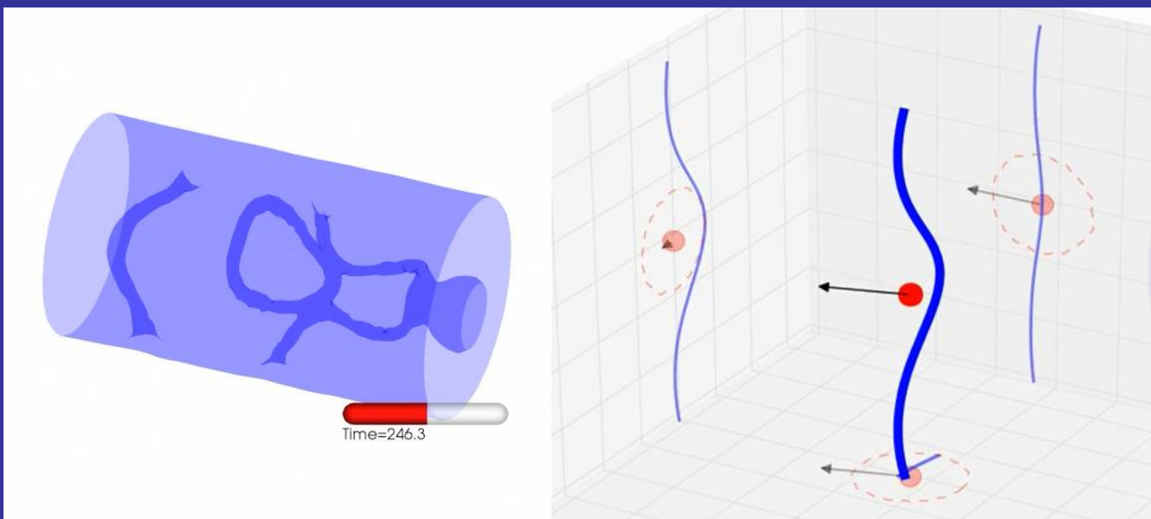
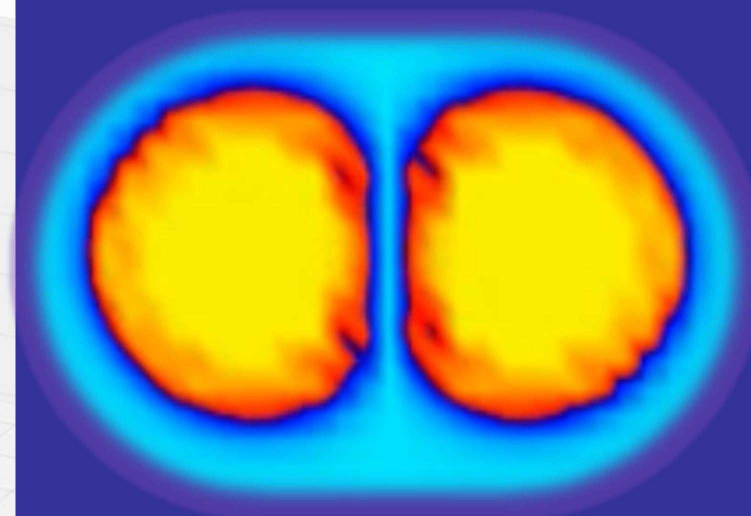
Dynamics of topological excitations: from ultracold atomic gases to atomic nuclei and neutron stars





Piotr Magierski

(Warsaw University of Technology)

100 years of superconductivity and superfluidity in Fermi systems

<u>Discovery</u>: H. Kamerlingh Onnes in 1911 cooled a metallic sample of mercury at T<4.2K

20 orders of magnitude over a century of (low temperature) physics

- ✓ **Dilute atomic Fermi gases** $T_c \approx 10^{-12} - 10^{-9}$ eV
- ✓ **Liquid ^{3}He** $T_c \approx 10^{-7}$ eV
- ✓ **Metals, composite materials** $T_c \approx 10^{-3} - 10^{-2}$ eV
- ✓ **Nuclei, neutron stars** $T_c \approx 10^{5} - 10^{6}$ eV
- **QCD color superconductivity** $T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV $\approx 10^4$ K)

Robert B. Laughlin, Nobel Lecture, December 8, 1998:

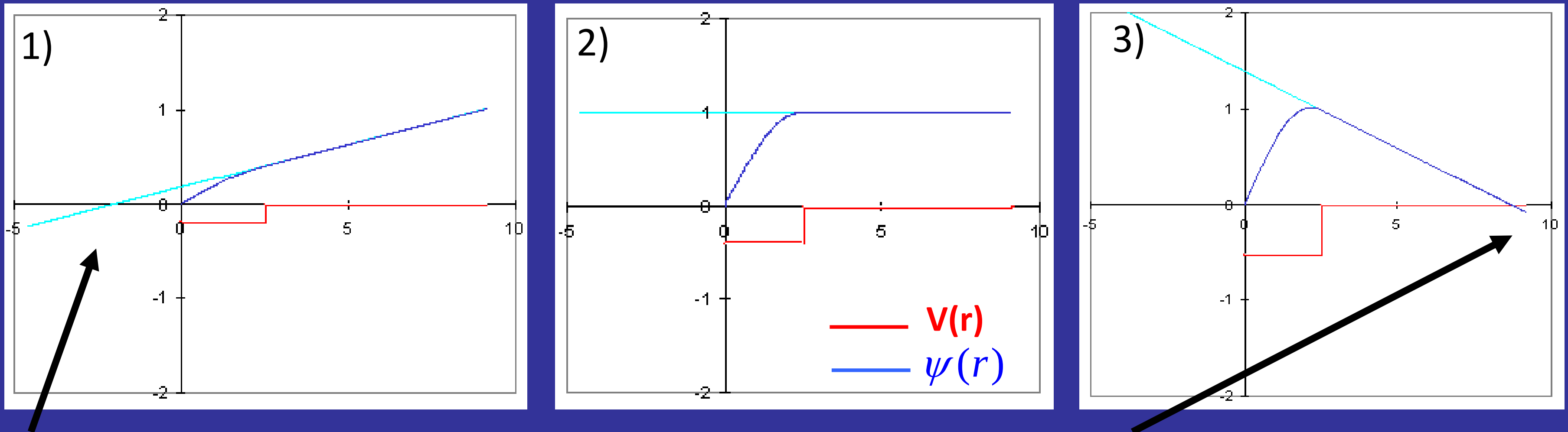
One of my favorite times in the academic year occurs [..] when I give my class of extremely bright graduate students [..] a take home exam in which they are asked <u>TO DEDUCE SUPERFLUIDITY FROM FIRST PRINCIPLES.</u>

There is no doubt a special place in hell being reserved for me at this very moment for this mean trick, for the task is <u>IMPOSSIBLE</u>. Superfluidity [..] is an EMERGENT phenomenon – a low energy collective effect of huge number of particles that <u>CANNOT</u> be deduced from the microscopic equations of motion in a <u>RIGOROUS WAY</u> and that <u>DISAPPEARS</u> completely when the system is taken apart.

[..]students who stay in physics long enough [..] eventually come to understand that the <u>REDUCTIONIST IDEA IS WRONG</u> a great deal of the time and perhaps <u>ALWAYS.</u>

Energy of dilute Fermi gas with attractive interaction

Dilute: scattering length 'a' determines the interaction



$a<0$ there is no bound state

$a=\pm\infty$

$a>0$ a bound state exists

What is the energy of the dilute Fermi gas? $E(k_F a)=?$

$\varepsilon_F=\frac{\hbar^2 k_F^{\,2}}{2m};\quad n=\frac{k_F^{\,3}}{3\pi^2}$ **- particle density**

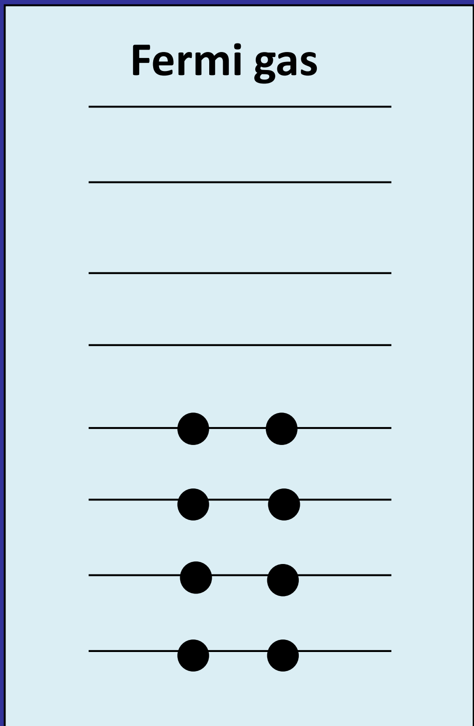
Perturbation series:

$$E/E_{FG}=1+\frac{10}{9\pi}(k_F a)\left[1+\frac{6}{35\pi}(k_F a)(11-2\ln 2)+\ldots\right]$$

$E_{FG}=\frac{3}{5}\varepsilon_F N$ - Energy of the noninteracting Fermi gas

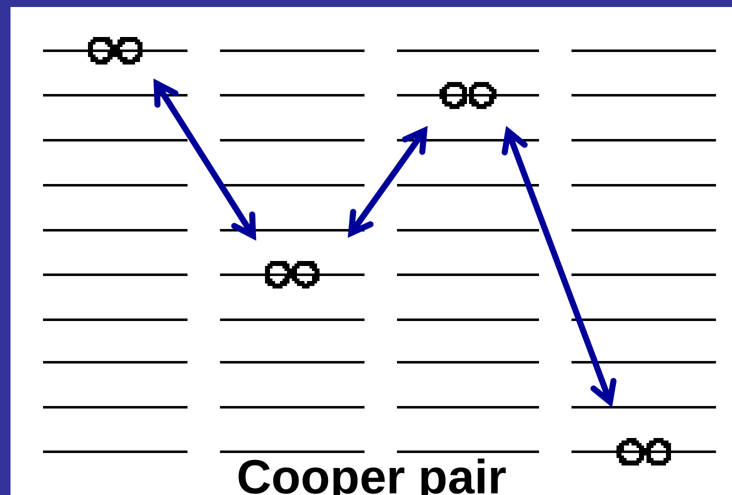
Pairing correlations and superconductivity

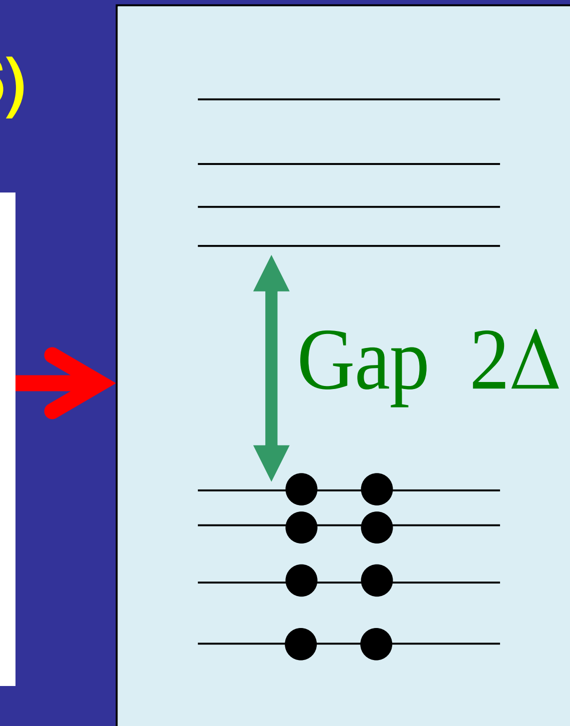
$a < 0$



Cooper's argument (1956)

Pair scattering:





$$\Delta_{BCS} = \frac{8}{e^2}\frac{\hbar^2 k_F^2}{2m}\exp\left(\frac{\pi}{2k_F a}\right), \quad \text{iff} \quad k_F\,|a| \ll 1 \ \text{ and } \ \frac{1}{k_F} \ll \eta = \frac{1}{k_F}\frac{\varepsilon_F}{\Delta}$$ - size of the Cooper pair

$$\frac{E_{HF+BCS}}{E_{FG}} = 1 + \frac{10}{9\pi}(k_F a) + \ldots - \frac{5}{8}\left(\frac{\Delta_{BCS}}{\varepsilon_F}\right)^2 = 1 + \frac{10}{9\pi}(k_F a) + \ldots - \frac{40}{e^4}\exp\left(\frac{\pi}{k_F a}\right)$$

Hartree-Fock term

BCS term

A little bit of history

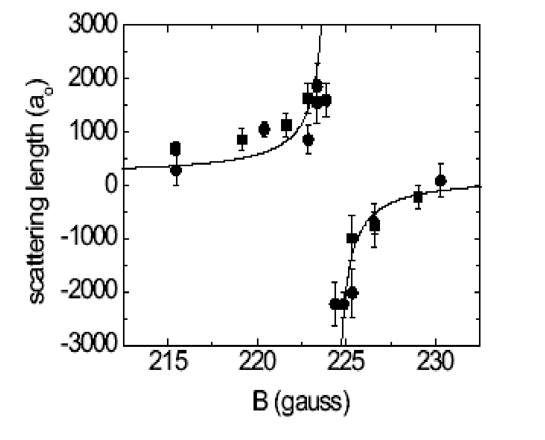
Bertsch Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

Why? Besides pure theoretical curiosity, this problem is relevant to neutron stars!

