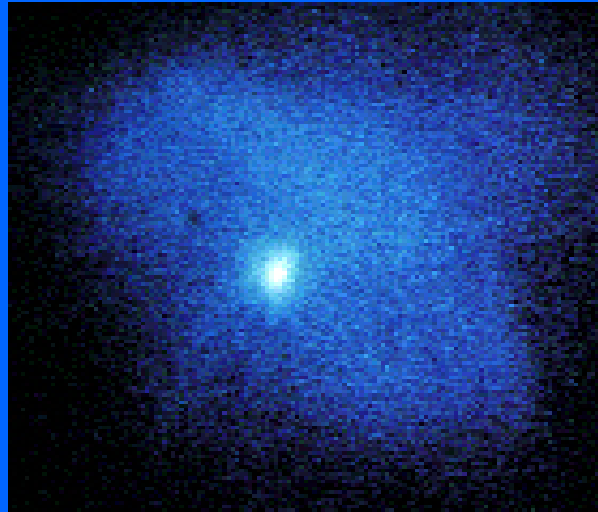


Nuclear structure and dynamics in the neutron star crust



Piotr Magierski (Warsaw)

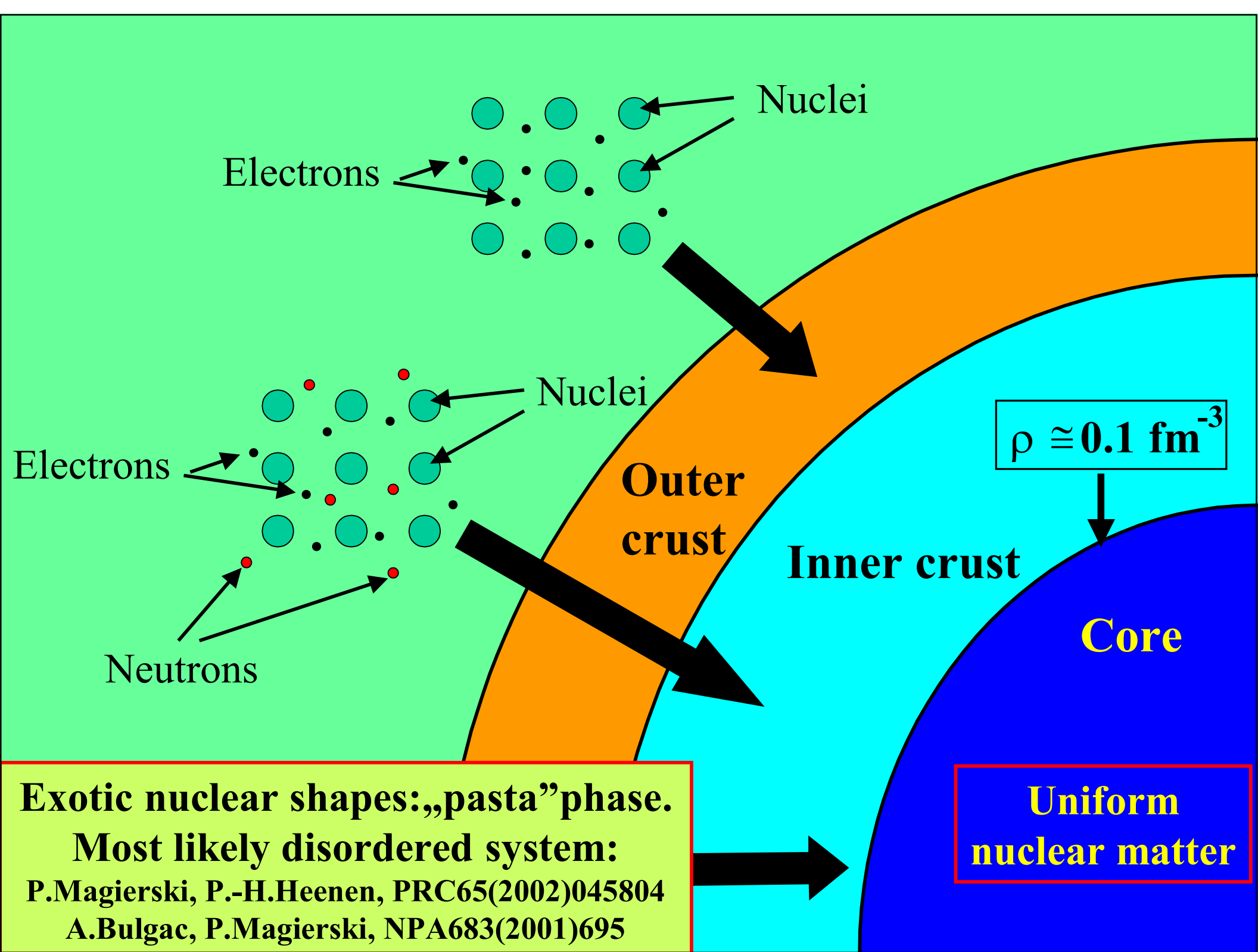
**Collaborators: Aurel Bulgac (Seattle), Paul-Henri Heenen (Brussels),
Andreas Wirzba (Bonn)**

