Quantum vortices in fermionic superfluids: from ultracold atoms to neutron stars

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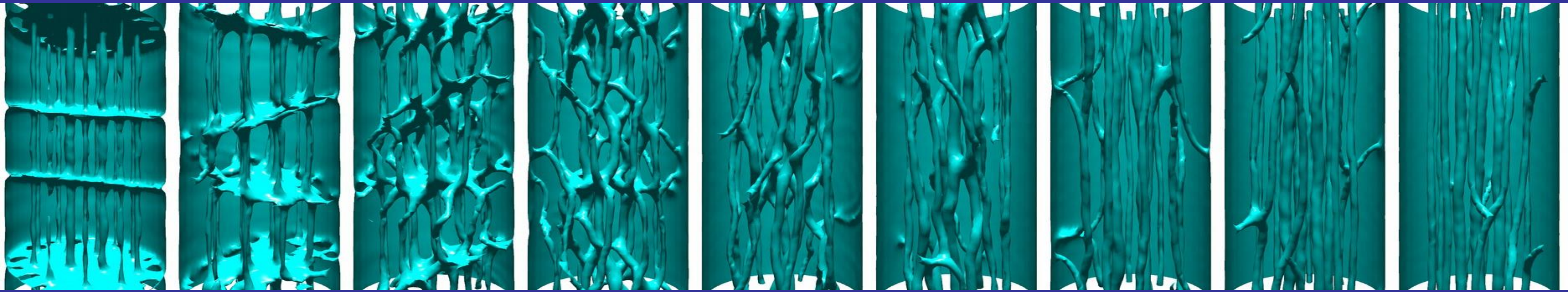
Warsaw University of Technology (WUT)



Generation and decay of fermionic turbulence

Collaborators:

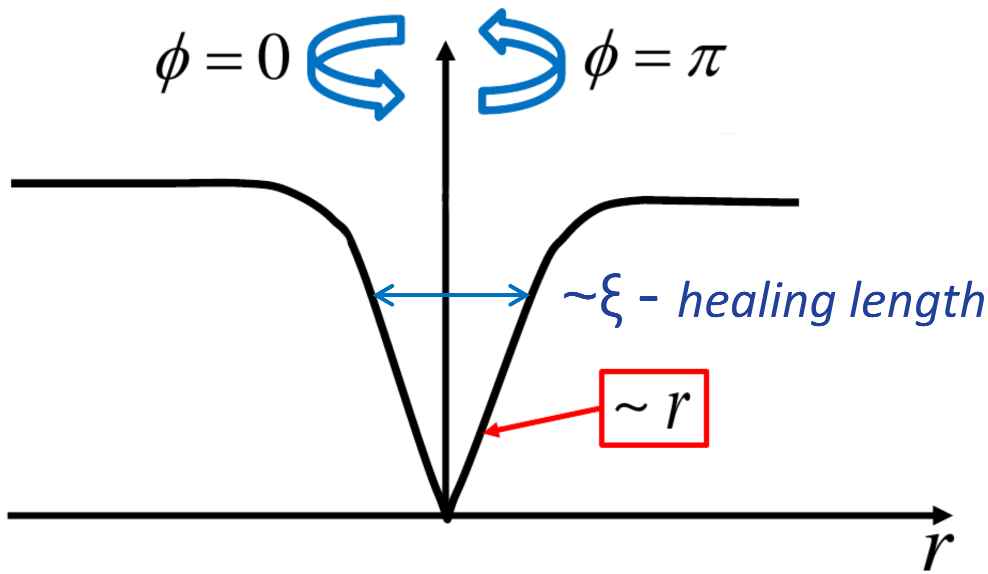
Andrea Barresi (WUT - PhD student)
Antoine Boulet (WUT)
Nicolas Chamel (ULB)
Konrad Kobuszewski (WUT - PhD student)
Andrzej Makowski (WUT - PhD student)
Daniel Pęcak (WUT)

Kazuyuki Sekizawa (Tokyo Inst. Technology)
Buğra Tüzemen (WUT -> IF PAN)
Gabriel Wlazłowski (WUT)
Tomasz Zawiślak (WUT -> Univ. Trento)
and LENS exp. group - Giacomo Roati et al.

Anatomy of the vortex core

Bosonic vortex structure:
weakly interacting Bose gas at T=0 → Gross-Pitaevskii eq. (GPE)

$$\left[-\frac{1}{2m}\nabla^2 + g|\psi(\vec{r})|^2 + V_{ext}(\vec{r})\right]\psi(\vec{r}) = \mu\psi(\vec{r})$$



