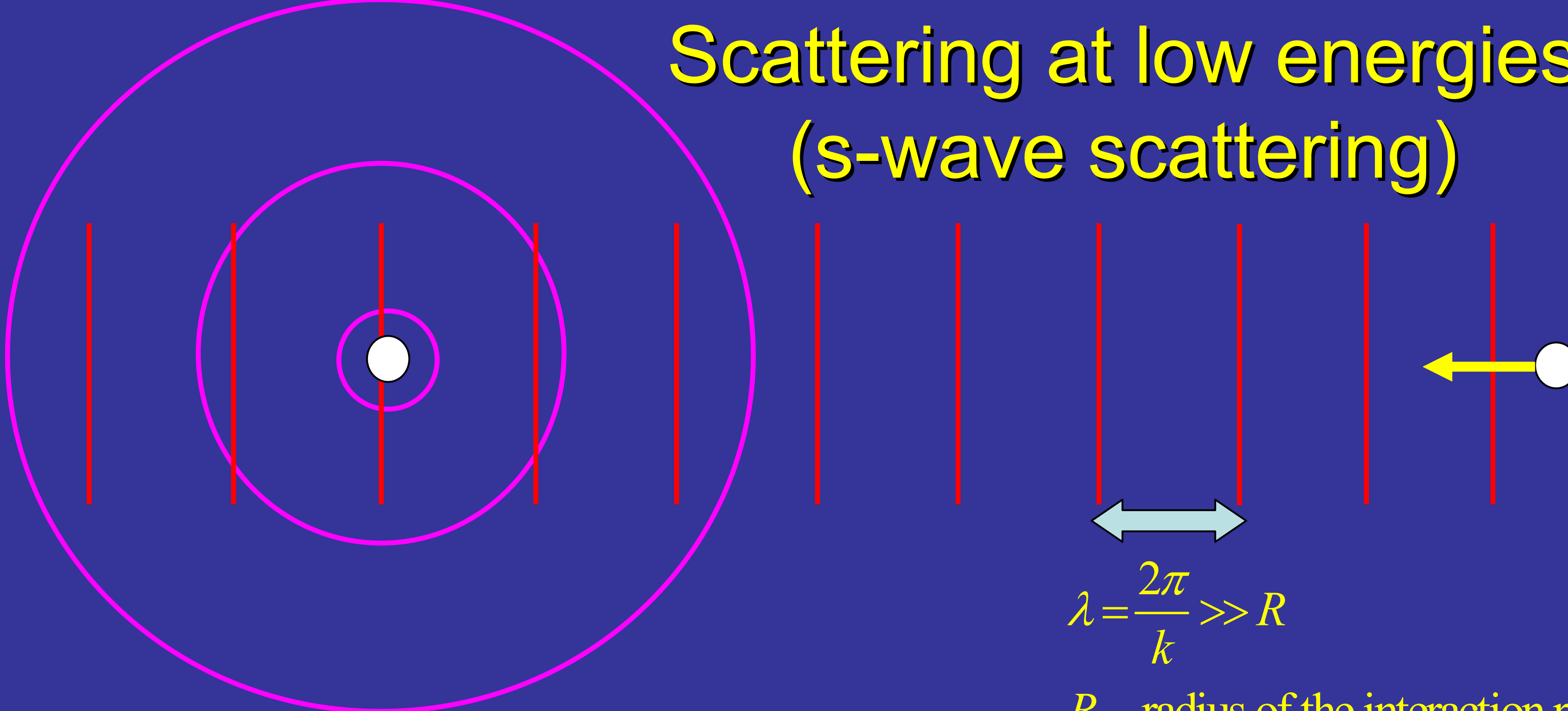
<u>In collaboration with:</u> Aurel Bulgac, Joaquin E. Drut
(University of Washington, Seattle)

Outline

- **BCS-BEC crossover. Universality of the unitary regime.**
- **Physical realization of the unitary regime:**
  ultra cold atomic gases.
- **Equation of state for the uniform Fermi gas in the unitary regime. Critical temperature.**
- **Measurements of the entropy and the critical temperature in a harmonic trap: experiment vs. theory.**

Scattering at low energies (s-wave scattering)



$$\lambda = \frac{2\pi}{k} >> R$$

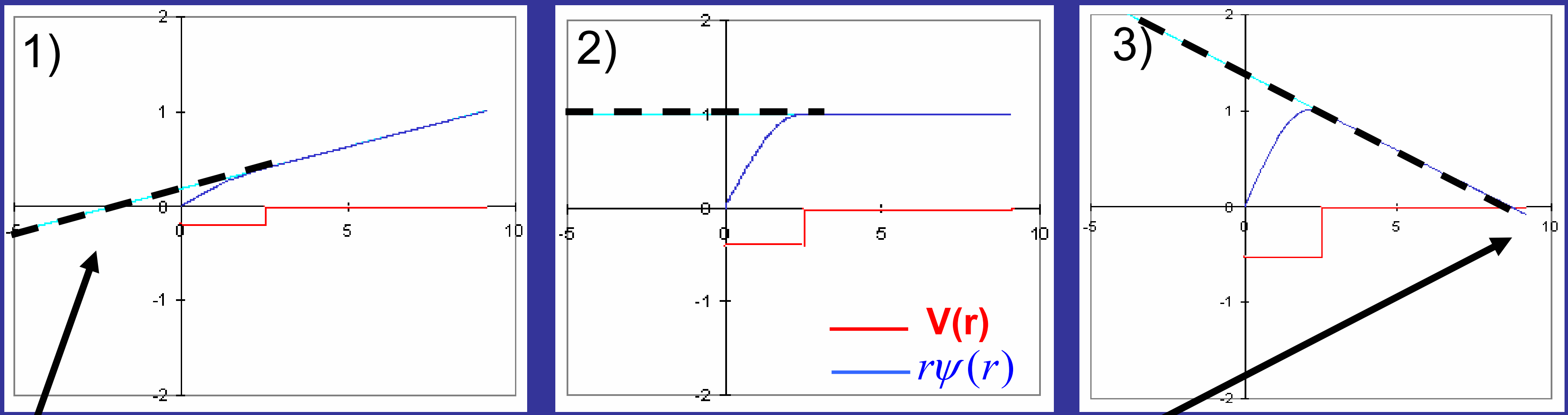
R - radius of the interaction potential

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + f(k)\frac{e^{ikr}}{r}; \quad f(k)$$ - scattering amplitude

$$f(k) \overset{k\to 0}{=} \frac{1}{-ik - 1/a + \frac{1}{2} r_0 k^2},$$ a - scattering length, r_0 - effective range