ENAM'04

The Fourth International Conference
on Exotic Nuclei and Atomic Masses

Content:

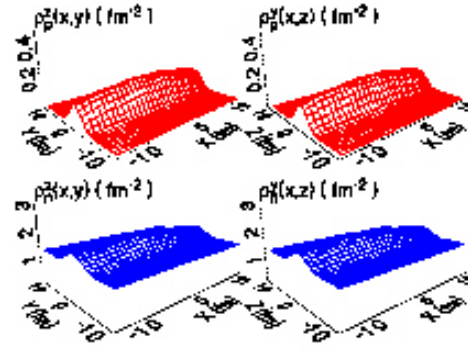
- Collective excitations of nuclei immersed in a superfluid neutron environment. Specific heat of the inner crust.
- Spherical symmetry breaking of nuclei in the inner crust.
- Nuclear clustering in the bottom of the inner crust: selfconsistent description of exotic 'pasta' phases.
- Fermionic Casimir effect.
- Neutron localization induced by the pairing field in the inner crust.



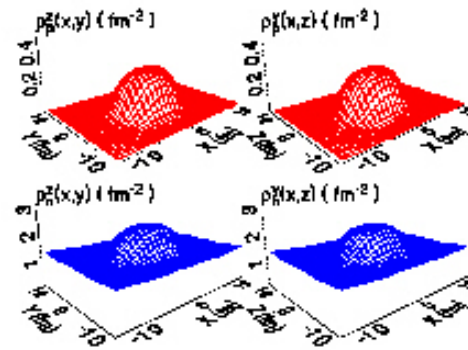
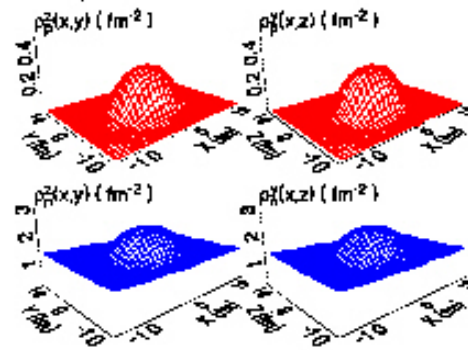
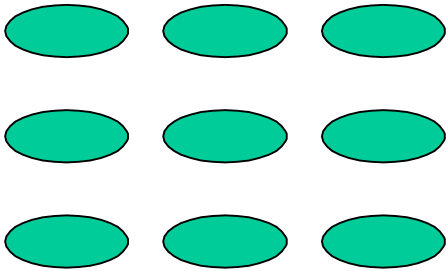
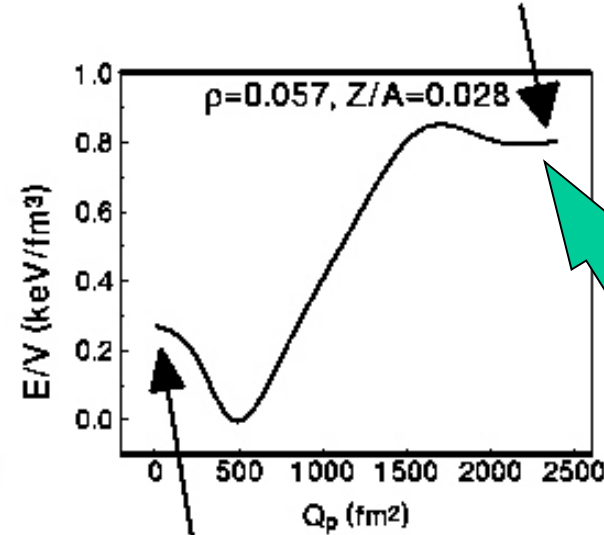
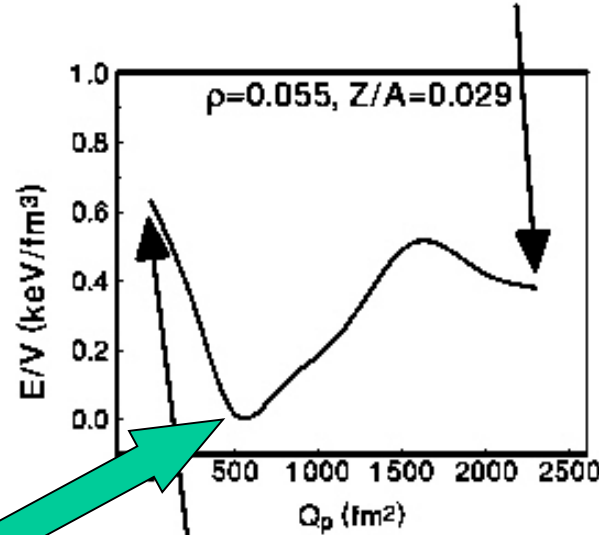
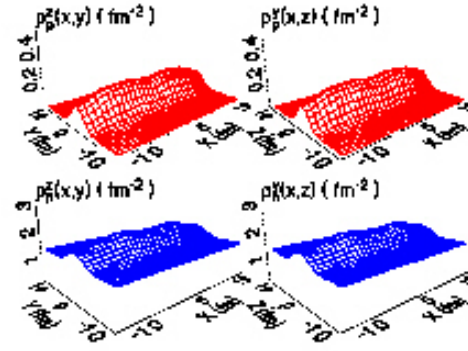
Exotic nuclear shapes: „pasta” phase.
Most likely disordered system:
 P.Magierski, P.-H.Heenen, PRC65(2002)045804
 A.Bulgac, P.Magierski, NPA683(2001)695