Order parameter:

$$\psi(\vec{r}) = \sqrt{n(\vec{r})}e^{i\phi}$$

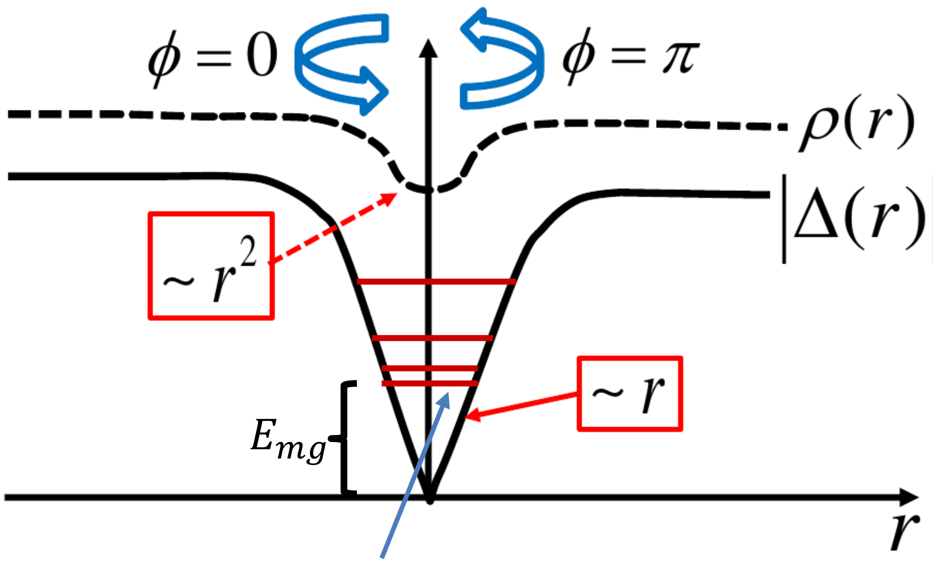
$$v_S = \frac{\hbar}{m}\nabla\phi$$

$$\kappa = \oint d\vec{l}\cdot\vec{v}_S = \frac{\hbar}{m}$$

Fermionic vortex structure:

Weakly interacting Fermi gas → Bogoliubov de Gennes (BdG) eqs.

$$\begin{pmatrix} h_\uparrow & \Delta \\ \Delta^* & -h_\downarrow^* \end{pmatrix} \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix} = \varepsilon_n \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix}$$



Form of the vortex-like solutions:

$$u_\eta(\mathbf{r}) = u_{nmk_z}(\rho) e^{im\varphi} e^{ik_z z}$$
$$v_\eta(\mathbf{r}) = v_{nmk_z}(\rho) e^{i(m+1)\varphi} e^{ik_z z}$$

CdGM (Andreev) states

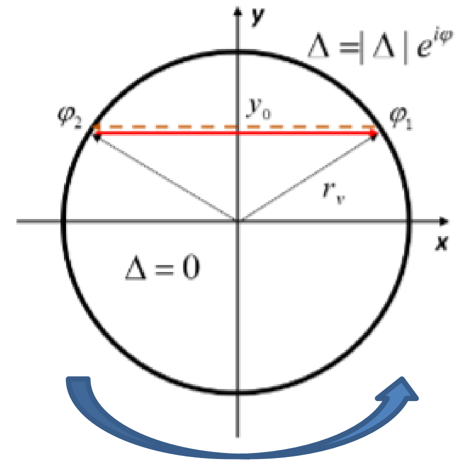
C. Caroli, P. de Gennes, J. Matricon, Phys. Lett. 9, 307 (1964):

Minigap: $E_{mg} \sim \dfrac{|\Delta_\infty|^2}{\varepsilon_F}$ - energy scale for vortex core excitations.

Density of states: $g(\varepsilon) \sim \dfrac{\varepsilon_F}{|\Delta_\infty|^2}\ ;\ \varepsilon \ll |\Delta_\infty|$

Vortex core structure in Andreev approximation:

Schematic section of the core

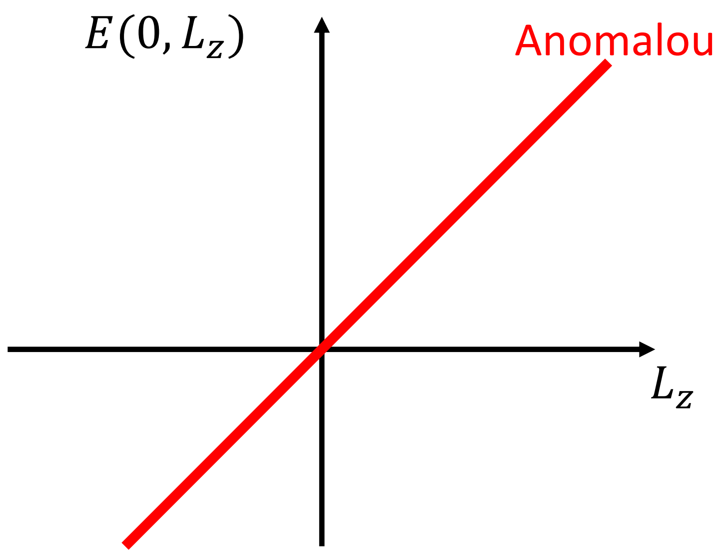


$$\frac{E(0,L_z)}{\varepsilon_F} k_F r_V \sqrt{1 - \left(\frac{L_z}{k_F r_V}\right)^2} + \arccos\left(\frac{-L_z}{k_F r_V}\right) - \arccos\left(\frac{E(0,L_z)}{|\Delta_\infty|}\right) = 0$$

$$E(0,L_z) = E(0) L_z, \; E \ll |\Delta_\infty|$$

$$E(0,L_z) \approx \frac{|\Delta_\infty|^2}{\varepsilon_F \frac{r_V}{\xi}\left(\frac{r_V}{\xi} + 1\right)} \frac{L_z}{\hbar}, \qquad \xi = \frac{\varepsilon_F}{k_F |\Delta_\infty|}$$