<u>In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not! A number of people argued that under such conditions fermionic matter is unstable.</u>

- **Thomas' Duke group (2002) demonstrated experimentally using ultracold atomic gas that such systems are (meta)stable.**



Feshbach resonance

Regal and Jin, PRL <u>90</u>, 230404 (2003)

In dilute atomic systems experimenters can control nowadays almost anything:

- The number of atoms in the trap: typically about 10^5-10^6 atoms divided among the lowest two hyperfine states.
- The density of atoms
- Mixtures of various atoms
- The temperature of the atomic cloud
- **<u>The strength of this interaction is fully tunable!</u>**

What is a unitary gas?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$\boxed{n\, r_0^3 \ll 1}$ $\boxed{n\, |a|^3 \gg 1}$

n - particle density
a - scattering length
r_0 - effective range

$$\boxed{\text{i.e. } r_0 \to 0, a \to \pm\infty}$$

NONPERTURBATIVE REGIME

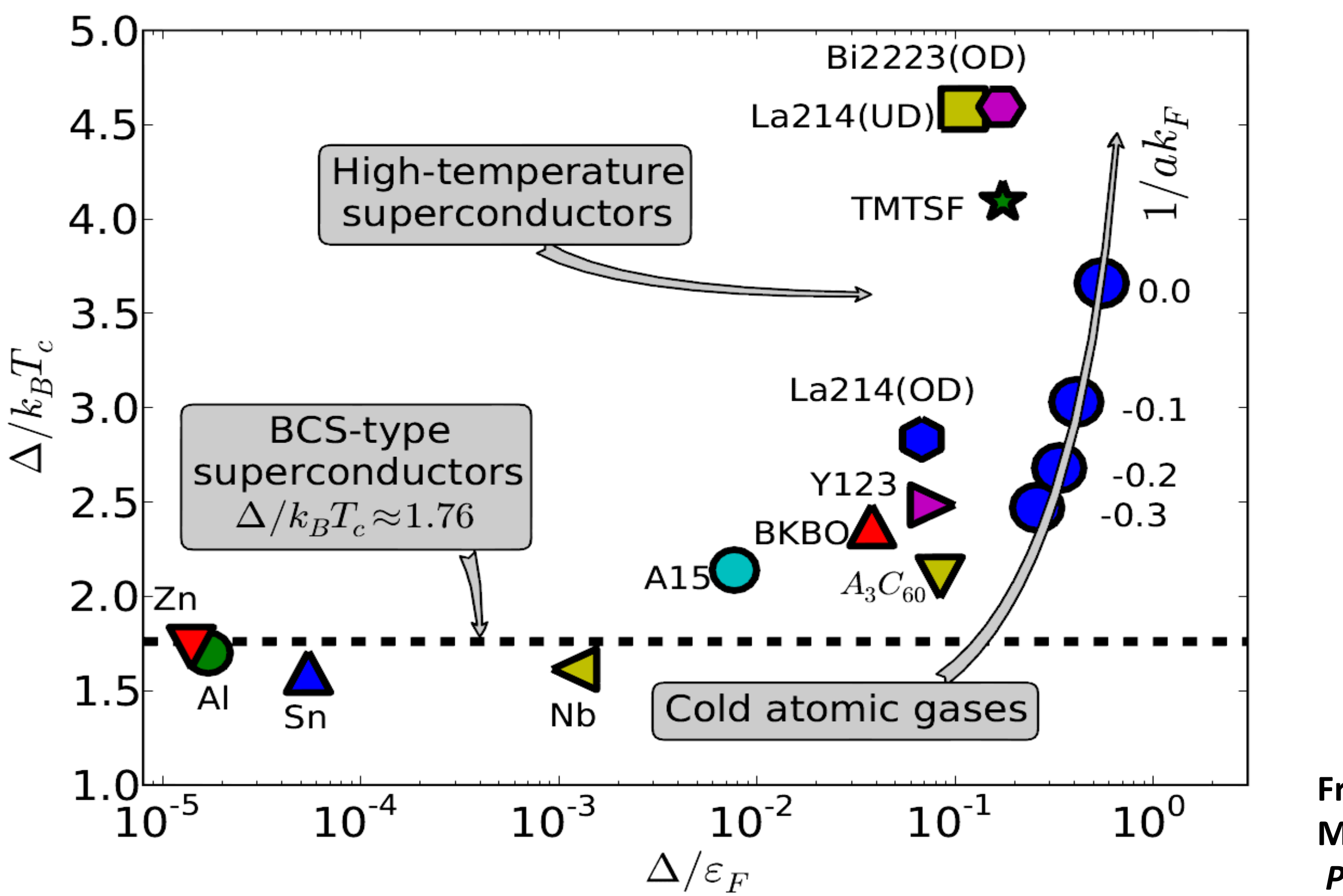
System is dilute but strongly interacting!

Universality: $E = \xi_0 E_{FG} \quad \text{for } T = 0$

$\xi_0 = 0.376(5)$ - **Exp. estimate**

E_{FG} - **Energy of noninteracting Fermi gas**

Cold atomic gases and high Tc superconductors

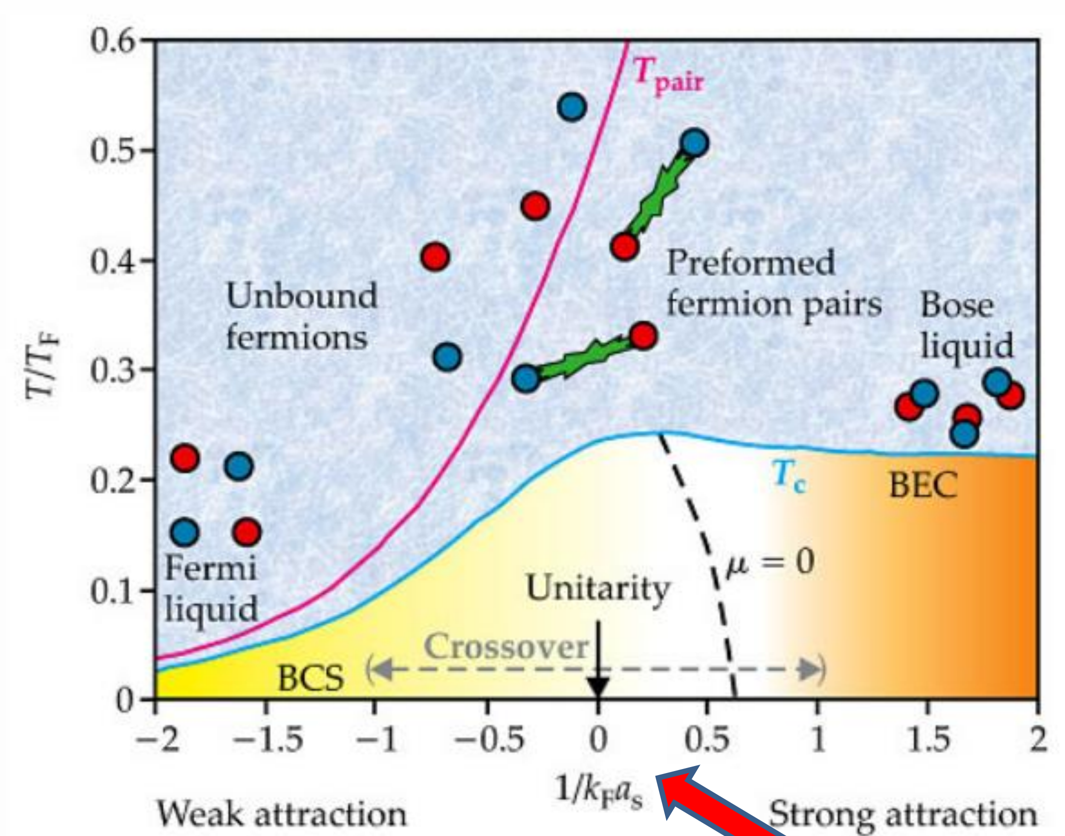


From:
Magierski, Wlazłowski, Bulgac, *Phys. Rev. Lett.* 107,145304(2011)

Δ/ε_F – Ratio of the strength of two interparticle correlations to the kinetic energy of the fastest particle in the system.

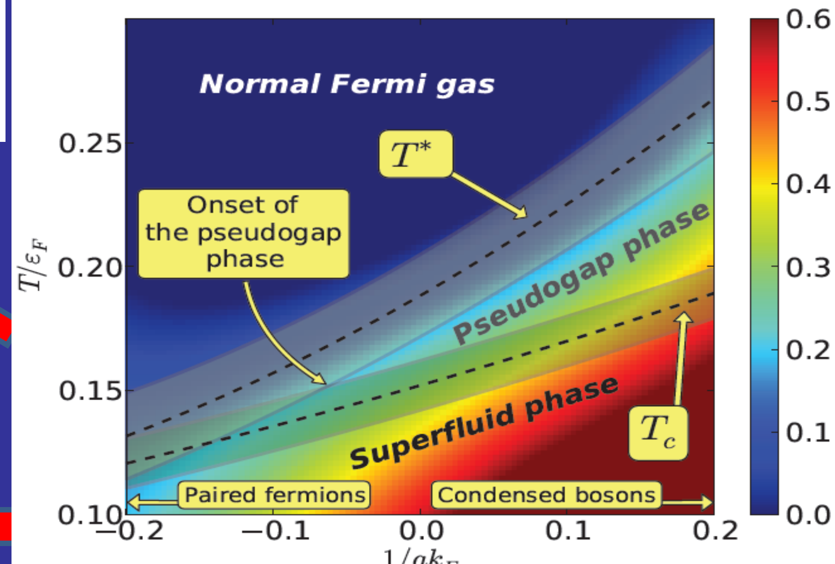
Standard theory of superconductivity (BCS theory) fails!
Qualitatively new phenomena occur like e.g. pseudogap characteristic for high-Tc superconductors

Magierski, Wlazłowski, Bulgac, Drut, *Phys. Rev. Lett.* 103,210403(2009)



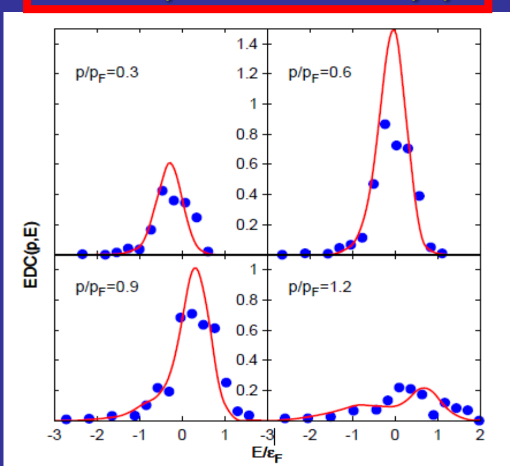
From Sa de Melo, Physics Today (2008)

Gap in the single particle fermionic spectrum – Quantum Monte Carlo results



Magierski, Wlazłowski, Bulgac, *Phys. Rev. Lett.*107,145304(2011)
Magierski, Wlazłowski, Bulgac, Drut, *Phys. Rev. Lett.*103,210403(2009)