Uniform nuclear matter

Proton density distribution



Neutron density distribution



'Spaghetti' phase

**Hamiltonian of a nucleus immersed
in a neutron superfluid ($E_{exc} < \Delta$):**

$$H = \sum_{l,m} \left(\frac{|\hat{\pi}_{lm}|^2}{M_l} + C_l |\hat{\alpha}_{lm}|^2 \right)$$

$$M_l = m \rho_{in} \frac{(\gamma - 1)^2}{\gamma(l+1) + l} R_N^5; \quad \gamma = \frac{\rho_{out}}{\rho_{in}}$$

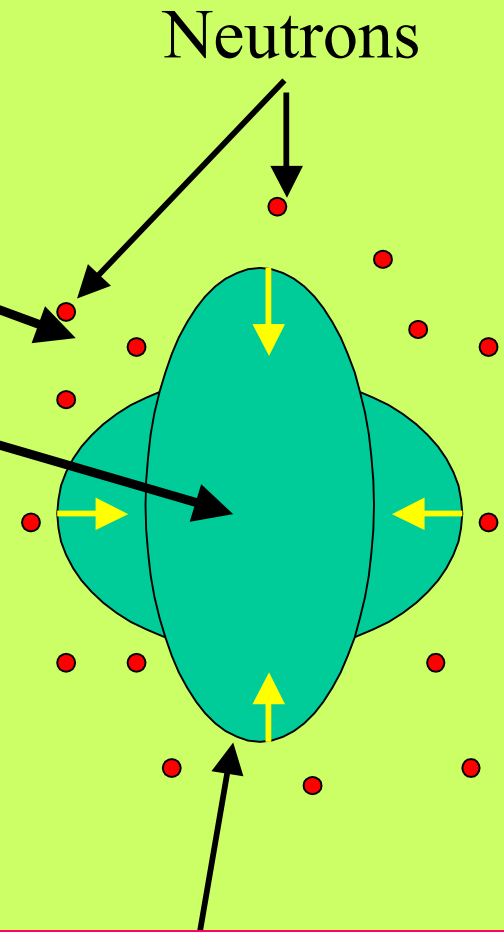
$$C_l = C_l^{surf} + C_l^{coul}$$

**Spreading width of a quadrupole
vibrational multiplet (l=2):**

$$\Gamma_{tot} \simeq 0.169 \frac{(Ze)^2}{R_C} \left(\frac{R_p}{R_C} \right)^2 \left(\frac{3\hbar\omega_2}{2C_2} \right)^{1/2}$$