Spectrum of in-gap states



Works well in deep BCS limit: $\frac{1}{k_F a_s} \ll 0$

M. Stone, Phys. Rev. B 54, 13222 (1996)
P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

Quasiparticle mobility along the vortex line

$$E(k_z) = \frac{E(0)}{\sqrt{1-\left(\frac{k_z}{k_F}\right)^2}}\ ;\ k_z < k_F$$

C. Caroli, P. de Gennes, J. Matricon, Phys. Lett. 9, 307 (1964):

In Andreev approximation:

$$\sqrt{\varepsilon_F + E}\,\sin\alpha = \sqrt{\varepsilon_F - E}\,\sin\beta$$

$$k_h = \sqrt{2(\varepsilon_F - E)}$$

$$k_p = \sqrt{2(\varepsilon_F + E)}$$

$$v_z = k_z \frac{\sqrt{k_p^2 - k_z^2} - \sqrt{k_h^2 - k_z^2}}{\sqrt{k_p^2 - k_z^2} + \sqrt{k_h^2 - k_z^2}}$$

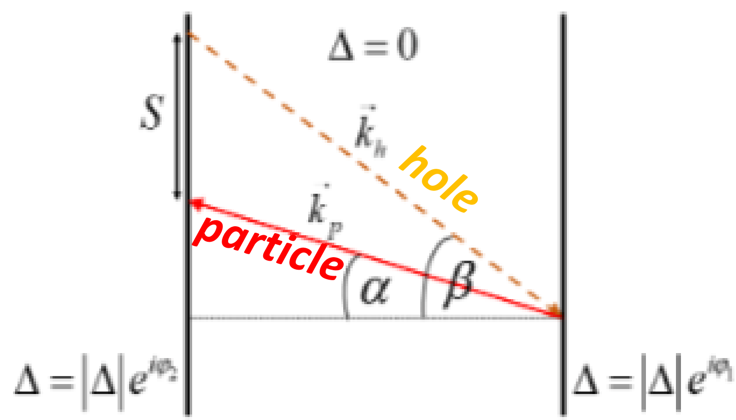
Velocity component along the vortex line

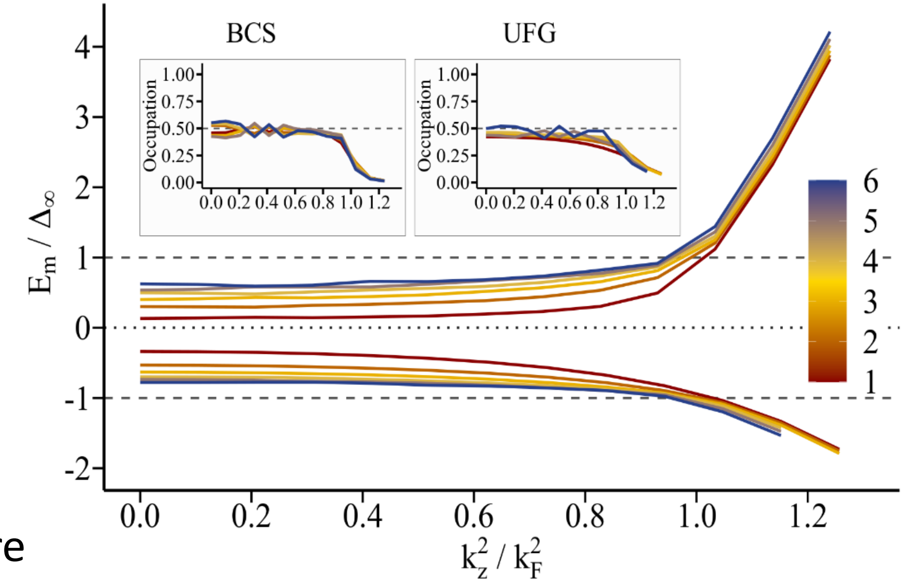
It gives the same dispersion relations as above up to the second order.

$$M_{eff}^{-1}(L_z) \approx \frac{2}{3}\left(\frac{|\Delta_\infty|}{\varepsilon_F}\right)^2 \frac{L_z}{\hbar}$$

Effective mass of quasiparticle in the core carrying ang. mom. Lz

Schematic picture of Andreev reflection of particle-hole moving along the vortex line