RF spectroscopy

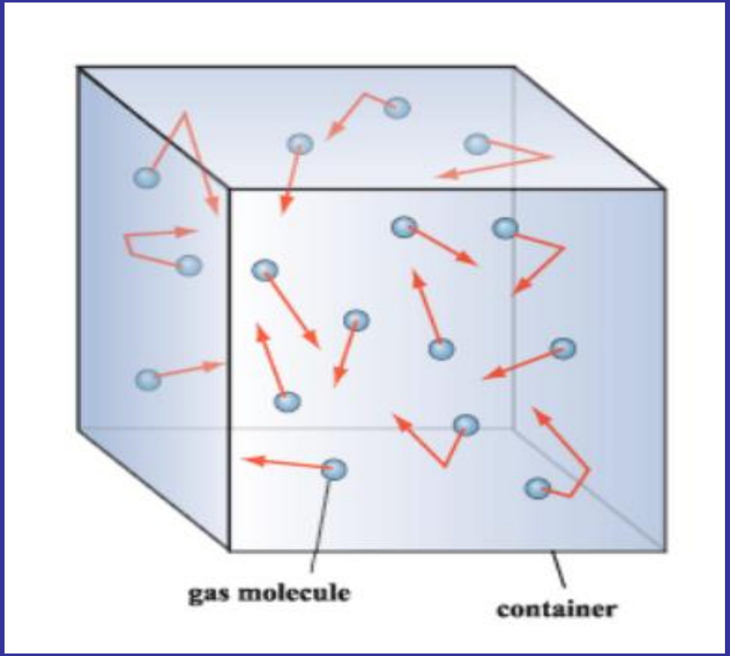


Experiment (blue dots): D. Jin's group
Gaebler et al. *Nature Physics 6, 569(2010)*

Viscosity in strongly correlated quantum systems:

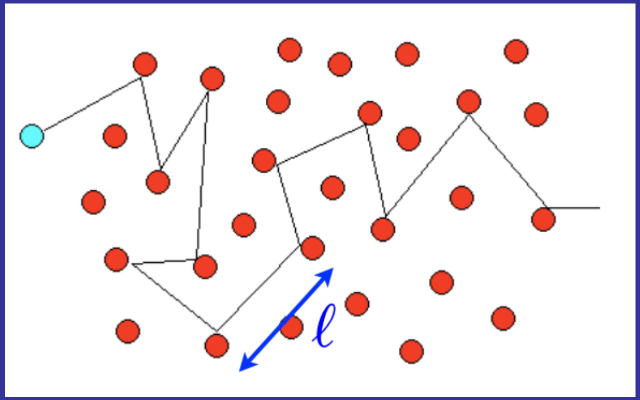
Water and honey flow with different rates:
different viscosity

Viscosity in strongly correlated quantum systems:



In the light of the kinetic theory of gases molecules are moving mostly along straight lines and occasionally bump onto each other.

Mean free path



This leads to the Maxwell's formula for viscosity (1860):

$$\eta \sim \rho v \ell = \text{mass density} \times \text{velocity} \times \text{mean free path}$$

Consequences:

- Non interacting gas is a pathological example of the system with an infinite viscosity
- Strongly interacting system can have low viscosity since the mean free path is short **but from Q. Mechanics:**

$$\frac{\eta}{\rho} \sim \bar{p} l \geq \hbar$$

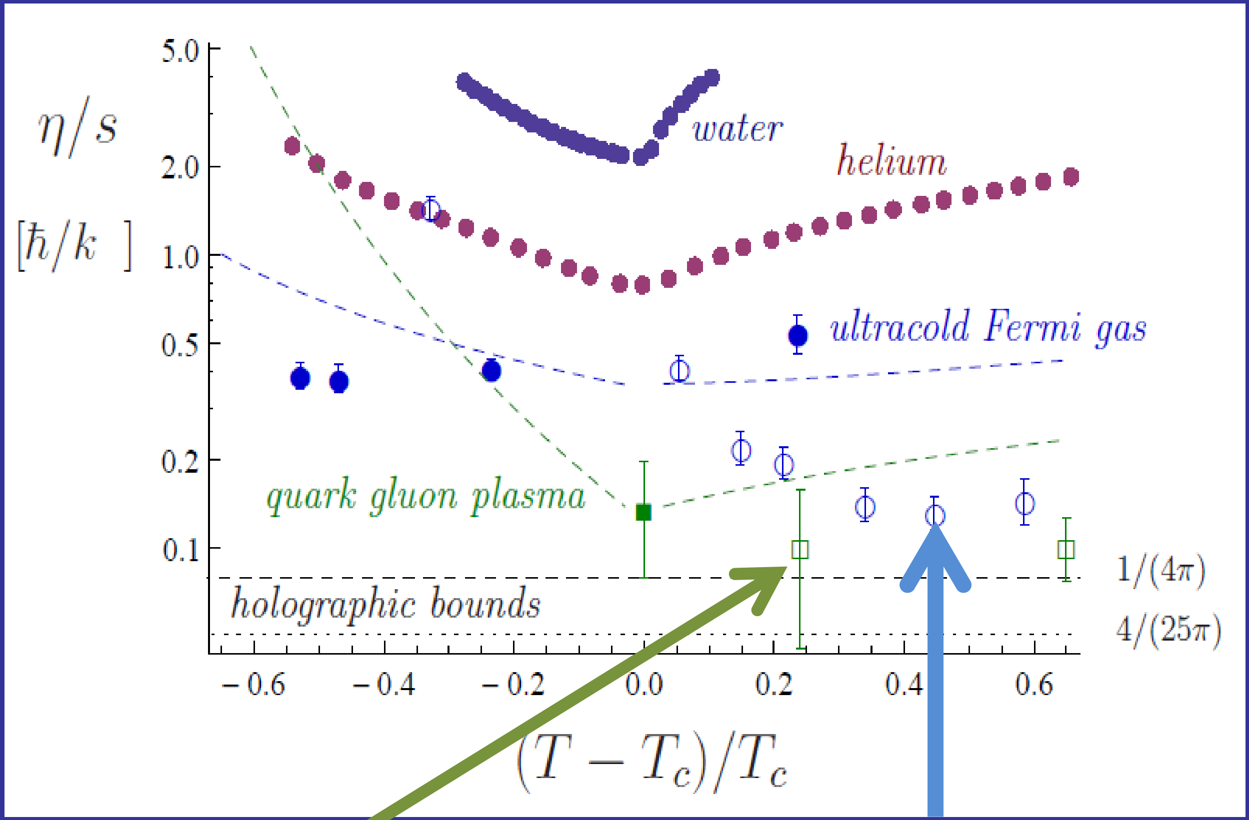
$\bar{p}$ - average momentum

Perfect fluid $\frac{\eta}{S} = \frac{\hbar}{4\pi k_B}$ - strongly interacting quantum system = No well defined quasiparticles

Candidates: quark gluon plasma, atomic gas

Shear viscosity to entropy ratio – experiment vs. theory

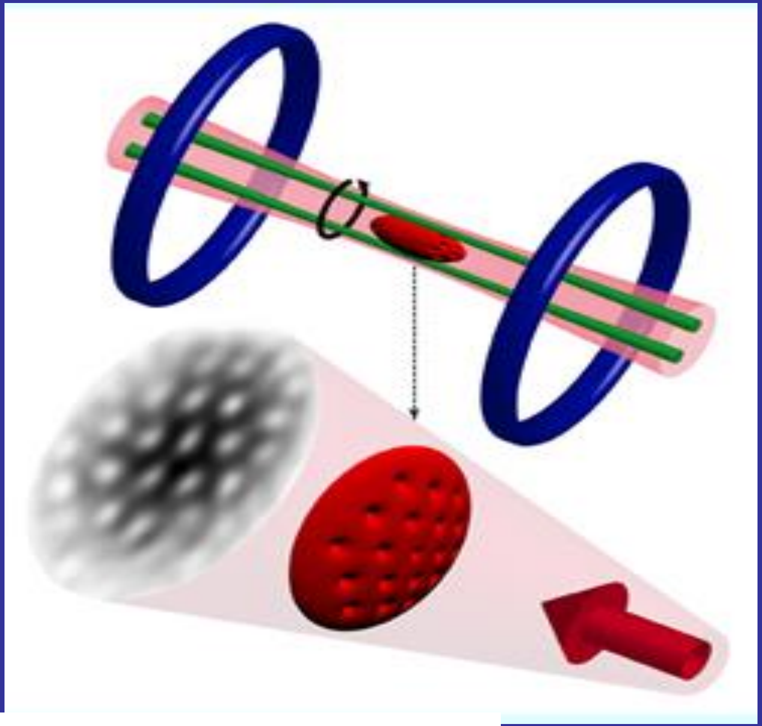
(from A. Adams et al. New Journal of Physics, "Focus on Strongly Correlated Quantum Fluids: from Ultracold Quantum Gases to QCD Plasmas„ arXive:***1205.5180)***



Lattice QCD (SU(3) gluodynamics): H.B. Meyer, Phys. Rev. D 76, 101701 (2007)

QMC calculations for UFG: G. Wlazłowski, P. Magierski, J.E. Drut, Phys. Rev. Lett. 109, 020406 (2012)

Vortex generation in ultracold Fermi gases

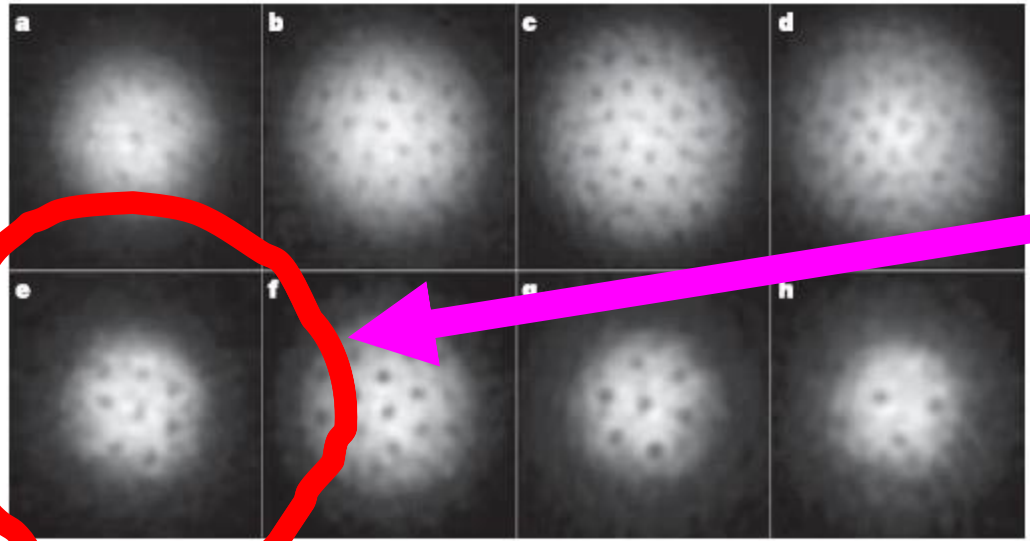


system of fermionic ^6Li atoms

Feshbach resonance:
B=834G

BEC side:
a>0

BCS side
a<0

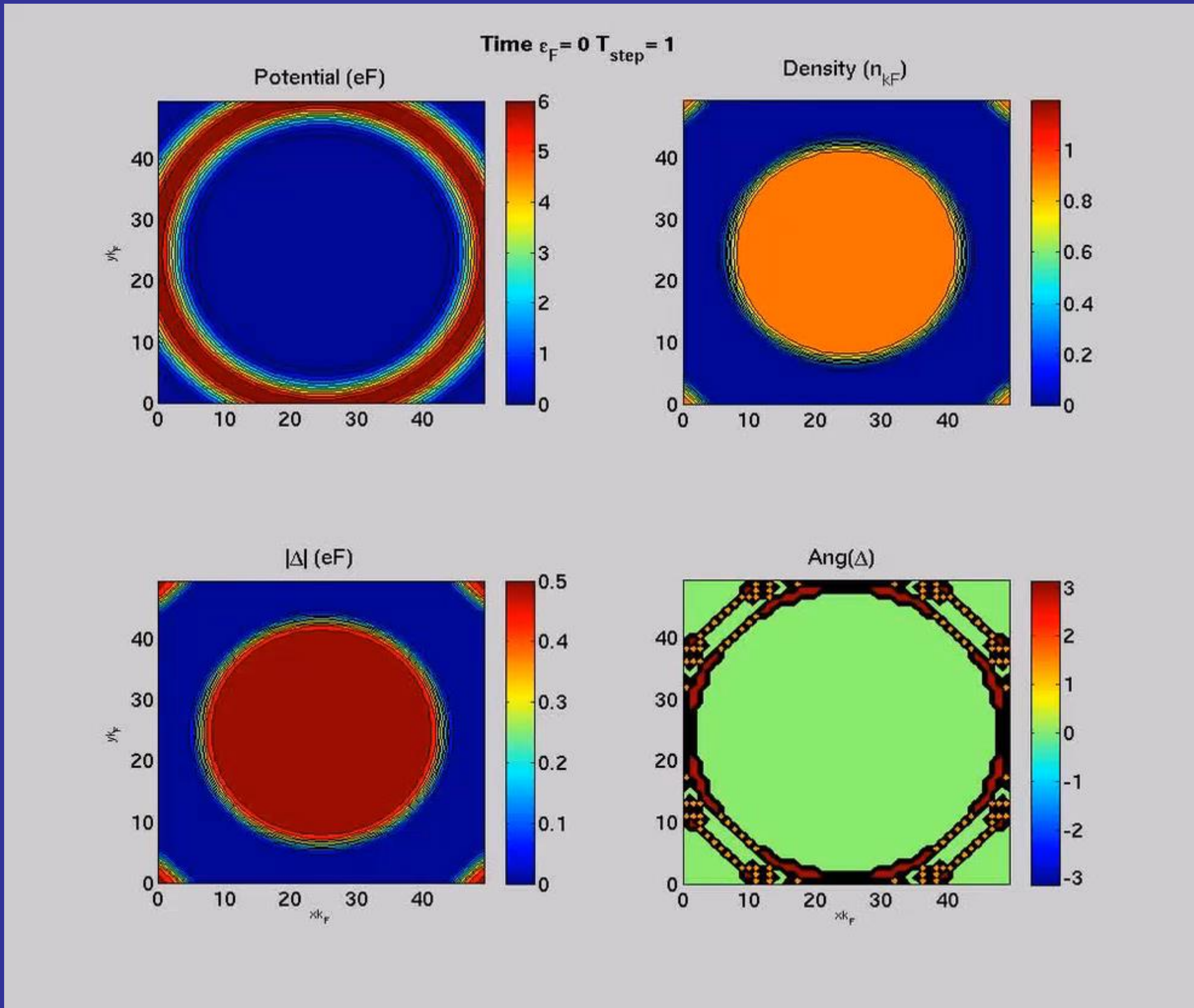


UNITARY REGIME

Figure 2 | Vortices in a strongly ... fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b–h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (s... magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**... 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of vie... 880 μm × 880 μm.