R_p – proton radius

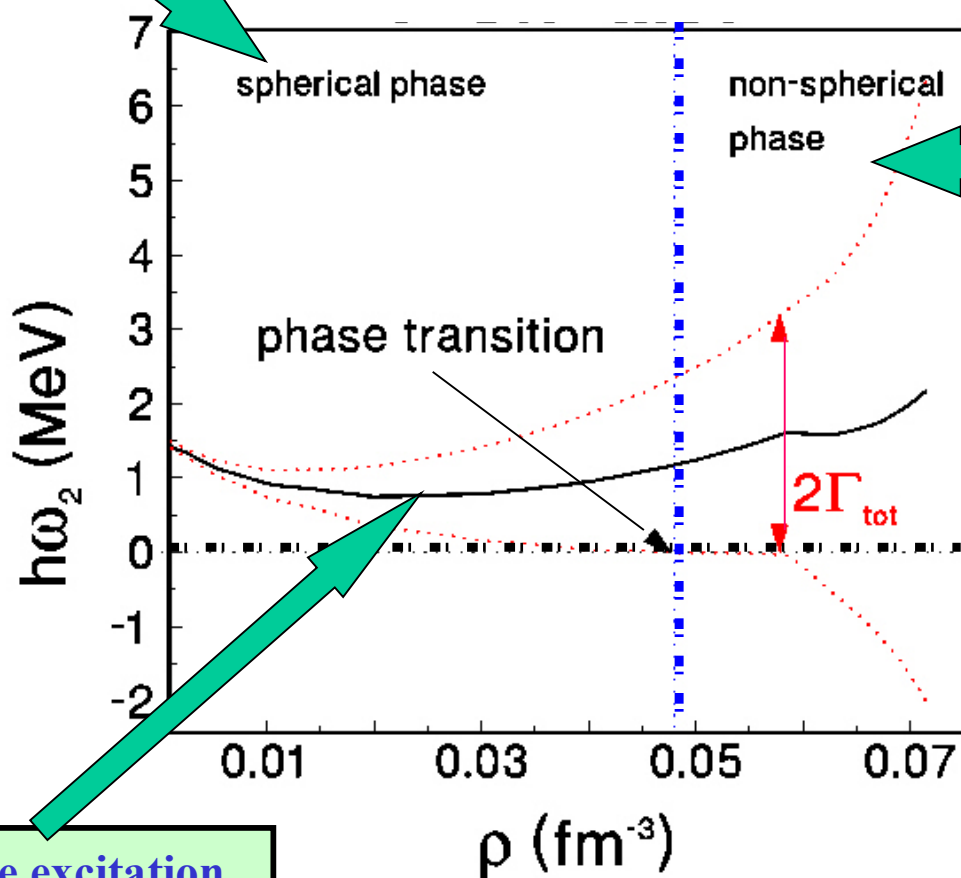
R_C – Wigner–Seitz cell radius



**Energy depends on the
orientation with respect
to the lattice vectors**

**Spherical symmetry breaking
due to the coupling between lattice and
nuclear vibrations**

spherical nuclei



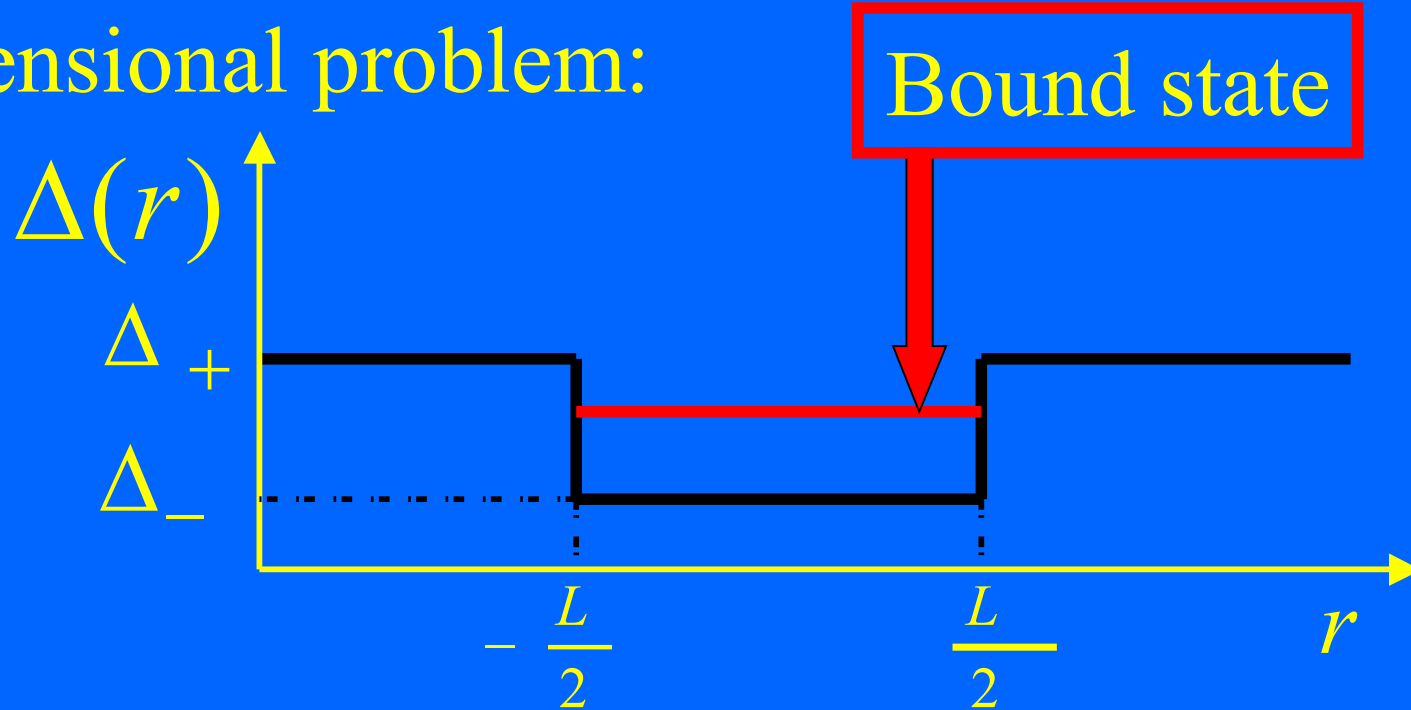
deformed nuclei

Nuclear quadrupole excitation
energy in the inner crust

$$\begin{pmatrix} h - \mu & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & -h + \mu \end{pmatrix} \begin{pmatrix} u(\vec{r}) \\ v(\vec{r}) \end{pmatrix} = E \begin{pmatrix} u(\vec{r}) \\ v(\vec{r}) \end{pmatrix} \quad \text{BdG eqs.}$$

$$h = -\frac{\hbar^2}{2m} \nabla^2; \quad \Delta(\vec{r} + \vec{a}) = \Delta(\vec{r})$$

1-dimensional problem:



Andreev approximation: $\begin{pmatrix} u(r) \\ v(r) \end{pmatrix} = \begin{pmatrix} \bar{u}(r) \\ \bar{v}(r) \end{pmatrix} e^{ik_F r}$

Quantization condition: $A(\varphi, \psi) e^{2iqL} = 1$

$$A(\varphi, \psi) = \frac{(e^{-\varphi} - e^{-i\psi})(e^{\varphi} - e^{i\psi})}{(e^{-\varphi} - e^{i\psi})(e^{\varphi} - e^{-i\psi})}; \quad \cos\psi = \frac{E}{\Delta_+}$$
$$\cosh\varphi = \frac{E}{\Delta_-}$$

$$q = \frac{m}{\hbar^2 k_F} \sqrt{E^2 - \Delta_-^2}$$

There is always at least one bound state!