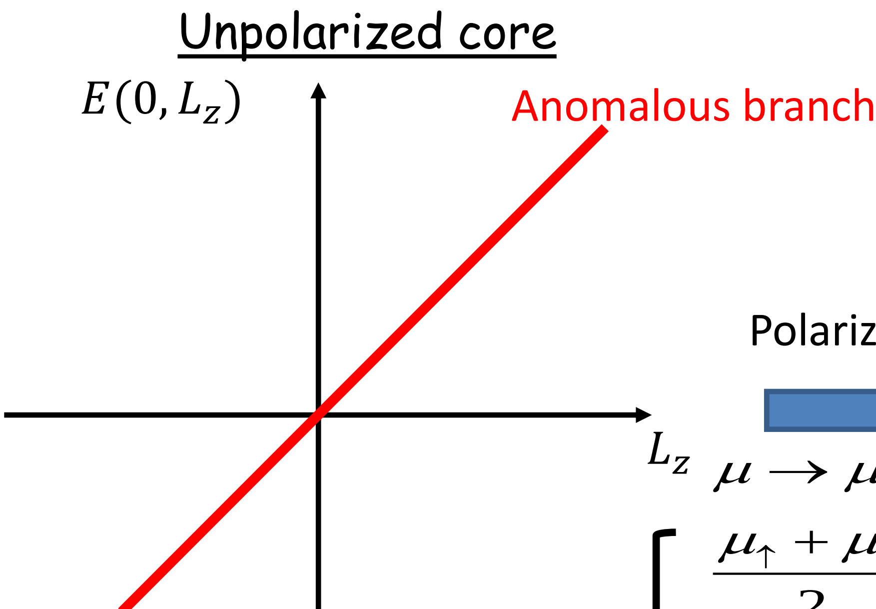




P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

Note that large value of effective mass along the vortex line originate from the fact that the occupations of hole and particle states below the gap are approximately equal.

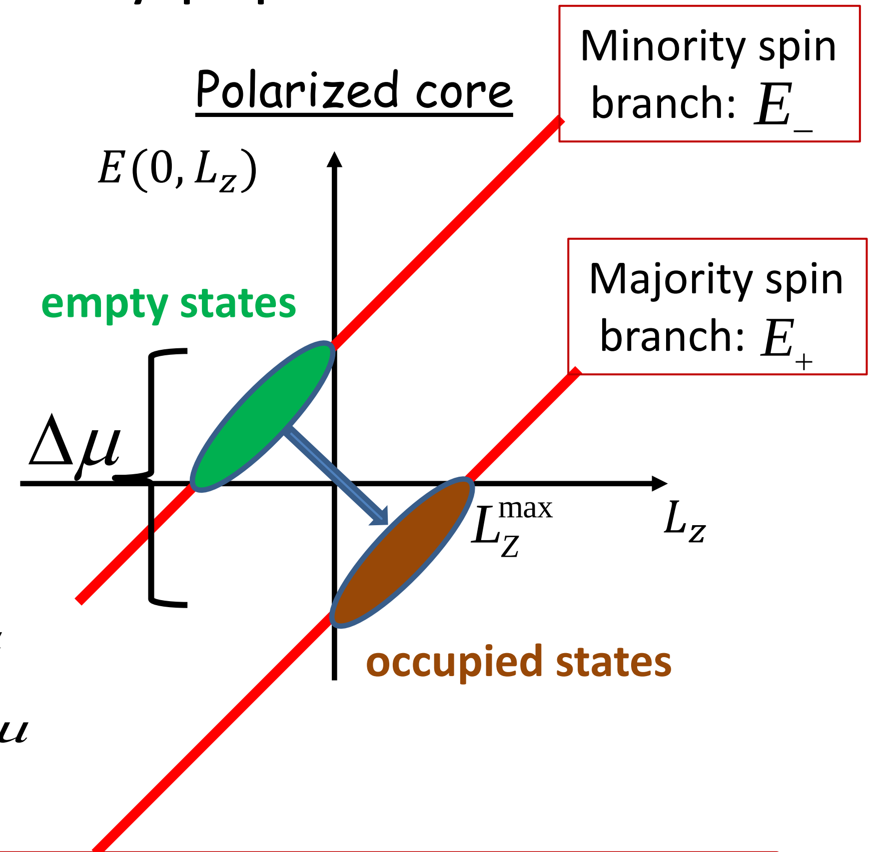
Changes of the core structure induced by spin polarization



Polarization

$$\mu \rightarrow \mu_\uparrow, \mu_\downarrow$$

$$\begin{cases} \dfrac{\mu_\uparrow + \mu_\downarrow}{2} = \mu \\ \mu_\uparrow - \mu_\downarrow = \Delta\mu \end{cases}$$



Branches are split proportionally to polarization

$$E_\pm(0, L_Z) \approx \frac{|\Delta_\infty|^2}{\varepsilon_F \dfrac{r_V}{\xi}\left(\dfrac{r_V}{\xi}+1\right)} \frac{L_Z}{\hbar} \mp \frac{\Delta\mu}{2}$$

Certain fraction of majority spin particles rotate in the opposite direction!

$$L_Z^{\max} \approx \frac{1}{2} \frac{\varepsilon_F}{|\Delta_\infty|^2} \frac{r_V}{\xi}\left(\frac{r_V}{\xi}+1\right) \hbar\Delta\mu$$

Two consequences of vortex core polarization:

1) Minigap vanishes.