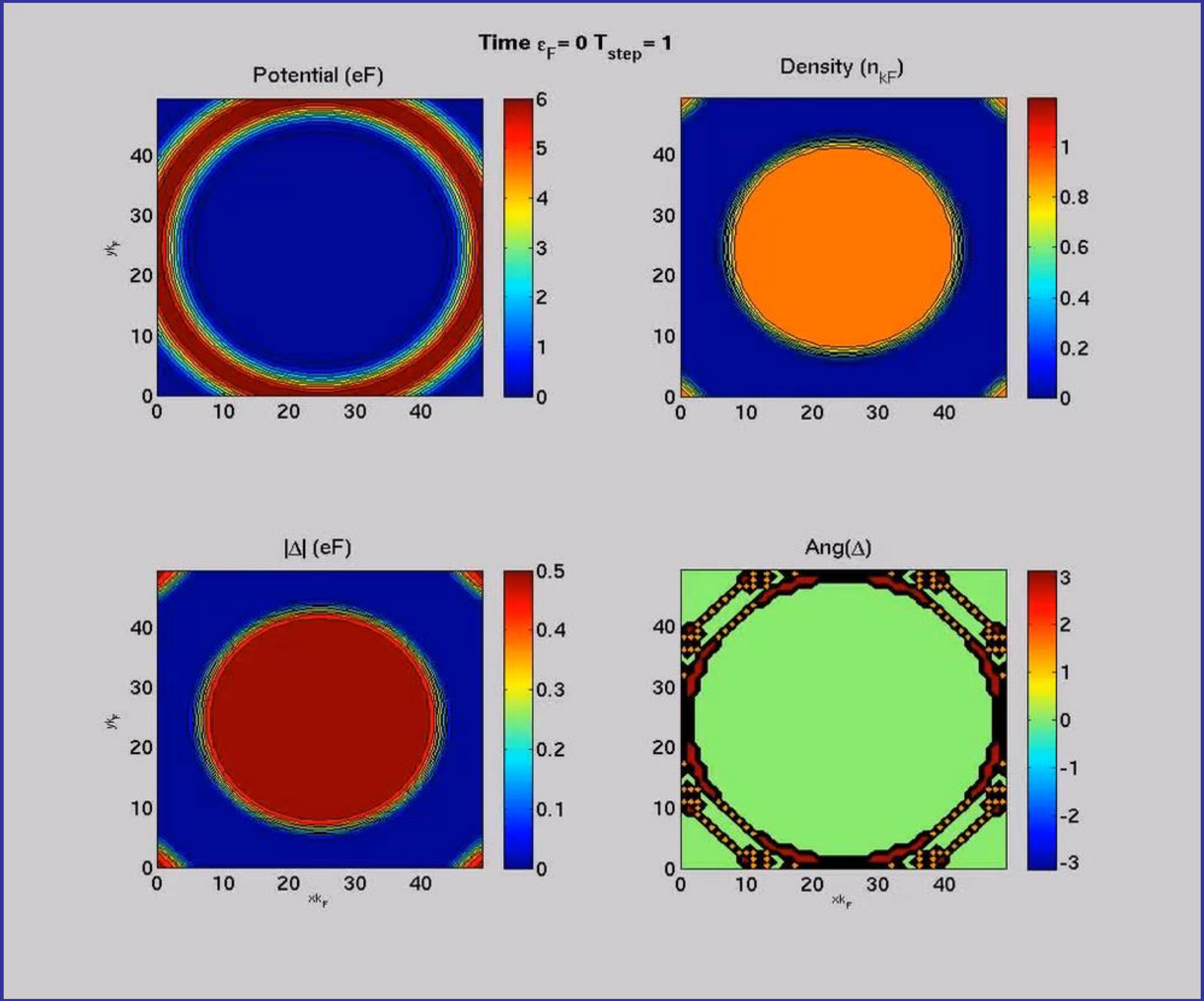
M.W. Zwierlein *et al*.,
Nature, 435, 1047 (2005)

Stirring the atomic cloud with stirring velocity **lower** than the critical velocity



Bulgac, Luo, Magierski, Roche, Yu, *Science* 332, 1288 (2011)

Stirring the atomic cloud with stirring velocity **exceeding** the critical velocity



Bulgac, Luo, Magierski, Roche, Yu, *Science* 332, 1288 (2011)

Vortex reconnections

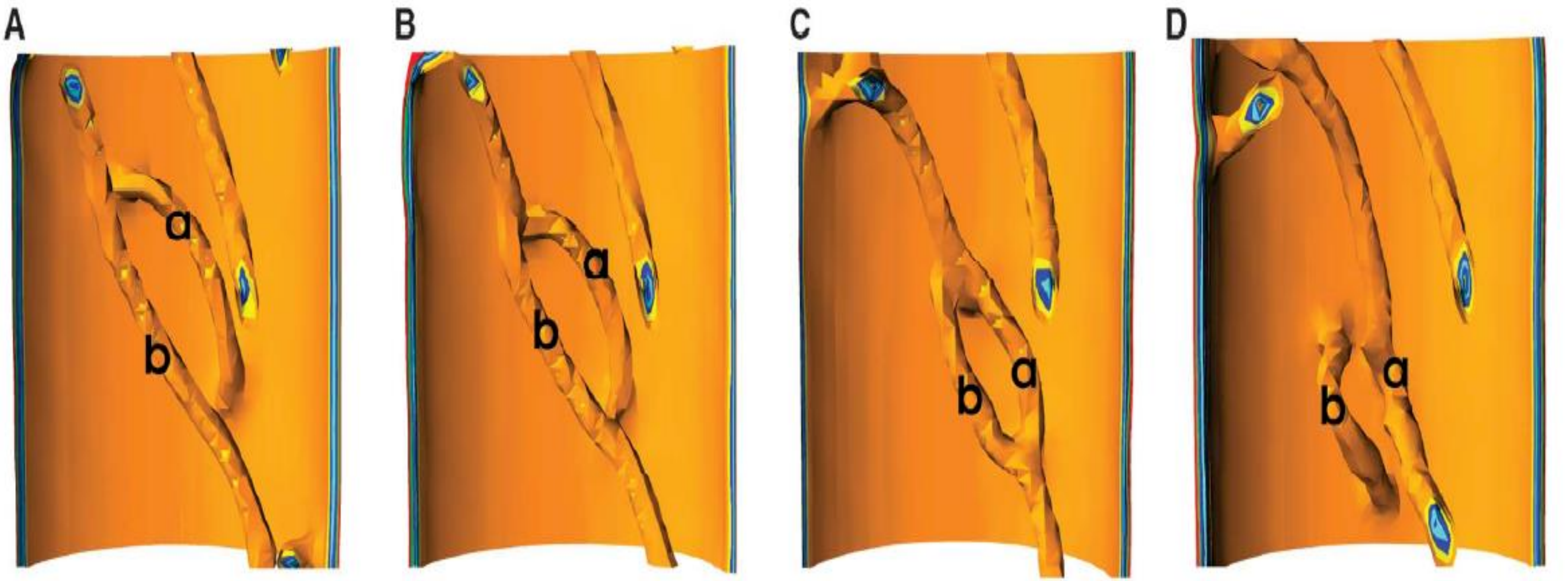


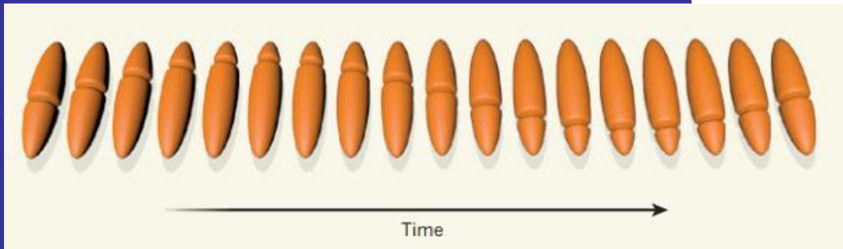
Fig. 3. (**A** to **D**) Two vortex lines approach each other, connect at two points, form a ring and exchange between them a portion of the vortex line, and subsequently separate. Segment (a), which initially belonged to the vortex line attached to the wall, is transferred to the long vortex line (b) after reconnection and vice versa.

Vortex reconnections are important for the energy dissipation mechanism in quantum turbulence.

TDSLDA can describe these processes as well as the energy transfer between collective and single particle degrees of freedom (which is a problem for simplified treatments based e.g. on Gross-Pitaevskii equation)

Bulgac, Luo, Magierski, Roche, Yu, *Science* 332, 1288 (2011)

Soliton dynamics vs ring vortex – a controversy



MIT Experiment:
Nature 499 (2013) 426

Theory prefers ring vortices:
A. Bulgac, M. M. Forbes, M.M. Kelley, K. J.Roche, G. Wlazłowski, Phys. Rev. Lett. 112, 025301 (2014)