Penetration length inside a barrier Δ_+

$$\xi = \hbar^2 k_F / (m \sqrt{\Delta_+^2 - E^2})$$

Localization condition: $\xi < R_C - R_N$

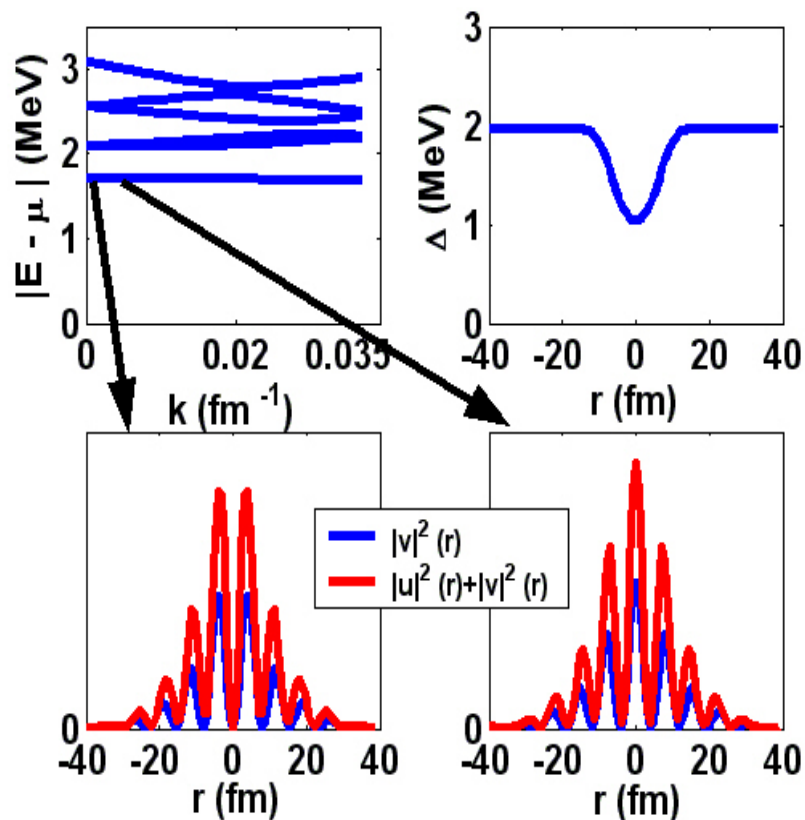
R_C – Wigner-Seitz cell radius

R_N – Nuclear radius

Localization condition: $F(\rho) > 1$

where:

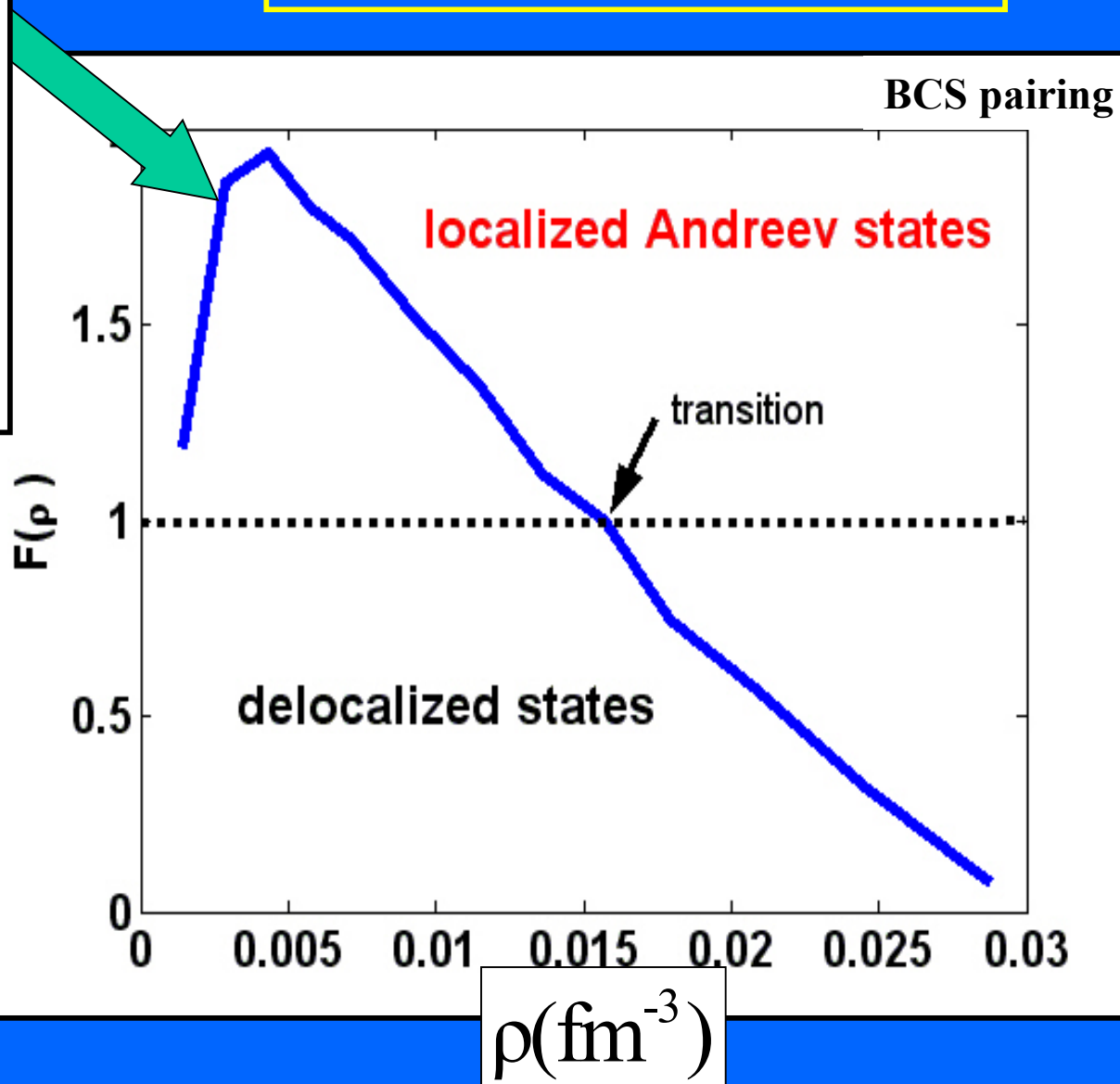
$$F(\rho) = \frac{1}{2} k_F R_N \sqrt{\left(\frac{\Delta_+}{\mu}\right)^2 - \left(\frac{E}{\mu}\right)^2} \left(\frac{R_C}{R_N} - 1\right)$$