If $k \to 0$ then the interaction is determined by the scattering length alone.

two-particle wave function for small $r \geq R$ (range of the potential): $r\psi(r) \sim (r-a)$



$a < 0$ there is no bound state $\qquad a = \pm\infty \qquad a > 0$ a bound state exists

Fermi gas: n - number density, a - scattering length

What is the energy of the dilute Fermi gas? $\qquad E(k_F a) = ?$

$(k_F r_0 << 1)$

$\varepsilon_F = \frac{\hbar^2 k_F^2}{2m}; \quad n = \frac{k_F^3}{3\pi^2}$ **- particle density**

$$E/E_{FG} = 1 + \frac{10}{9\pi}(k_F a)\left[1 + \frac{6}{35\pi}(k_F a)(11 - 2ln2) + \ldots\right] + \text{pairing}$$

$$E_{FG} = \frac{3}{5}\varepsilon_F N$$ - Energy of the noninteracting Fermi gas

Perturbation series

(works if: $|k_F a| < 1$)

➢ What is the unitary regime?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$n\, r_0^3 \ll 1$ $\quad$ $n\, |a|^3 \gg 1$

n - particle density
a - scattering length
r_0 - effective range

$$i.e.\ \ r_0 \to 0,\ a \to \pm\infty$$

NONPERTURBATIVE REGIME

System is dilute but strongly interacting!

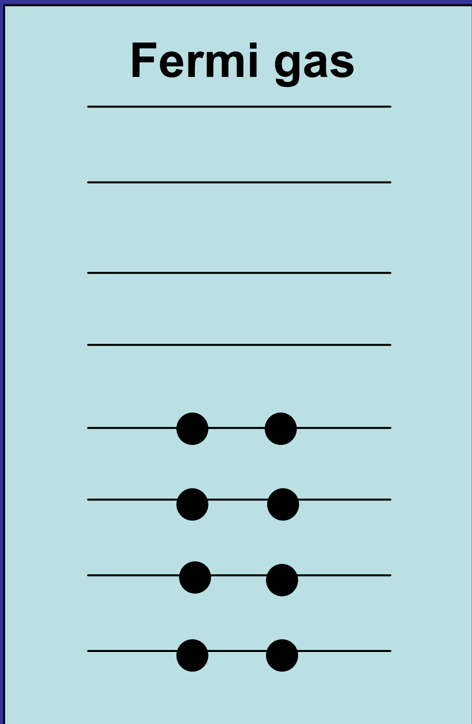
The only scale: $$E_{FG}/N = \frac{3}{5}\varepsilon_F$$

UNIVERSALITY: $$E(T) = \xi\left(\frac{T}{\varepsilon_F}\right) E_{FG}$$

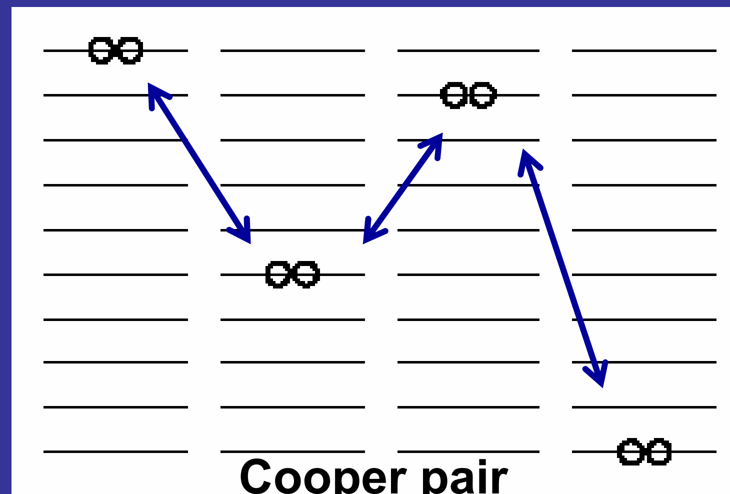
QUESTIONS: **What is the shape of** $\xi(T/\varepsilon_F)$ **?**
What is the critical temperature for the superfluid-to-normal transition?
...

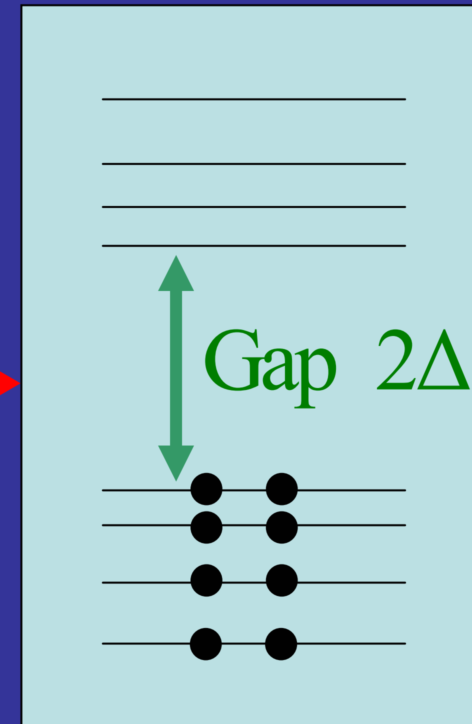
How pairing emerges?