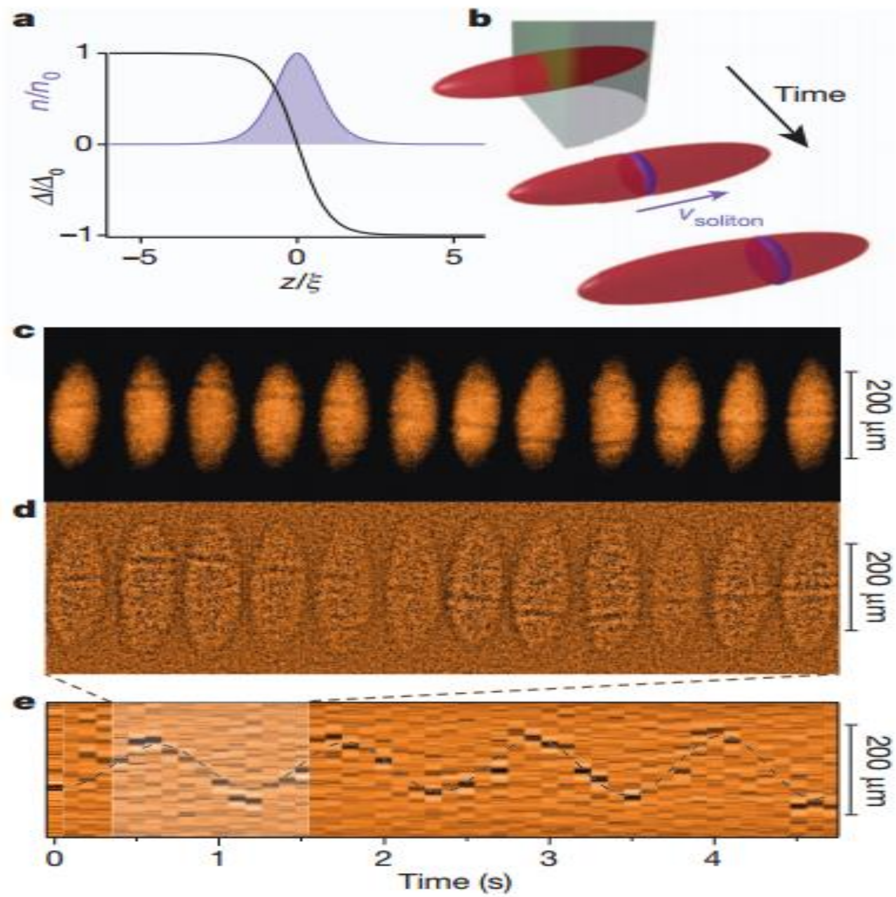
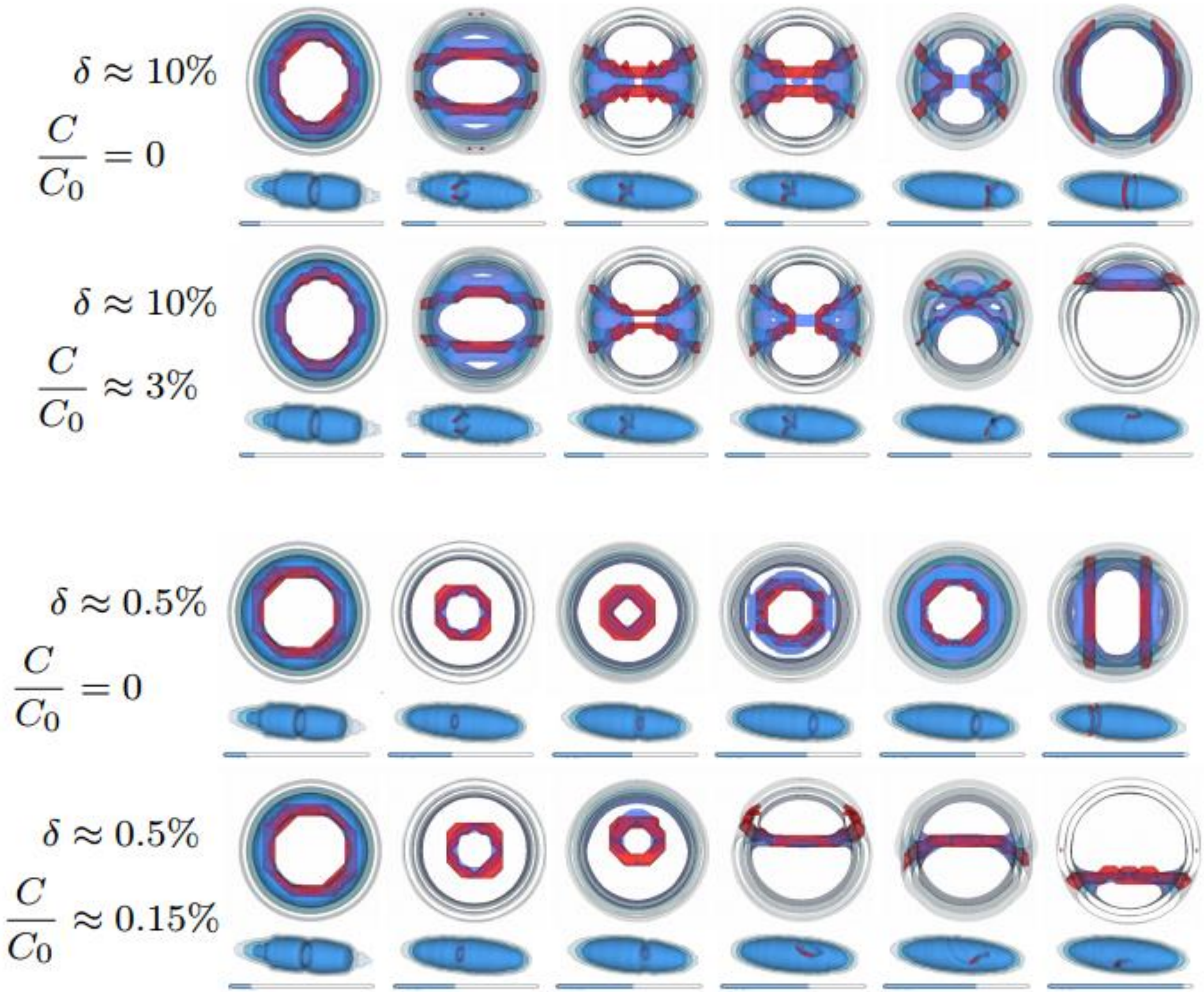


Figure 1 | Creation and observation of solitons in a fermionic superfluid. **a**, Superfluid pairing gap $\Delta(z)$ for a stationary soliton, normalized by the bulk pairing gap Δ_0, and density $n(z)$ of the localized bosonic (fermionic) state versus position z, in the BEC (BCS) regime of the crossover, in units of the BEC healing length (BCS coherence length) ξ. **b**, Diagram of the experiment. A phase-imprinting laser beam twists the phase of one-half of the trapped superfluid by approximately π. The soliton generally moves at non-zero velocity v_{soliton}. **c**, Optical density and **d**, residuals (optical density minus a smoothed copy of the same image) of atom clouds at 815 G, imaged via the rapid ramp method[34], showing solitons at various hold times after creation. One period of soliton oscillation is shown. The in-trap aspect ratio was $\lambda = 6.5(1)$. **e**, Radially integrated residuals as a function of time revealing long-lived soliton oscillations. The soliton period is $T_s = 12(2)T_z$, much longer than the trapping period of $T_z = 93.76(5)$ ms, revealing an extreme enhancement of the soliton's relative effective mass, M^*/M.



Moreover with TDDFT we can reproduce the sequence of topological excitations observed experimentally (M.H.J. Ku et al. Phys. Rev. Lett. 113, 065301 (2014)).

Wlazłowski, et al., Phys. Rev. A91, 031602 (2015)

Time Dependent Density Functional Theory

Runge Gross mapping

$$i\hbar\frac{\partial}{\partial t}\left|\psi(t)\right\rangle = \hat{H}\left|\psi(t)\right\rangle, \quad \left|\psi_0\right\rangle = \left|\psi(t_0)\right\rangle$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\vec{j} = 0$$

$$\left.\begin{matrix}\rho(\vec{r},t)\\ \left|\psi(t_0)\right\rangle\end{matrix}\right\} \leftrightarrow e^{i\alpha(t)}\left|\psi(t)\right\rangle$$

Up to an arbitrary function α(t)

and consequently the functional exists:

$$F[\psi_0,\rho] = \int_{t_0}^{t_1}\left\langle\psi[\rho]\right|\left(i\hbar\frac{\partial}{\partial t} - \hat{H}\right)\left|\psi[\rho]\right\rangle dt$$

E. Runge, E.K.U Gross, PRL 52, 997 (1984)
B.-X. Xu, A.K. Rajagopal, PRA 31, 2682 (1985)
G. Vignale, PRA77, 062511 (2008)

Kohn-Sham approach

Suppose we are given the density of an interacting system.
There exists a unique noninteracting system with the same density.

Interacting system

$$i\hbar \frac{\partial}{\partial t}\left|\psi(t)\right\rangle = (\hat{T}+\hat{V}(t)+\hat{W})\left|\psi(t)\right\rangle$$

Noninteracting system

$$i\hbar \frac{\partial}{\partial t}\left|\varphi(t)\right\rangle = (\hat{T}+\hat{V}_{KS}(t))\left|\varphi(t)\right\rangle$$

$$\rho(\vec{r},t) = \left\langle \psi(t)\right|\hat{\rho}(\vec{r})\left|\psi(t)\right\rangle = \left\langle \varphi(t)\right|\hat{\rho}(\vec{r})\left|\varphi(t)\right\rangle$$

Hence the DFT approach is essentially exact.

However as always there is a price to pay:

- Kohn-Sham potential in principle depends on the past (memory).
  Very little is known about the memory term and usually it is disregarded.
- Only one body observables can be reliably evaluated within standard DFT.

Formalism for Time Dependent Phenomena: TDSLDA

Local density approximation

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k\uparrow}(\mathbf{r},t) \\ u_{k\downarrow}(\mathbf{r},t) \\ v_{k\uparrow}(\mathbf{r},t) \\ v_{k\downarrow}(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow,\uparrow}(\mathbf{r},t) & h_{\uparrow,\downarrow}(\mathbf{r},t) & 0 & \Delta(\mathbf{r},t) \\ h_{\downarrow,\uparrow}(\mathbf{r},t) & h_{\downarrow,\downarrow}(\mathbf{r},t) & -\Delta(\mathbf{r},t) & 0 \\ 0 & -\Delta^*(\mathbf{r},t) & -h^*_{\uparrow,\uparrow}(\mathbf{r},t) & -h^*_{\uparrow,\downarrow}(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & 0 & -h^*_{\uparrow,\downarrow}(\mathbf{r},t) & -h^*_{\downarrow,\downarrow}(\mathbf{r},t) \end{pmatrix}\begin{pmatrix} u_{k\uparrow}(\mathbf{r},t) \\ u_{k\downarrow}(\mathbf{r},t) \\ v_{k\uparrow}(\mathbf{r},t) \\ v_{k\downarrow}(\mathbf{r},t) \end{pmatrix}$$

Density functional contains normal densities, anomalous density (pairing) and currents:

$$E(t) = \int d^3r \left[\varepsilon(n(\vec{r},t), \tau(\vec{r},t), \nu(\vec{r},t), \underline{\vec{j}(\vec{r},t)}) + V_{ext}(\vec{r},t)n(\vec{r},t) + \ldots \right]$$

- **The system is placed on a large 3D spatial lattice.**
- **No symmetry restrictions**
- **Number of PDEs is of the order of the number of spatial lattice points**

Current capabilities of the code:

- **volumes of the order of (L = 80^3) capable of simulating time evolution of 42000 neutrons at saturation density (natural application: neutron stars)**
- **For nuclear systems: capable of simulating up to times of the order of 10^{-19} s (a few million time steps)**
- **CPU vs GPU on Titan ≈ 15 speed-up (likely an additional factor of 4 possible)**

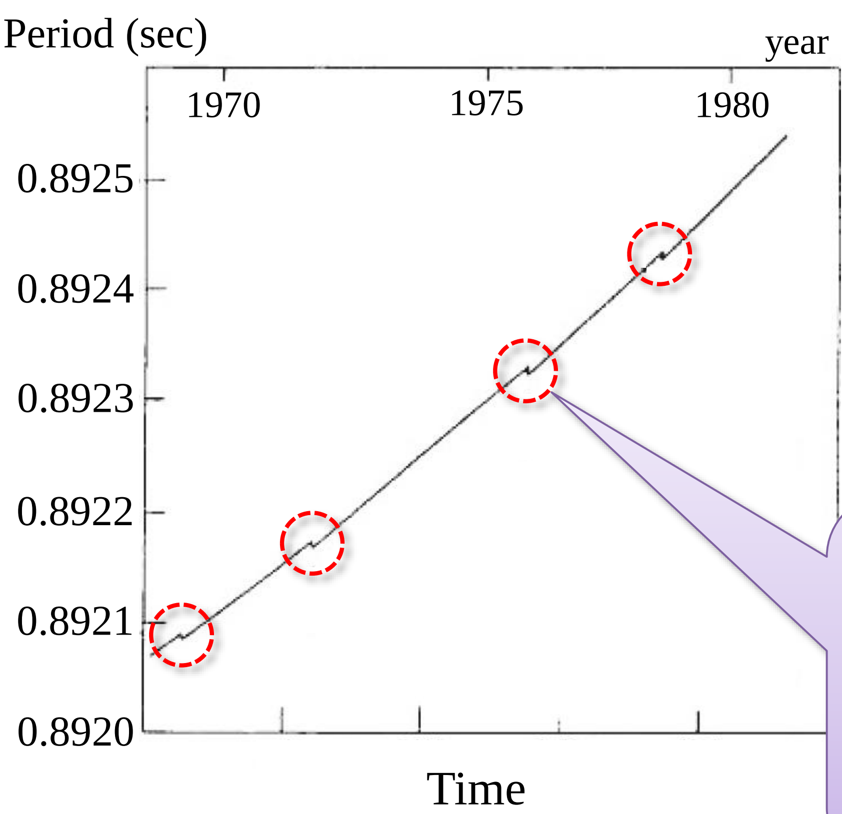
Eg. for 137062 two component wave functions:

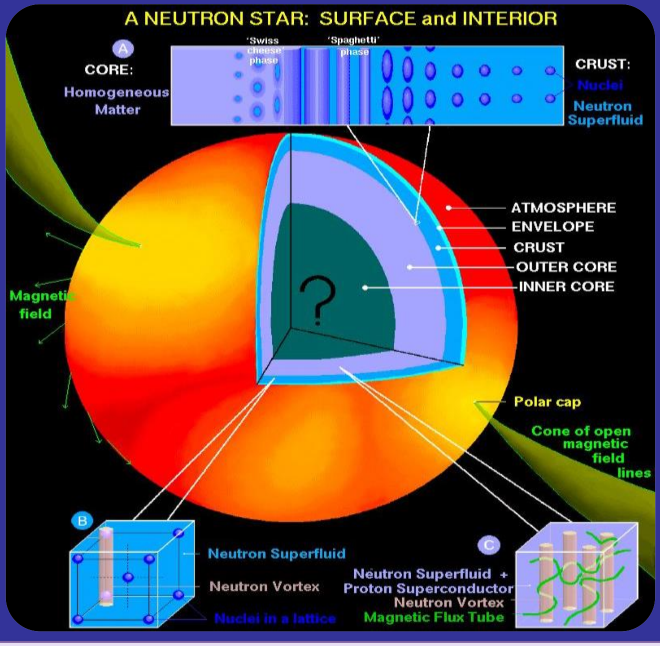
CPU version (4096 nodes x 16 PEs) - 27.90 sec for 10 time steps

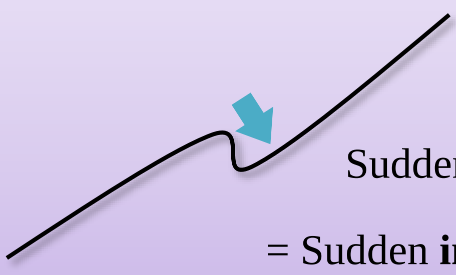
GPU version (4096 PEs + 4096GPU) - 1.84 sec for 10 time step

Glitch: a sudden increase of the rotational frequency

☐ Glitches in the Vela pulsar







Sudden **decrease** of the period

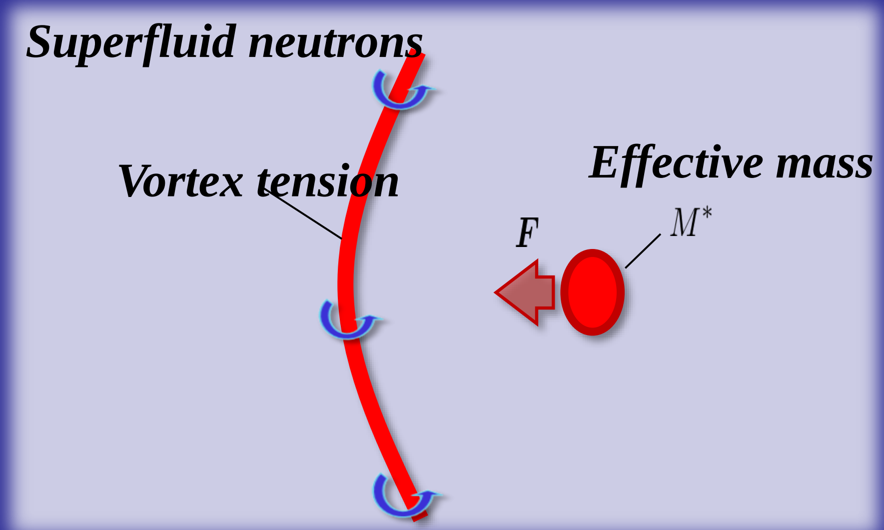
= Sudden **increase** of the frequency

V.B. Bhatia, A Textbook of Astronomy and Astrophysics with Elements of Cosmology, Alpha Science, 2001.

Vortex dynamics and vortex-impurity interaction

The effective equations of motion for the vortex dynamics
(per unit length of the vortex):

$$M_{vor}\frac{d^2\vec{r}}{dt^2} = \vec{F}_M + \vec{F}_D + \vec{F}_{vor-impurity}$$



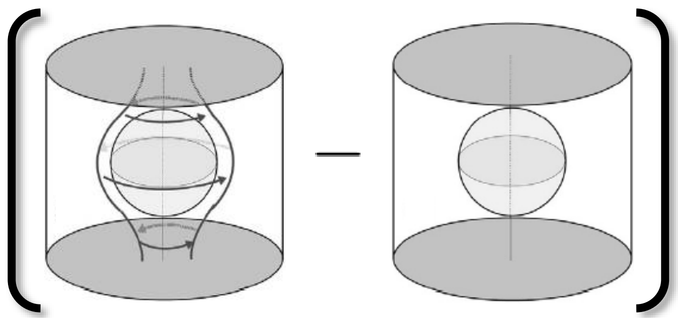
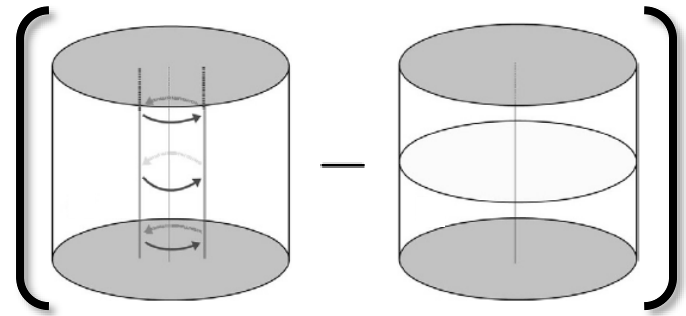
$\vec{F}_M = \rho_s \vec{\Gamma} \times \left(\frac{d\vec{r}}{dt} - \vec{v}_s\right)$ - Magnus force; $\vec{\Gamma}$ – local vorticity;

$\frac{d\vec{r}}{dt}$ - local vortex velocity, ρ_s – superfluid density, $\vec{v}_s$ – superfluid velocity

$\vec{F}_D$ – frictional force (negligible at small T)

$\vec{F}_{vor-impurity}$ - vortex-impurity force

Microscopic, static HFB calculations were performed assuming axial symmetry

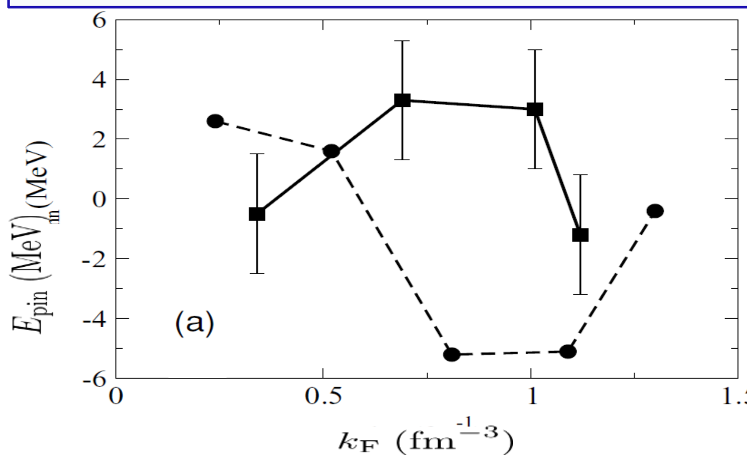
$E_{\text{pin}} = E$  $- E$ 

Energy to create a vortex line on a nuclear impurity

Energy to create a vortex line in a uniform matter

E.g.) 0.026 fm^{-3} (SLy4)

6.19 MeV	13058.04	12954.02	13714.88	13617.05



HFB (Avogadro et al.)
TF+LDA (Donati & Pizzochero)

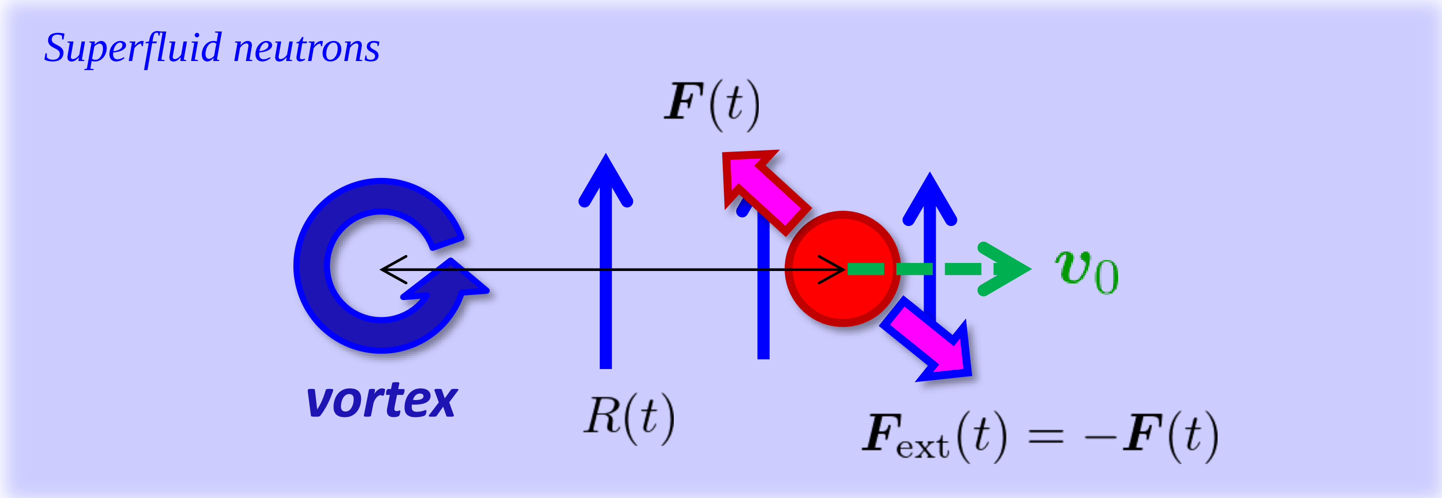
P. Avogadro, F. Barranco, R.A. Broglia, and E. Vigezzi, PRC**75**(2007)012805(R); NPA**788**(2007)130; NPA**811**(2008)378

We directly measure the force $\boldsymbol{F}(R)$ in dynamical simulation

- Newton's law

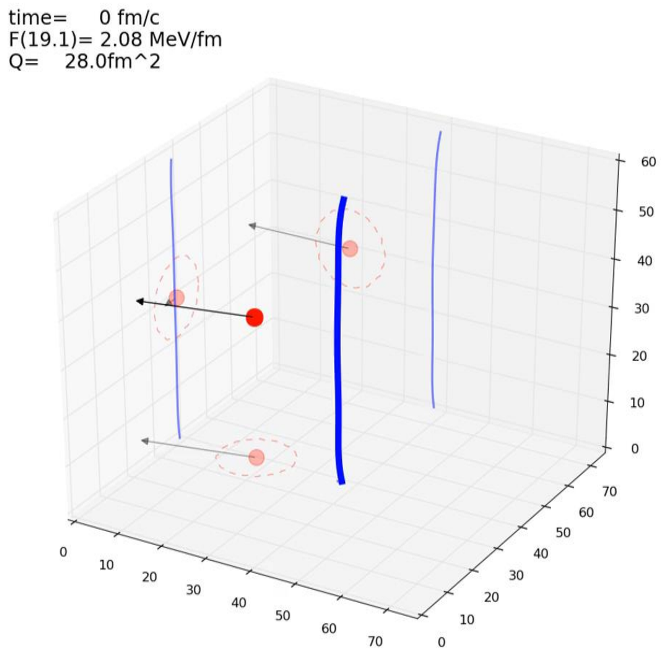
$$\boldsymbol{F} = M\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \quad \Rightarrow \quad \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = 0 \quad \text{if} \quad \boldsymbol{F} = 0$$

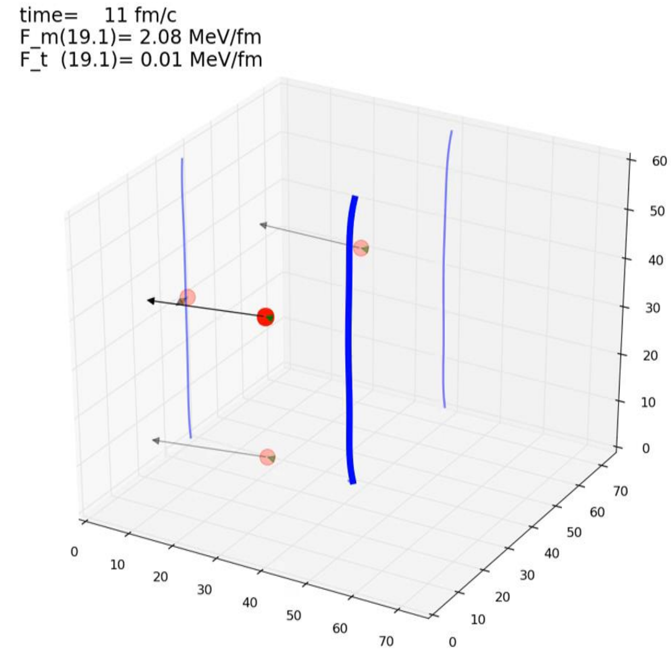
- We keep a nuclear motion in a constant velocity v_0 $(\ll v_{\mathrm{crit}})$



Vortex – impurity interaction

The extrenal potential keeps the nucleus moving along the straight line with a constant velocity below the critical velocity.