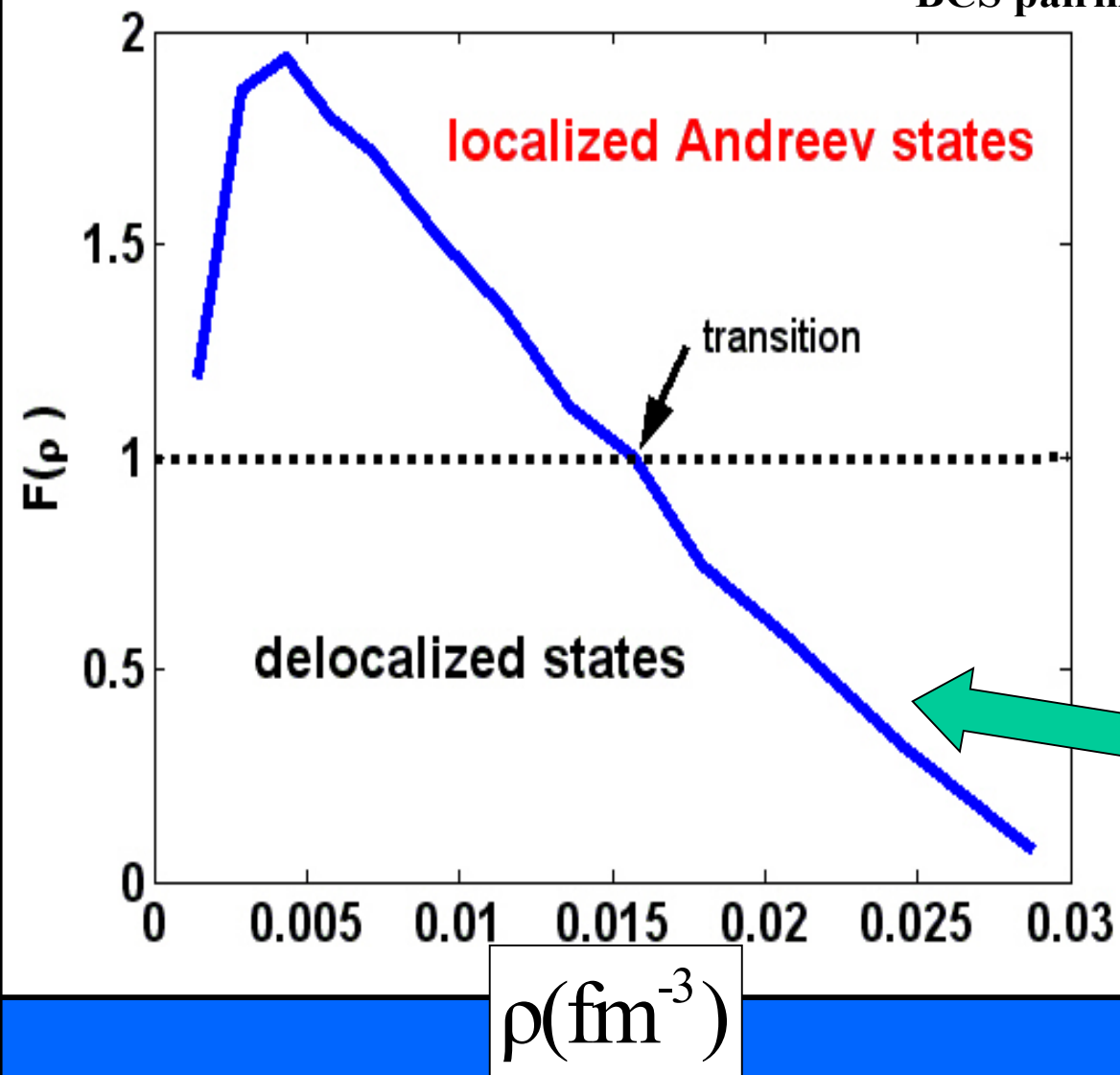
$$\Delta(r+a) = \Delta(r)$$

$$a = 80 \text{ fm}$$

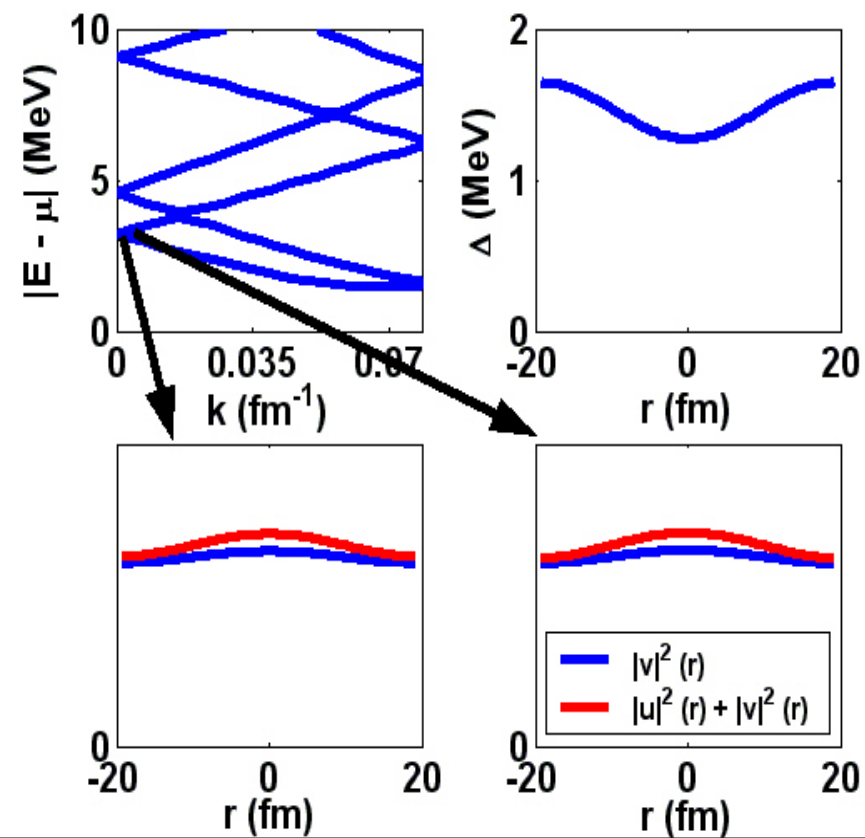
$$F(\rho) > 1$$



BCS pairing



$$\Delta(r+a) = \Delta(r)$$
$$a = 40 \text{ fm}$$



$$F(\rho) < 1$$

Conclusions

- Due to the coupling between the nuclear surface vibrations and the ion lattice part of the crust is filled with non-spherical nuclei. The phase transition takes place at densities far lower than the predicted density for the transition to the exotic „pasta phases”.
- There is a substantial renormalization effect of a nuclear/ion mass in the inner crust of a neutron star, due to the presence of a superfluid neutron liquid.
- The contribution to the specific heat associated with nuclear shape vibrations seems to be important at densities around 0.02 fm^{-3} where the pairing correlations are predicted to reach their maximum.
- At low densities in the inner crust neutrons around the Fermi level are localized due to the inhomogeneity of the pairing field.

Summarizing, due to these effects the transport properties (thermal and electric conductivities) across the crust are expected to be modified.