$a < 0$



Cooper's argument (1956)





BCS pairing gap

$$\Delta_{BCS} = \frac{8}{e^2}\frac{\hbar^2 k_F^2}{2m}\exp\left(\frac{\pi}{2k_F a}\right), \quad \text{iff } k_F|a| \ll 1 \text{ and } \frac{1}{k_F} \ll \eta = \frac{1}{k_F}\frac{\varepsilon_F}{\Delta} \text{ - size of the Cooper pair}$$

$$\frac{E_{HF+BCS}}{E_{FG}} = 1 + \frac{10}{9\pi}(k_F a) + \ldots - \frac{5}{8}\left(\frac{\Delta_{BCS}}{\varepsilon_F}\right)^2 = 1 + \frac{10}{9\pi}(k_F a) + \ldots - \frac{40}{e^4}\exp\left(\frac{\pi}{k_F a}\right)$$

Mean-field term **BCS term**

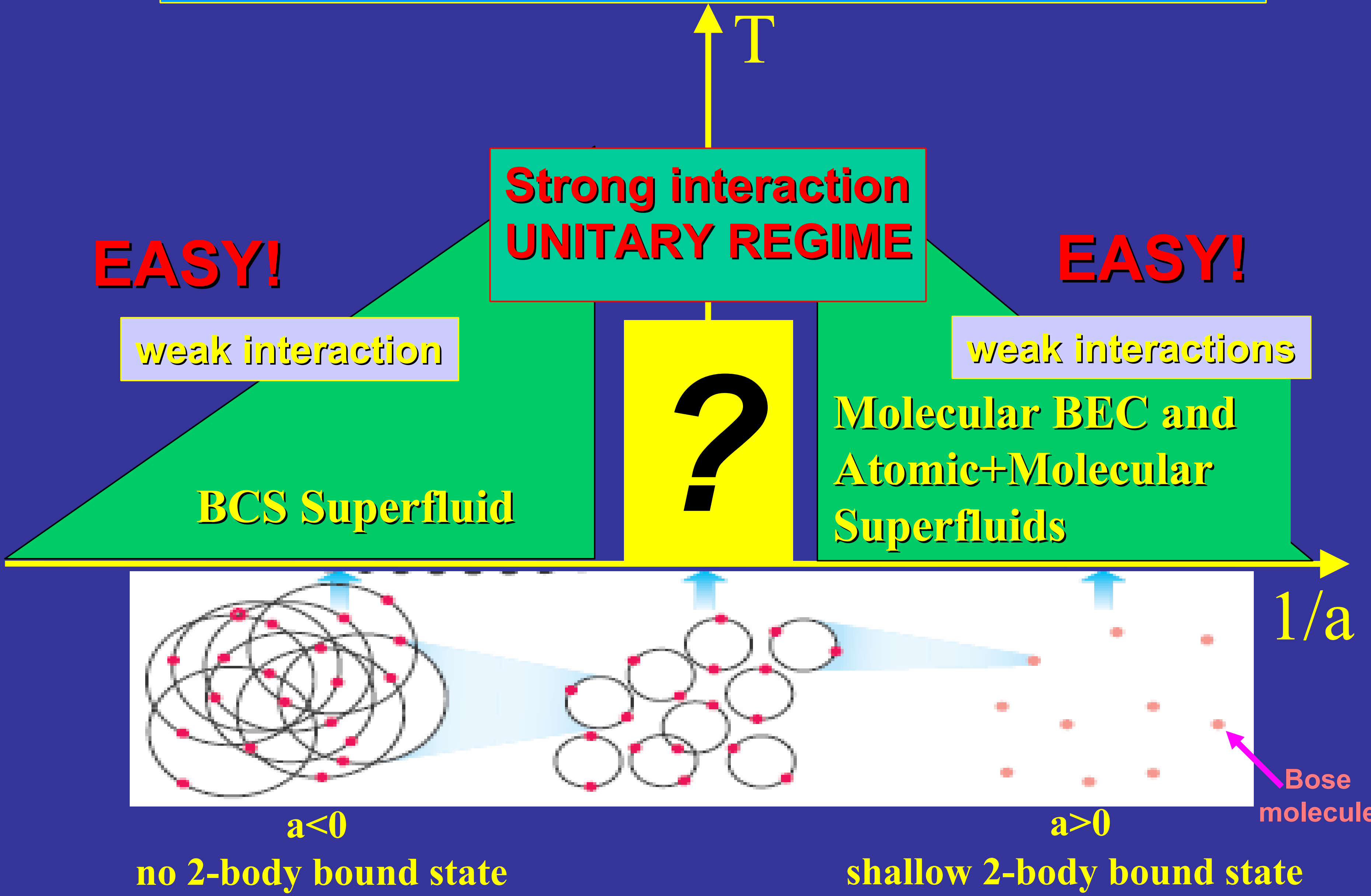
Superconductivity and superfluidity in Fermi systems

20 orders of magnitude over a century of (low temperature) physics

- ✓ **Dilute atomic Fermi gases** $T_c \approx 10^{-12} - 10^{-9}$ eV
- ✓ **Liquid ^{3}He** $T_c \approx 10^{-7}$ eV
- ✓ **Metals, composite materials** $T_c \approx 10^{-3} - 10^{-2}$ eV
- ✓ **Nuclei, neutron stars** $T_c \approx 10^{5} - 10^{6}$ eV
- **QCD color superconductivity** $T_c \approx 10^{7} - 10^{8}$ eV

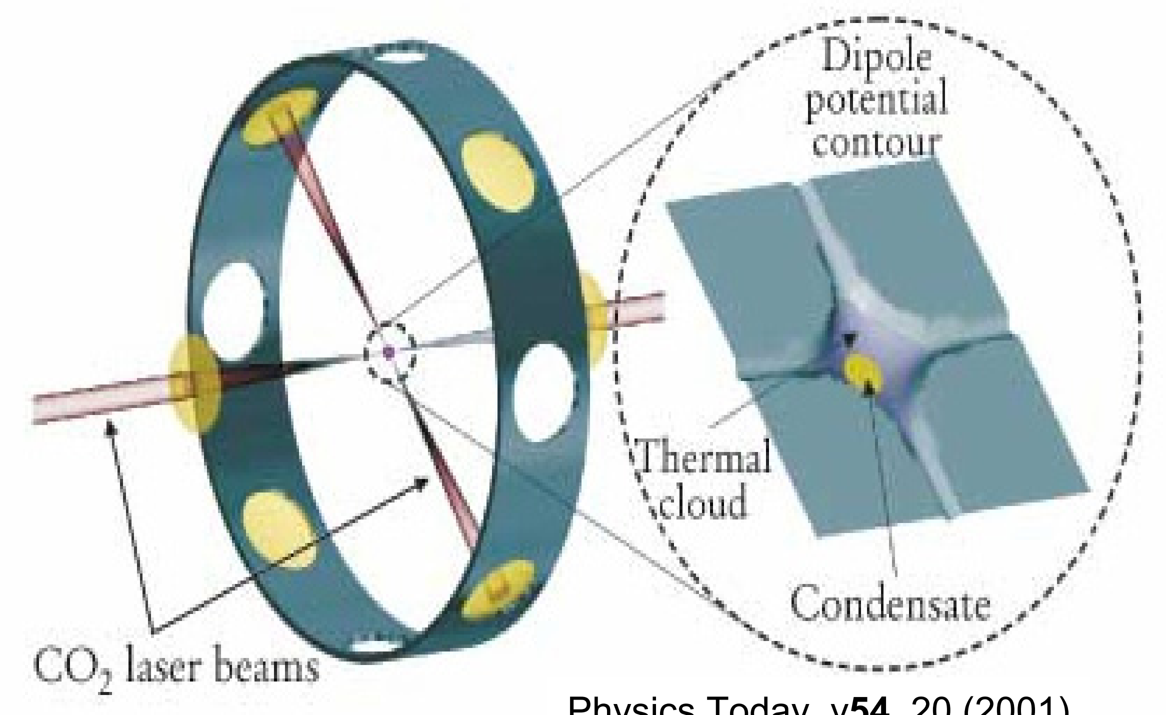
units (1 eV $\approx 10^4$ K)