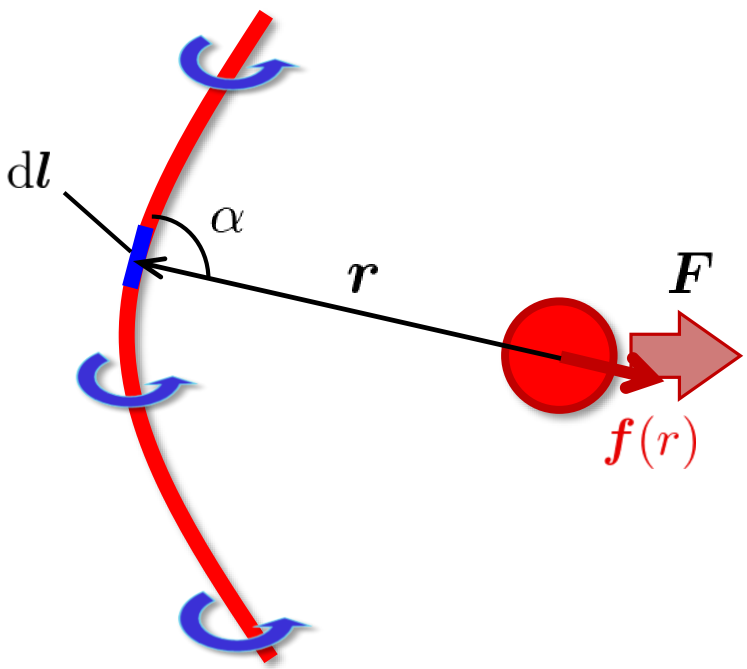
G. Wlazłowski, K. Sekizawa, P. Magierski, A. Bulgac, M.M. Forbes, to appear in Phys. Rev. Lett.

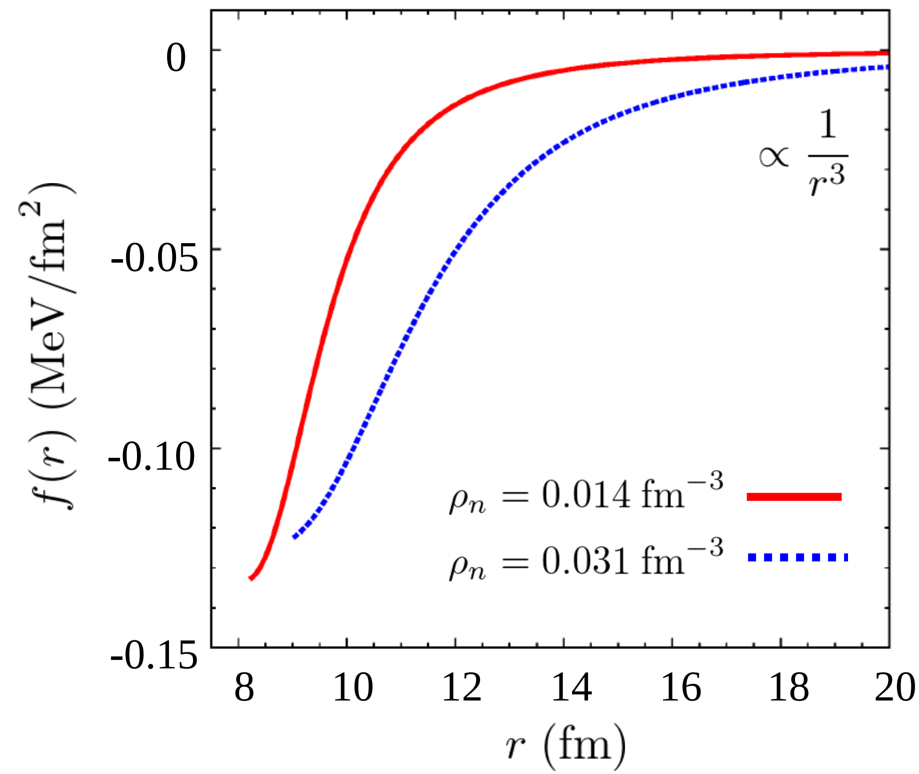
We can predict the force for any vortex-nucleus configuration

- Force per unit length

$$\boldsymbol{F} = \int_L f(r)\sin\alpha\, \boldsymbol{e}_r\, \mathrm{d}l \qquad f(r) = \frac{\sum_{k=0}^{n} a_k r^k}{1+\sum_{k=1}^{n+3} b_k r^k}$$

Padé approximant
(n=2 was used)

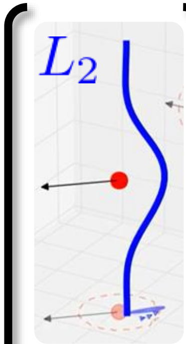




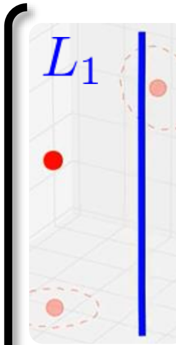
Vortex tension

We can evaluate the vortex tension from the dynamical simulations

$$T \lesssim \frac{E^*}{\Delta L} = \Bigg(E\Bigg[$$



$$\Bigg] - E\Bigg[$$



$$\Bigg]\Bigg) \Big/ \Delta L$$

Work done by $\boldsymbol{F}_{\mathrm{ext}}$

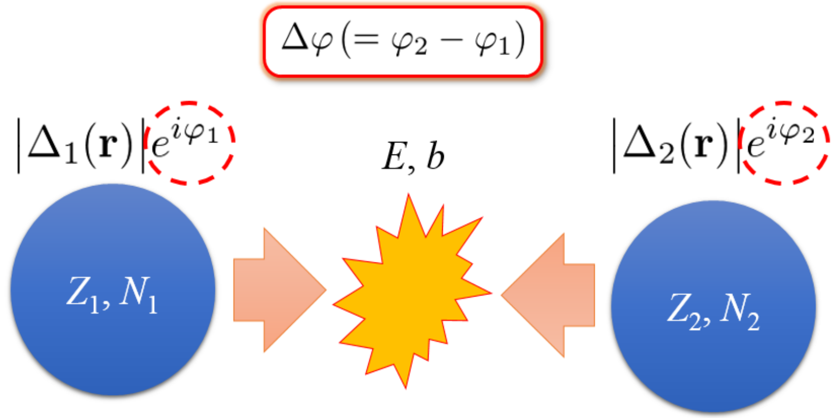
$$\int_{t_0}^{t_1} \boldsymbol{F}(t) \cdot \boldsymbol{v}(t) dt$$

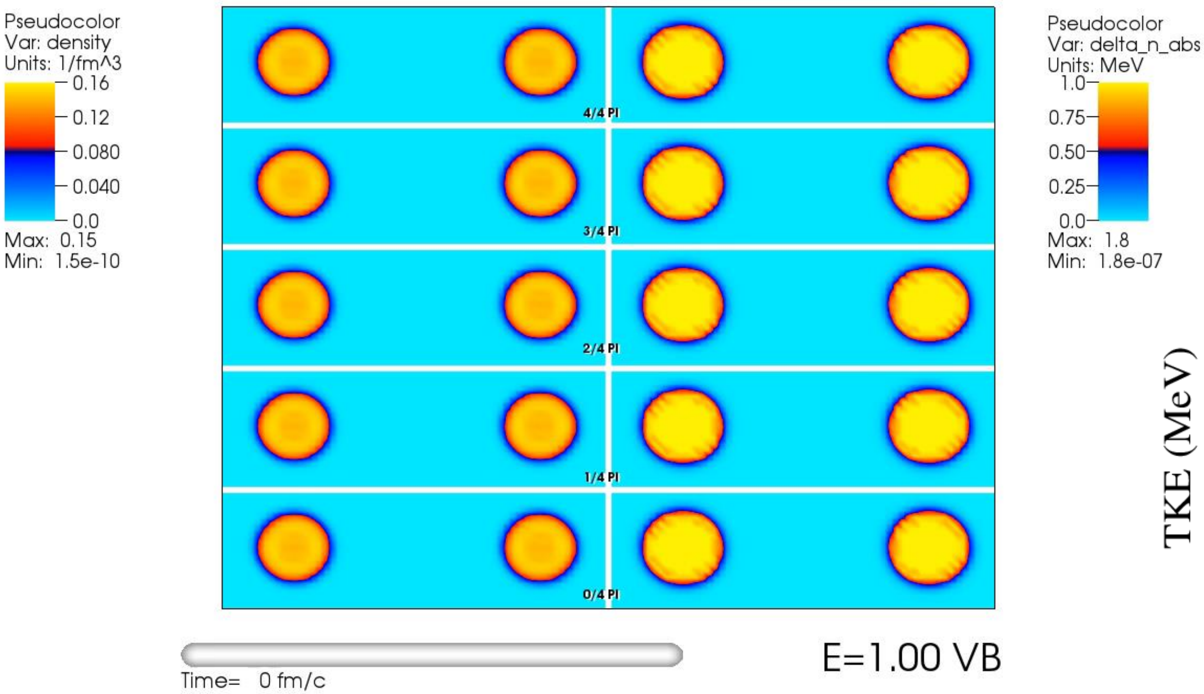
$$T \lesssim \frac{5}{3.5} = 1.4\,\mathrm{MeV/fm} \qquad n = 0.014\ \mathrm{fm}^{-3}$$

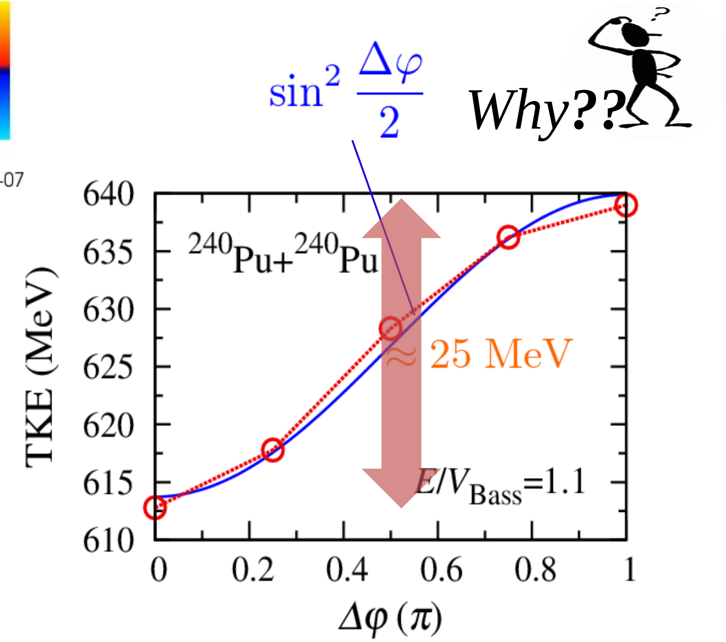
$$T \lesssim \frac{11}{1.5} = 7.3\,\mathrm{MeV/fm} \qquad n = 0.031\ \mathrm{fm}^{-3}$$

$$\frac{1.4}{7.3} = 0.19$$ cf. hydrodynamic approx.: 0.77

Solitonic excitations in nuclear reactions



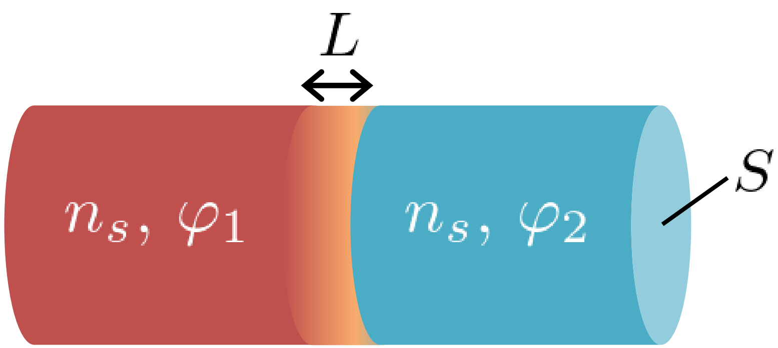




Additional energy is required to attach two superfluids with different phases

➢ The additional energy (derived from Ginzburg-Landau theory)

$$E = \frac{S}{L}\frac{\hbar^2}{2m} n_s \sin^2\frac{\Delta\varphi}{2}$$



$$\Delta\varphi\ (\equiv \varphi_1 - \varphi_2)$$

*It does not depend on the absolute value of the pairing!