2) Direction of the current in the core reverses.

1) Since the polarization correspond to relative shift of anomalous branches therefore the quasiparticle spectrum of spin-up and spin-down components is asymmetric for $k_z = 0$.

However the symmetry of the spectrum has to be restored in the limit of $k_z \to \infty$.
Since for a straight vortex one can decouple the degree of freedom along the vortex line:

$$H = \begin{pmatrix} h_{2D}(\mathbf{r}) + \frac{1}{2}k_z^2 - \mu_\uparrow & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h_{2D}^*(\mathbf{r}) - \frac{1}{2}k_z^2 + \mu_\downarrow \end{pmatrix}$$

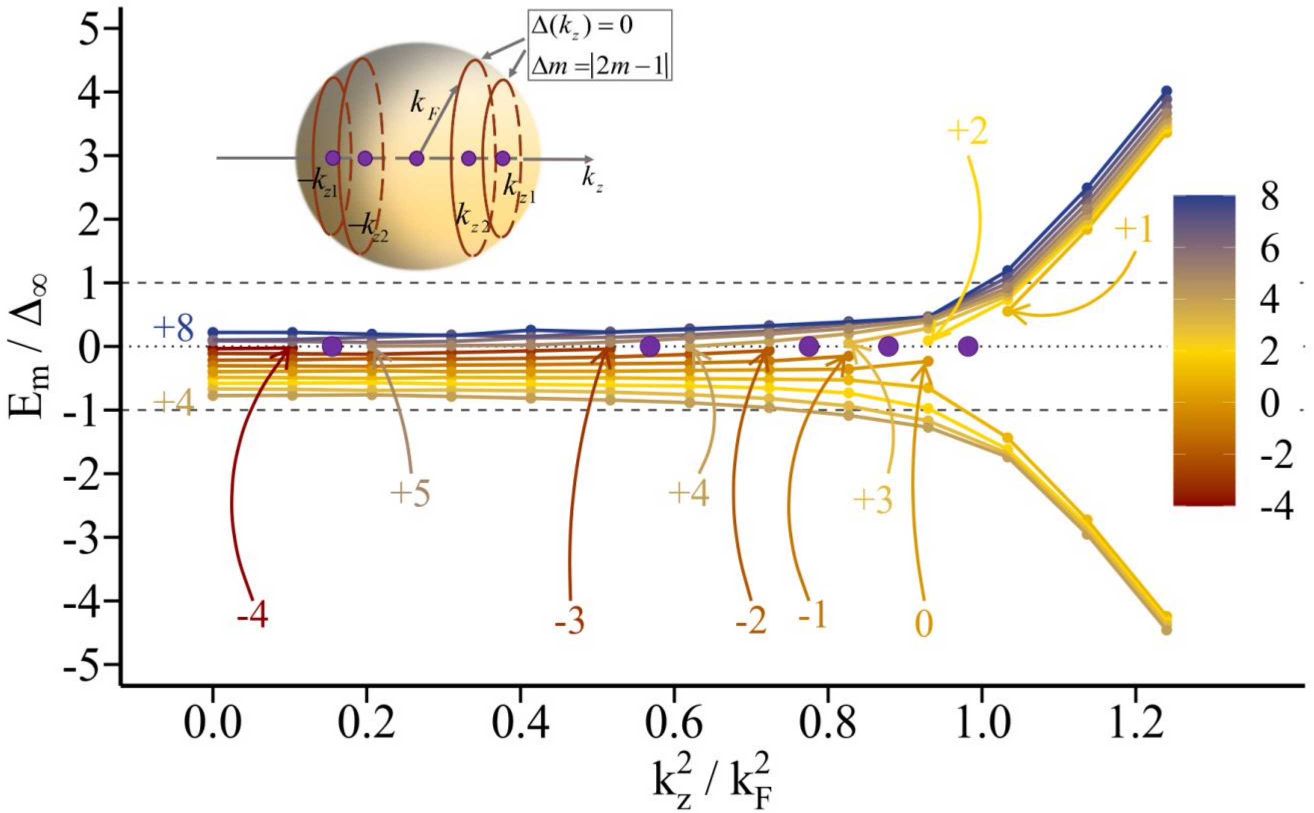
therefore $E(k_z) \propto \pm k_z^2$ when $k_z \to \infty$

As a result there must exist a sequence of values: $k_z = \pm k_{z1}, \pm k_{z2}, \ldots$
for which:

$$\boxed{E(\pm k_{zi}) = 0}$$

Moreover the crossings occur between levels of particular projection of angular momentum on the vortex line.

Namely, the crossing occurs in such a way that the particle state: $v_{\uparrow}$ of ang. momentum ***m*** is converted into a hole $u_{\uparrow}$ of momentum ***–m+1***
Hence the configuration changes by $\Delta m = |2m-1|$

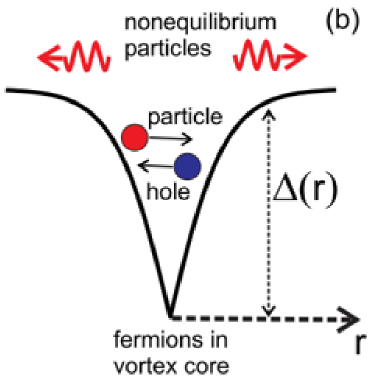


P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

How can we measure the influence of core states in ultracold gases?