Expected phases of a two species dilute Fermi system
BCS-BEC crossover



In dilute atomic systems experimenters can control nowadays almost anything:

- The number of atoms in the trap: typically about 10^5-10^6 atoms divided 50-50 among the lowest two hyperfine states.
- The density of atoms
- Mixtures of various atoms
- The temperature of the atomic cloud
- **The strength of this interaction is fully tunable!**

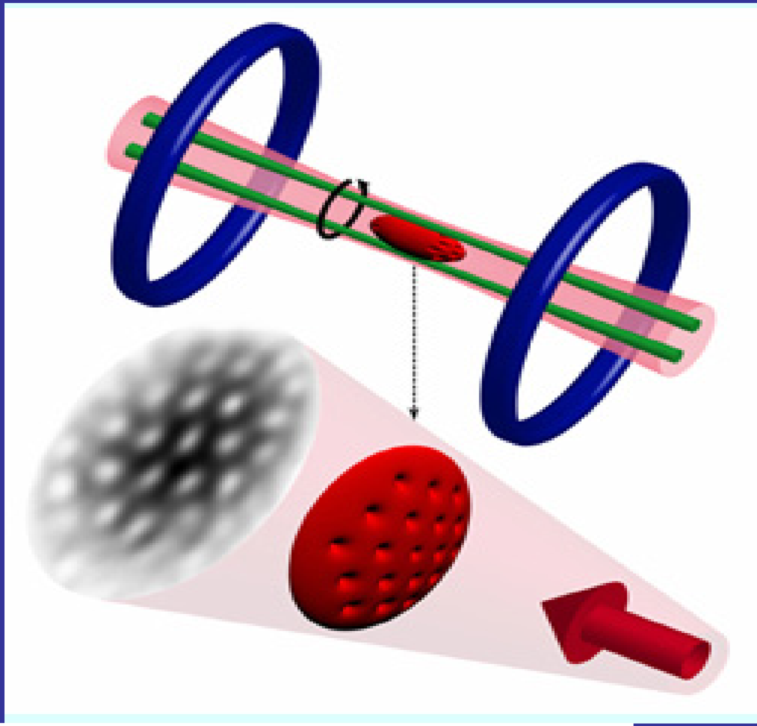


Physics Today, v**54**, 20 (2001)

Who does experiments?

- **Jin's group at Boulder**
- **Grimm's group in Innsbruck**
- **Thomas' group at Duke**
- **Ketterle's group at MIT**
- **Salomon's group in Paris**
- **Hulet's group at Rice**

Evidence for fermionic superfluidity: vortices!

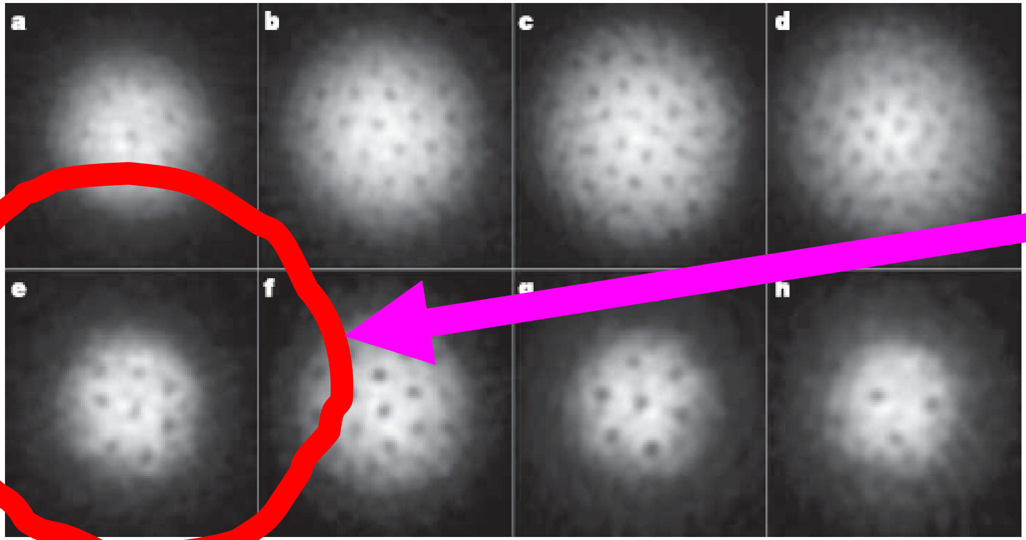


system of fermionic ^{6}Li atoms

Feshbach resonance: B=834G

BEC side: a>0

BCS side: a<0



UNITARY REGIME

M.W. Zwierlein *et al*., Nature, 435, 1047 (2005)

Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b**–**h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**), 812 G (**d**), 833 G (**e**), 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of view of each image is 880 μm × 880 μm.

Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r})\left(-\frac{\hbar^2\Delta}{2m}\right)\hat{\psi}_s(\vec{r}) \; - \; g \int d^3r \; \hat{n}_\uparrow(\vec{r})\hat{n}_\downarrow(\vec{r})$$

$$\hat{N} = \int d^3r \; \left(\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r})\right); \;\; \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r})\hat{\psi}_s(\vec{r})$$

Theoretical approach: Fermions on 3D lattice

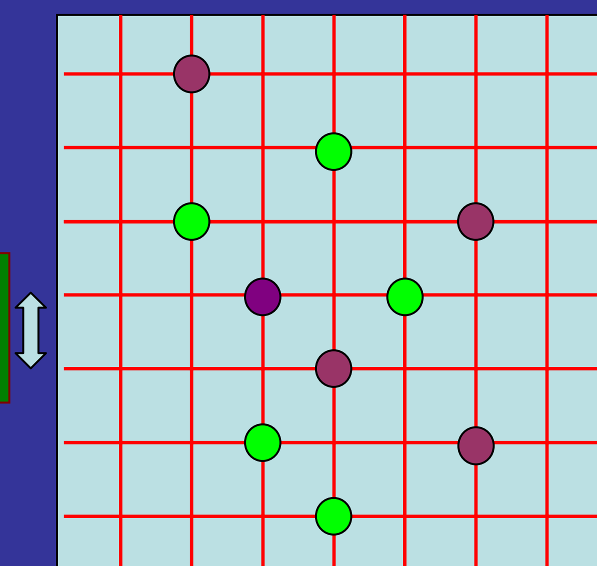
$$Volume = L^3$$

$$lattice\ spacing = \Delta x$$

Coordinate space

L –limit for the spatial correlations in the system

$$k_{cut} = \frac{\pi}{\Delta x}\;;\;\; \Delta x$$



- Spin up fermion: ↑
- Spin down fermion: ↓

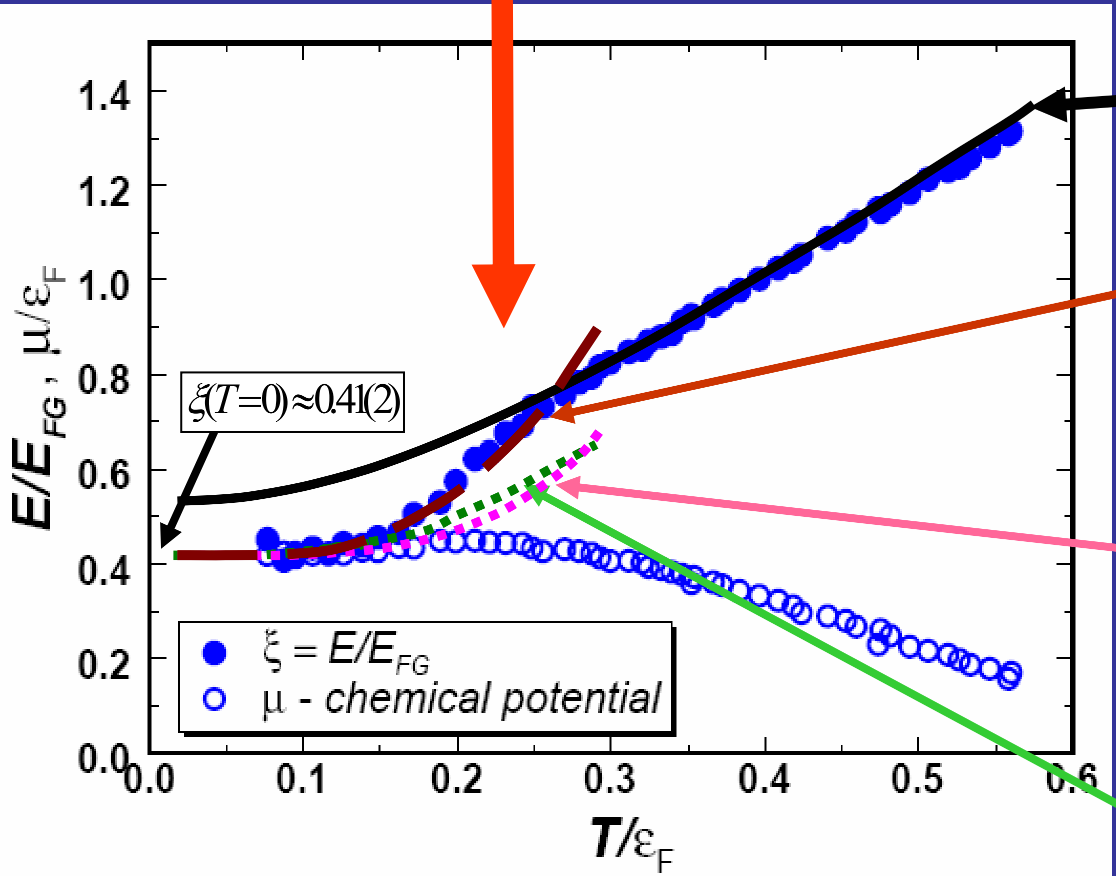
External conditions:

T - temperature

μ - chemical potential

Periodic boundary conditions imposed

a = ±∞ Superfluid to Normal Fermi Liquid Transition $T_c = 0.23(2)\varepsilon_F$



Normal Fermi Gas
(with vertical offset, solid line)

Bogoliubov-Anderson phonons and quasiparticle contribution
(dashed line)

Bogoliubov-Anderson phonons contribution only (dotted line)

Quasi-particle contribution only
(dotted line)

$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}} \exp\left(-\frac{\Delta}{T}\right)$$

$$\Delta = 0.5\varepsilon_F$$

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.44$$

A. Bulgac, J.E. Drut, P. Magierski,
Phys. Rev. Lett. 96,090404(2006)

Low temperature behaviour of a Fermi gas in the unitary regime

$$F(T) = \frac{3}{5}\varepsilon_F N \varphi\left(\frac{T}{\varepsilon_F}\right) = E - TS \quad \text{and} \quad \frac{\mu(T)}{\varepsilon_F} \approx \xi_s \approx 0.41(2) \text{ for } T < T_C$$

$$\mu(T) = \frac{dF(T)}{dN} = \varepsilon_F\left[\varphi\left(\frac{T}{\varepsilon_F}\right) - \frac{2}{5}\frac{T}{\varepsilon_F}\varphi'\left(\frac{T}{\varepsilon_F}\right)\right] \approx \varepsilon_F \xi_s$$

$$\varphi\left(\frac{T}{\varepsilon_F}\right) = \varphi_0 + \varphi_1\left(\frac{T}{\varepsilon_F}\right)^{5/2}$$

$$E(T) = \frac{3}{5}\varepsilon_F N\left[\xi_s + \varsigma_s\left(\frac{T}{\varepsilon_F}\right)^n\right]$$