e.g.) $S=\pi R^2$, $L\sim R=6$ fm, $n_s=0.08$ fm^{-3} → $E\sim30$ MeV

S: Attaching area

L: Length scale over which the phase varies

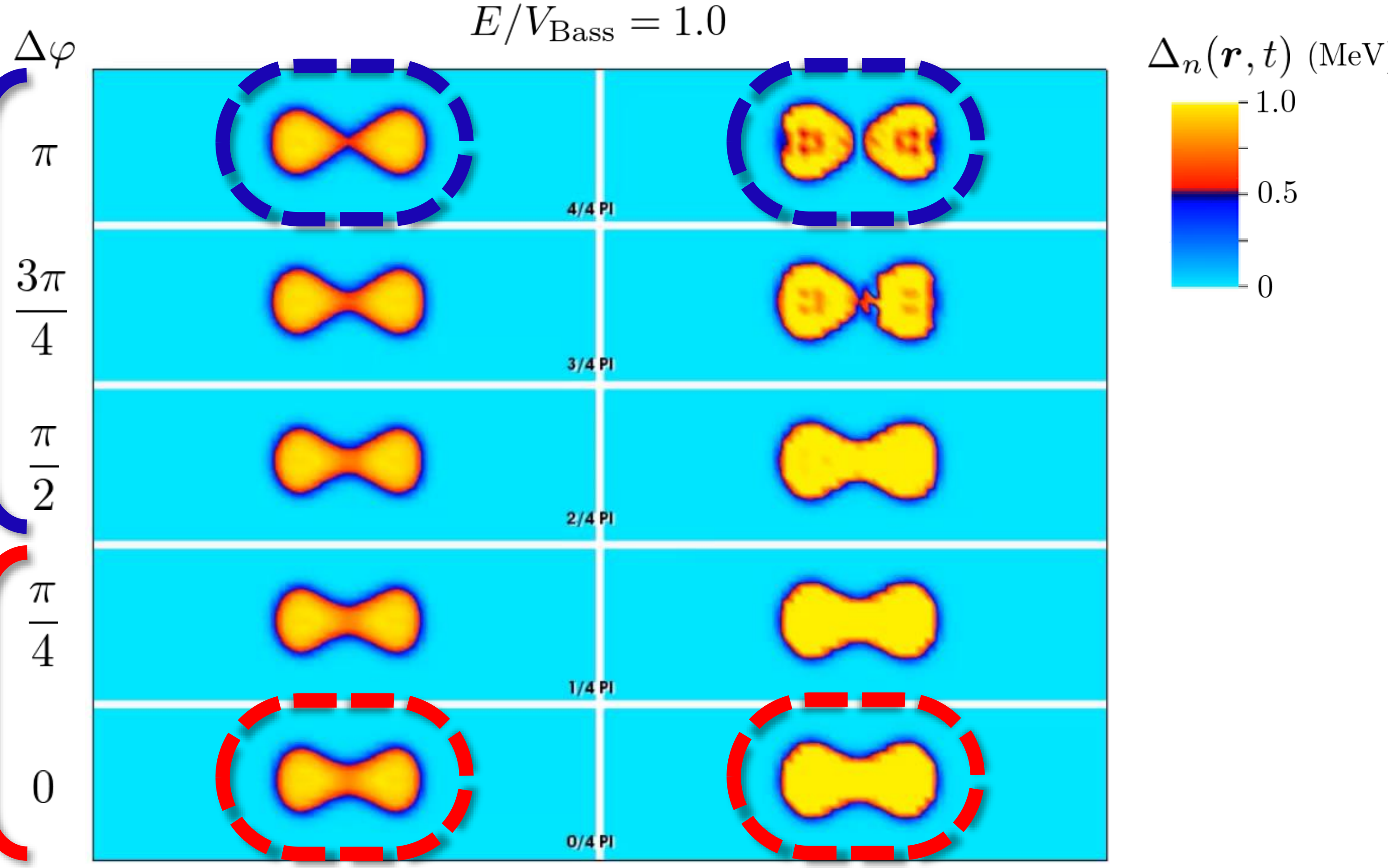
n_s: Superfluid density

K.S., G. Wlazłowski, P. Magierski, in preparation (will be submitted to PRL)

Fusion reaction is suppressed by the phase difference

Non-Fusion

Fusion



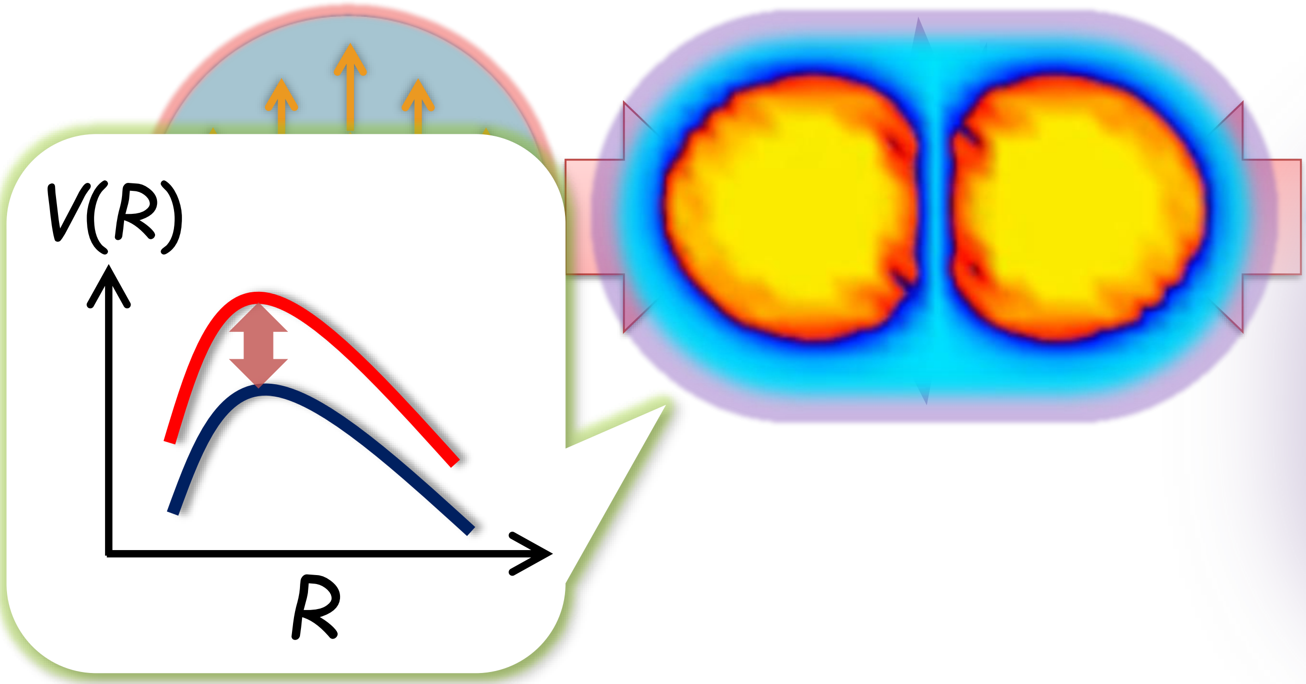
K.S., G. Wlazłowski, P. Magierski, in preparation (will be submitted to PRL)

When two superfluid nuclei with different phases collide solitonic excitations could be induced

$$\Delta\varphi\ (\equiv \varphi_1 - \varphi_2)$$

$|\Delta_1(\boldsymbol{r},t)|e^{\mathrm{i}\varphi_1(\boldsymbol{r},t)}$

$|\Delta_2(\boldsymbol{r},t)|e^{\mathrm{i}\varphi_2(\boldsymbol{r},t)}$



It may affect:

- ☐ TKE (~10-30 MeV)
- ☐ Fusion dynamics
- ☐ Neck formation
- ☐ Contact time
- ☐ Scattering angle

Instead of summary...

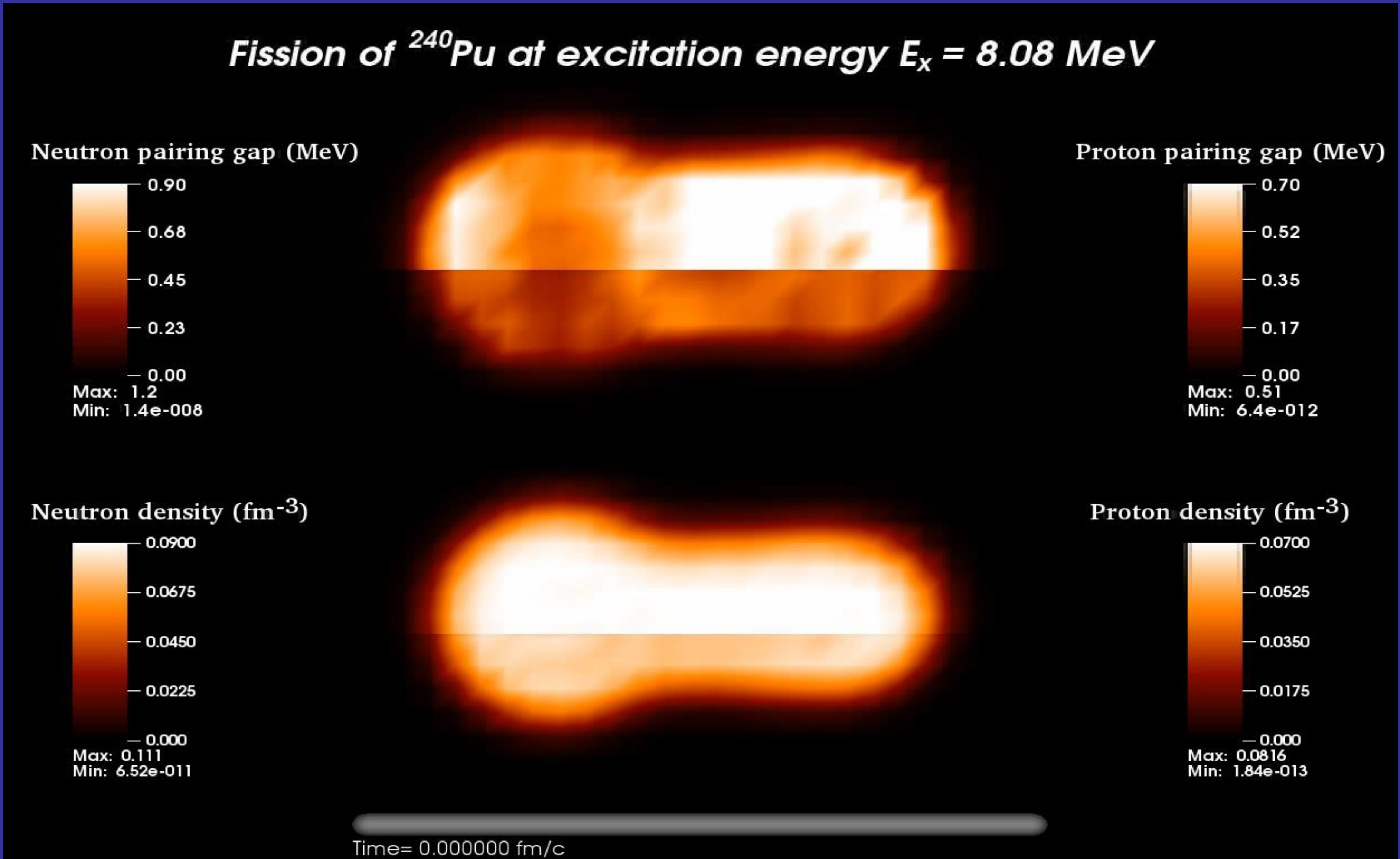
GOAL:

Description of superfluid dynamics far from equilibrium within the framework of Time Dependent Density Functional Theory (TDDFT).

We would like to describe the time evolution of (externally perturbed) spatially inhomogeneous, superfluid Fermi system and in particular such phenomena as:

- **Vortex dynamics in ultracold Fermi gases and neutron matter.**
- **Vortex impurity interaction, vortex reconnections.**
- **Quantum turbulence.**
- **Atomic cloud collisions.**
- **Nuclear dynamics: large amplitude collective motion, induced nuclear fission, reactions, fusion, excitation of nuclei with gamma rays and neutrons.**

Induced nuclear fission by neutron capture: pairing dynamics



Bulgac,Magierski,Roche,Stetcu, Phys. Rev. Lett. 116, 122504 (2016)

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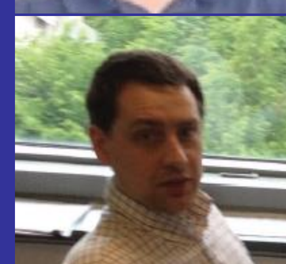

Gabriel Wlazłowski
(WUT)