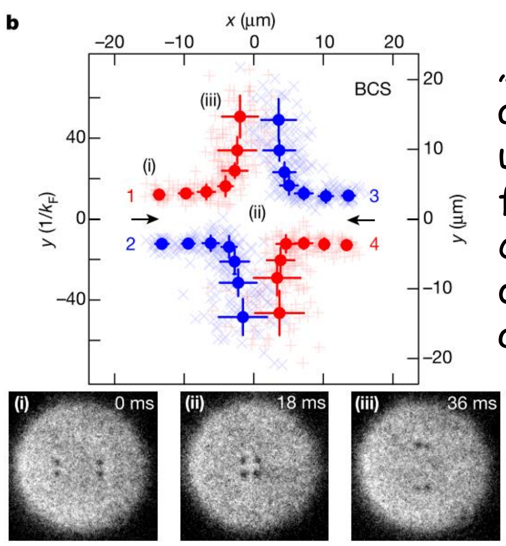
Dissipative processes involving vortex dynamics.

Silaev, Phys. Rev. Lett. 108, 045303 (2012)
Kopnin, Rep. Prog. Phys. 65, 1633 (2002)
Stone, Phys. Rev. B54, 13222 (1996)
Kopnin, Volovik, Phys. Rev. B57, 8526 (1998)
....



Classical treatment of states in the core (Boltzmann eq.).
More applicable in deep BCS limit unreachable in ultracold atoms.

Vortex-antivortex scattering in 2D



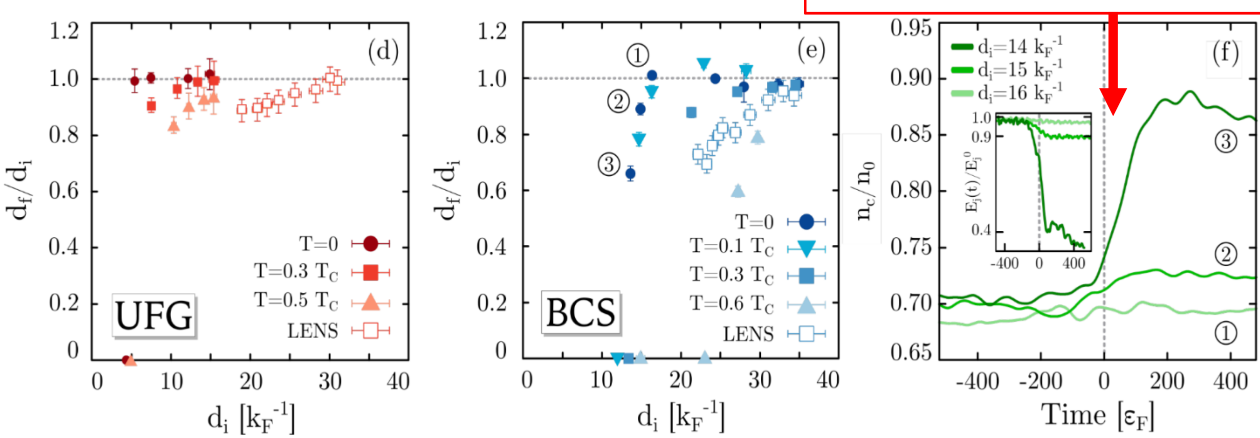
„Further, our few-vortex experiments extending across different superfluid regimes reveal non-universal dissipative dynamics, suggesting that fermionic quasiparticles localized inside the vortex core contribute significantly to dissipation, thereby opening the route to exploring new pathways for quantum turbulence decay, vortex by vortex."

W.J. Kwon et al. Nature **600**, 64 (2021)

Exciting quasiparticles in the vortex core

Indeed quasiparticles in the core are excited due to vortex acceleration but the effect is too weak to account for the total dissipation rate.