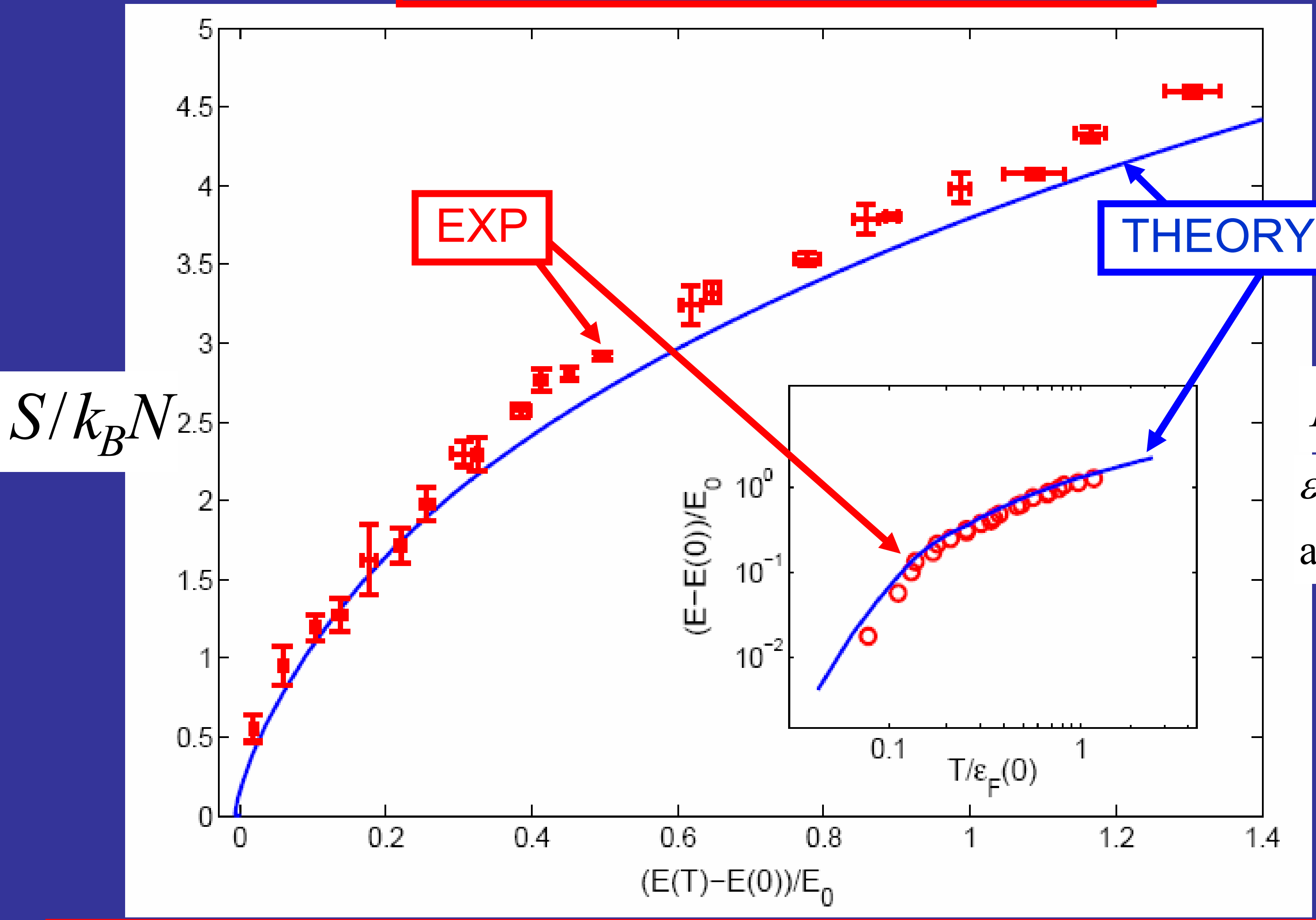
Lattice results disfavor either $n \geq 3$ or $n \leq 2$ and suggest $n = 2.5(0.25)$

This is the same behavior as for a gas of noninteracting (!) bosons below the condensation temperature.