A. Barresi, A. Boulet, P.M., G. Wlazłowski, Phys. Rev. Lett. 130, 043001 (2023)

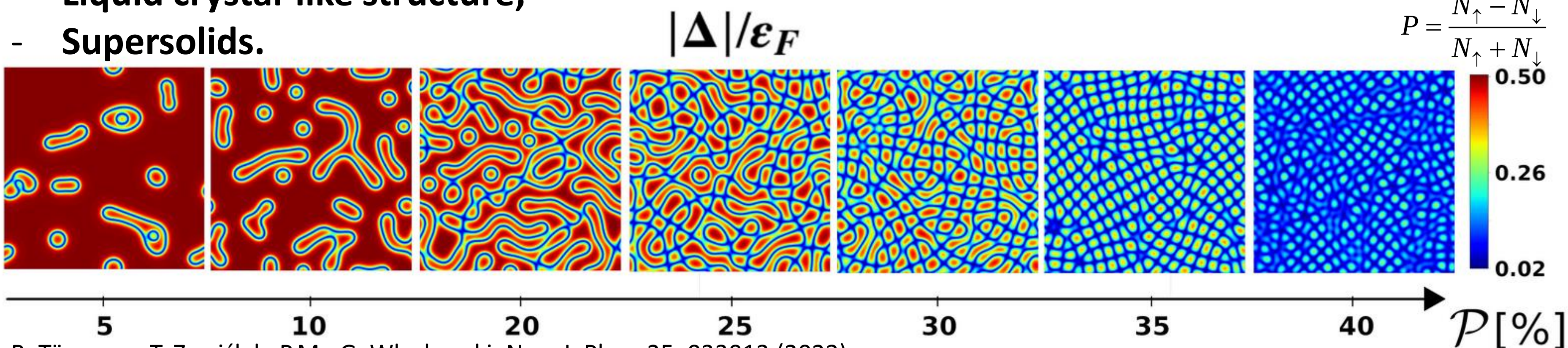


What is going to happen if we introduce spin imbalance?

In general it will generate distortions of Fermi spheres locally and triggering the appearance of **pairing field inhomogeneity** leading to various patterns involving:

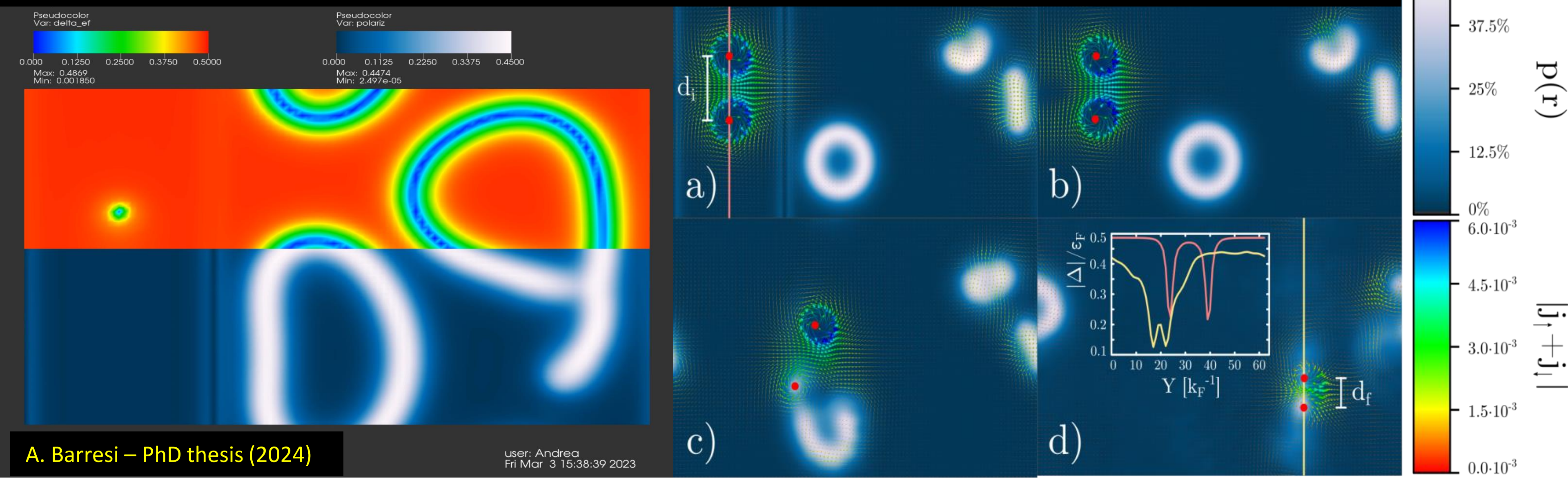
- **Separate impuritites (ferrons),**
- **Liquid crystal-like structure,**
- **Supersolids.**

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$



B. Tüzemen, T. Zawiślak, P.M., G. Wlazłowski, New J. Phys. 25, 033013 (2023).

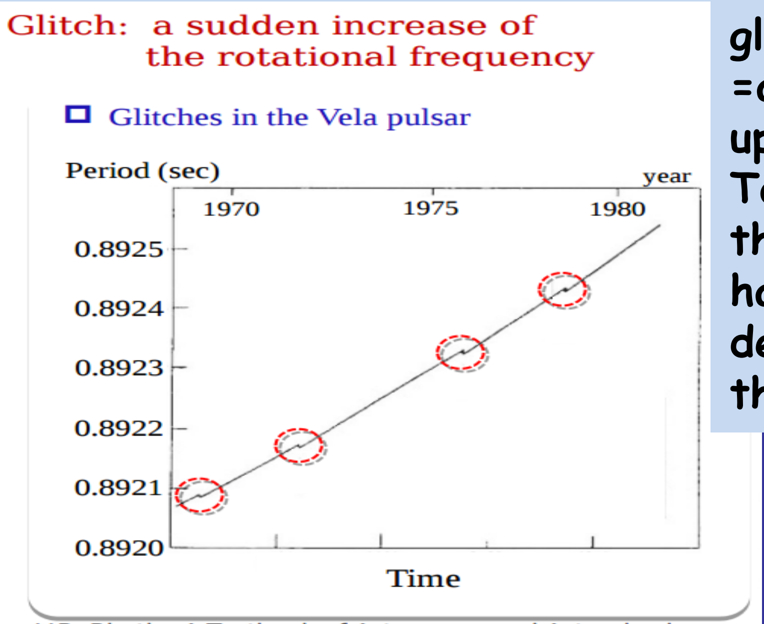
Dynamics of a vortex dipole in spin imbalanced Fermi superfluid.
Strong enhancement of vortex dipole energy dissipation.