Comparison with experiment

John Thomas' group at Duke University,

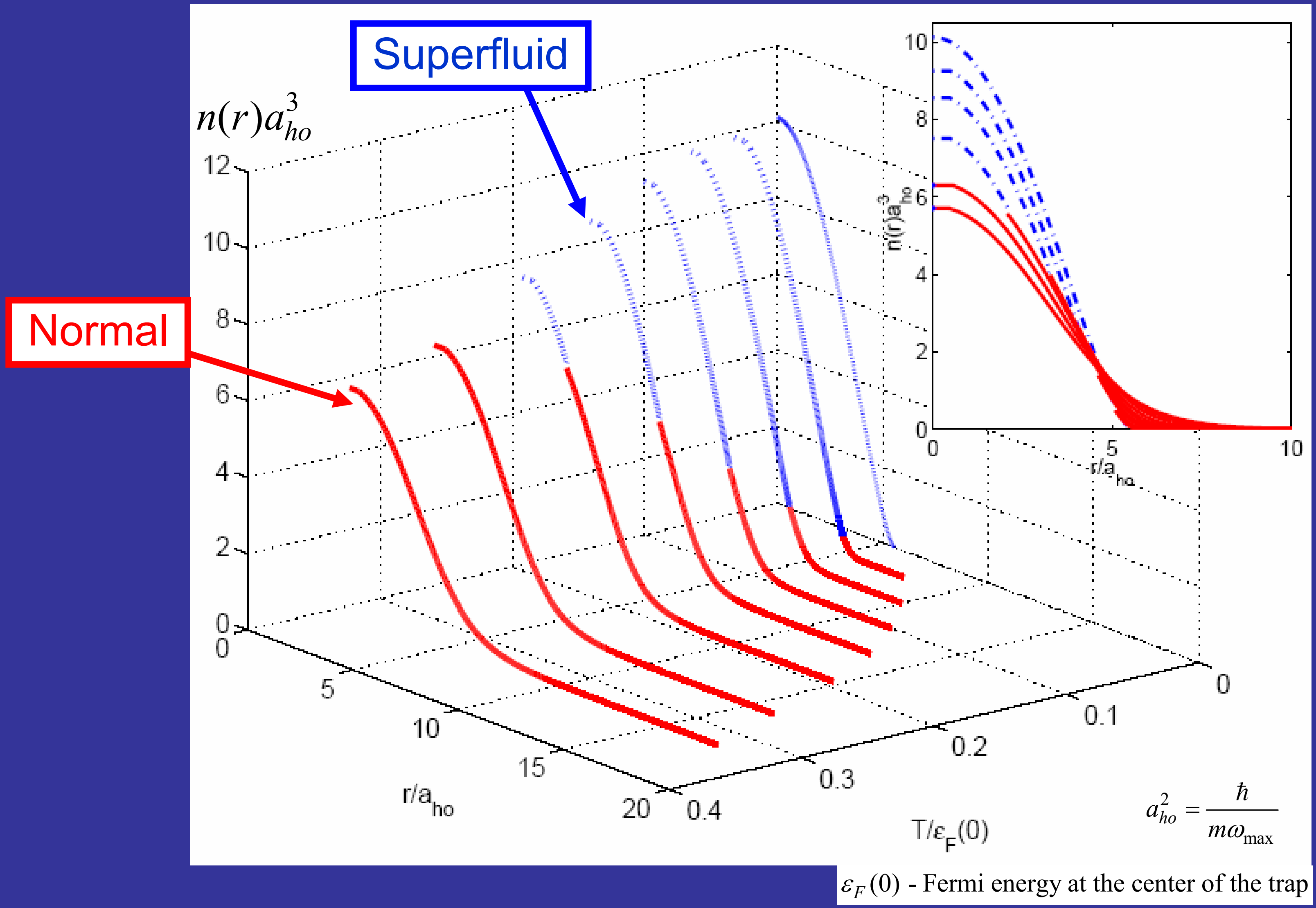
L.Luo, et al. Phys. Rev. Lett. 98, 080402, (2007)



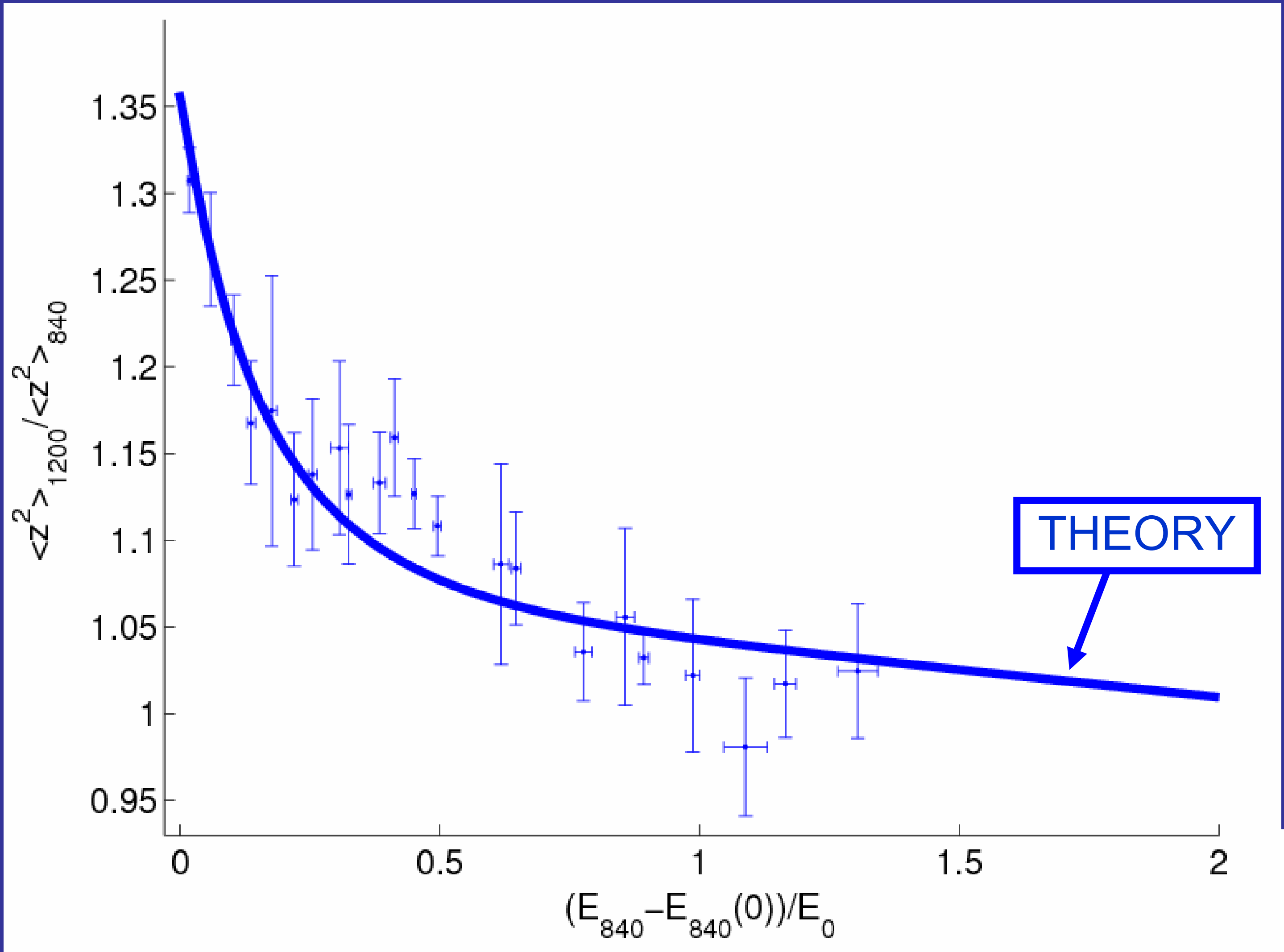
$E_0 = N\varepsilon_F^{ho}$

$\varepsilon_F(0)$ - Fermi energy at the center of the trap

Entropy as a function of energy (relative to the ground state) for the unitary Fermi gas in the harmonic trap. Inset: log-log plot of energy as a function of temperature.



The radial (along shortest axis) density profiles of the atomic cloud in the Duke group experiment at various temperatures.



$$E_0 = N\varepsilon_F^{ho}$$

Ratio of the mean square cloud size at B=1200G to its value at unitarity (B=840G) as a function of the energy. Experimental data are denoted by point with error bars.

$$B = 1200G \Rightarrow 1/k_F a \approx -0.75 \qquad B = 840G \Rightarrow 1/k_F a \approx 0$$

Summary

We presented the first model-independent comparison of recent measurements of the entropy and the critical temperature, performed by the Duke group: L.Luo, et al. Phys. Rev. Lett. 98, 080402, (2007), with our recent finite temperature Monte Carlo calculations.

EXP.

$$\begin{cases} E(T_c) - E(0) \approx 0.41(5) N \varepsilon_F^{ho}, \\ S_c / N \approx 2.7(2) k_B, \\ T_c \approx 0.29(3) \varepsilon_F^{ho} \end{cases}$$

THEORY

$$\begin{cases} E(T_c) - E(0) \approx 0.34(2) N \varepsilon_F^{ho}, \\ S_c / N \approx 2.4(3) k_B, \\ T_c \approx 0.27(3) \varepsilon_F^{ho} \end{cases}$$

A.Bulgac, J.E. Drut, P. Magierski,
cond-mat/0701786, Phys. Rev.Lett (in press)

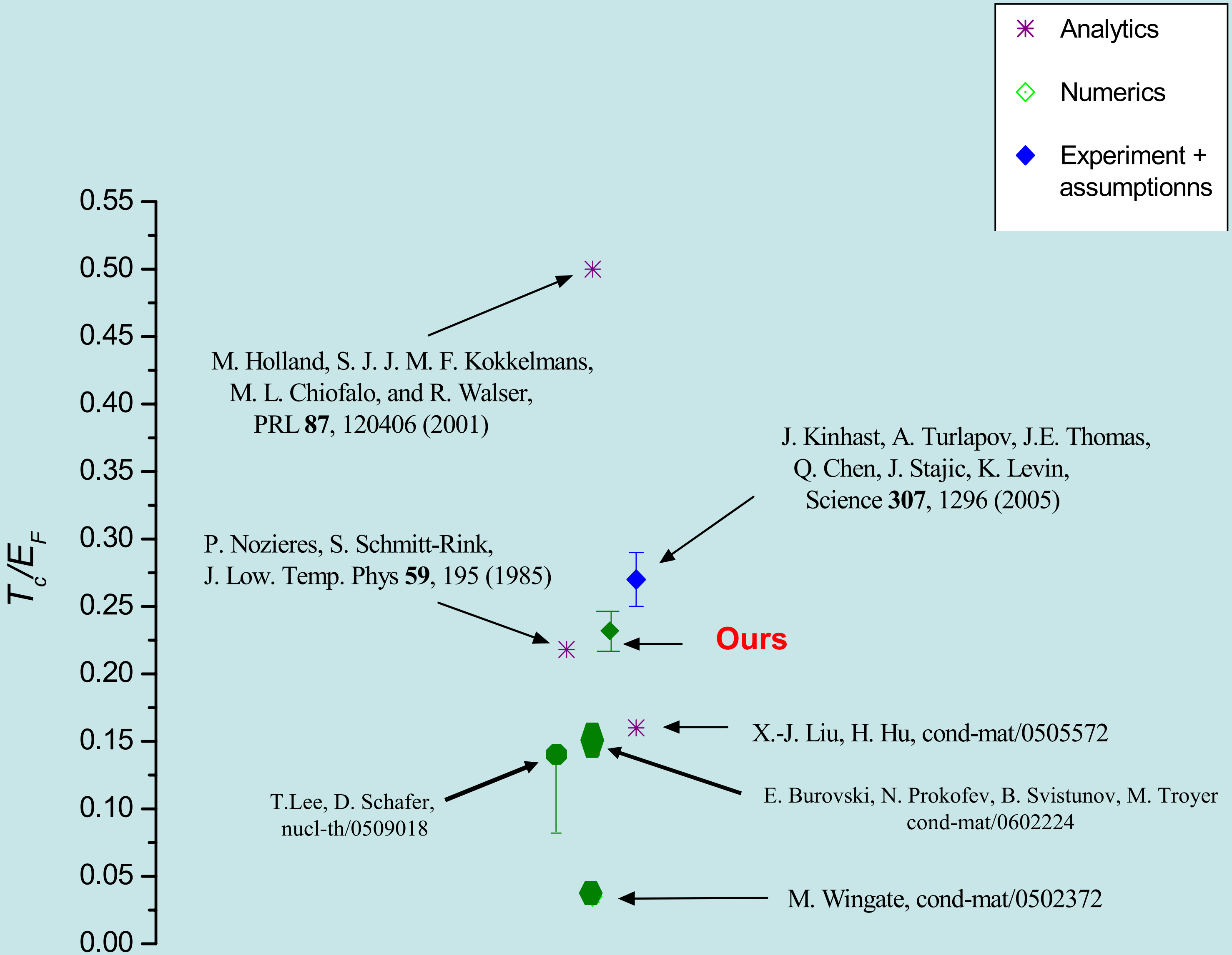
The results are consistent with the predicted value of the critical temperature for the uniform unitary Fermi gas: 0.23(2)ε_F

Conclusions

- ✓ Fully non-perturbative calculations for a spin ½ many fermion system in the unitary regime at finite temperatures are feasible and apparently the system undergoes a phase transition in the bulk at $T_c = 0.23\ (2)\ \varepsilon_F$
- ✓ Chemical potential is constant up to the critical temperature – note similarity with Bose systems!
- ✓ Below the transition temperature, both phonons and fermionic quasiparticles contribute almost equaly to the specific heat. In more than one way the system is at crossover between a Bose and Fermi systems.

There are reasons to believe that below the critical temperature this system is a new type of fermionic superfluid, with unusual properties.

Quest for unitary point critical temperature



Boris Svistunov's talk (updated), Seattle 2005