A. Barresi – PhD thesis (2024)

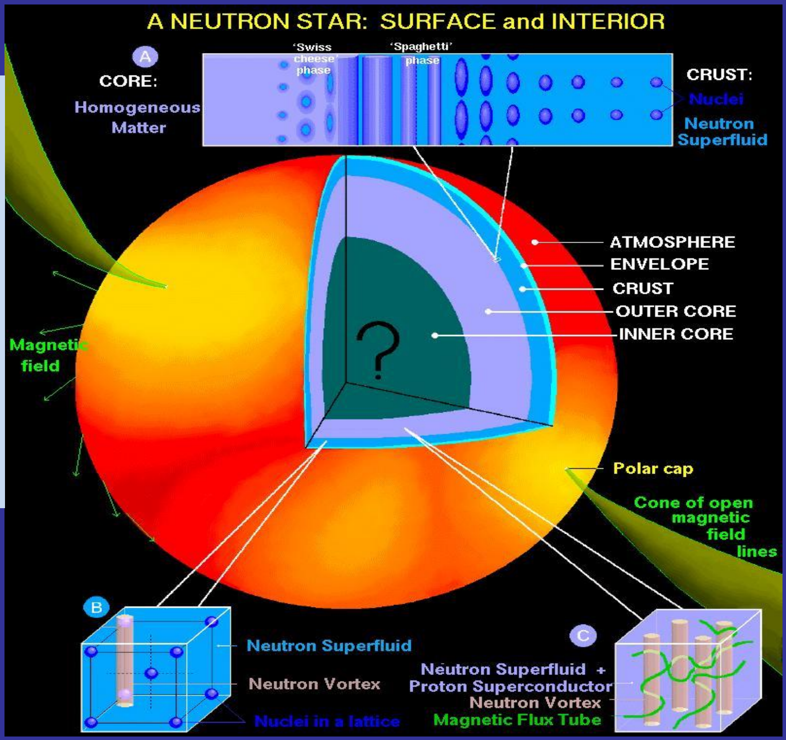
Modelling neutron star interior

Neutron star is a huge superfluid



V.B. Bhatia, A Textbook of Astronomy and Astrophysics with Elements of Cosmology, Alpha Science, 2001.

glitch phenomenon =a sudden speed up of rotation. To date more than 300 glitches have been detected in more than 100 pulsars



Glitch phenomenon is commonly believed to be related to rearrangement of vortices in the interior of neutron stars (Anderson,Itoh,Nature 256,25 (1975)) It would require however a correlated behavior of huge number of quantum vortices and the mechanism of such collective rearrangement is still a mystery.

Large scale dynamical model of neutron star interior (in particular neutron star crust), based on microscopic input from nuclear theory, is required.
In particular: vortex-impurity interaction, deformation modes of nuclear lattice, effective masses of nuclear impurities and couplings between lattice vibrations and neutron superfluid medium, need to be determined.

Properties of a vortex across the neutron star crust

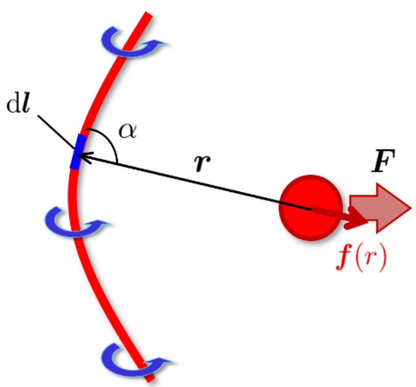
ρ_∞ (fm^{-3})	0.00036	0.0059	0.0112	0.0189	0.0231	0.0333
k_F^{-1} (fm)	4.52	1.79	1.45	1.21	1.14	1.01
ξ (fm)	8.44	5.53	5.97	7.00	7.78	10.28
$R_{\rm VFM}$ (fm)	15.0	10.5	10.5	12.0	13.5	16.5
Δ_∞ (MeV)	0.35	1.33	1.53	1.55	1.50	1.28
$T_{\rm crit}$ (MeV)	0.20	0.76	0.87	0.88	0.85	0.73
$\varepsilon_{\rm F}$ (MeV)	1.01	6.48	9.93	14.09	16.10	20.53
μ (MeV)	0.80	4.21	5.80	7.30	7.91	9.09
$E_{\rm mg}$ (MeV)	0.090	0.308	0.261	0.199	0.152	0.009
$B_{\rm crit}$ (10^{15} G)	7.76	26.5	22.5	17.2	13.1	0.82

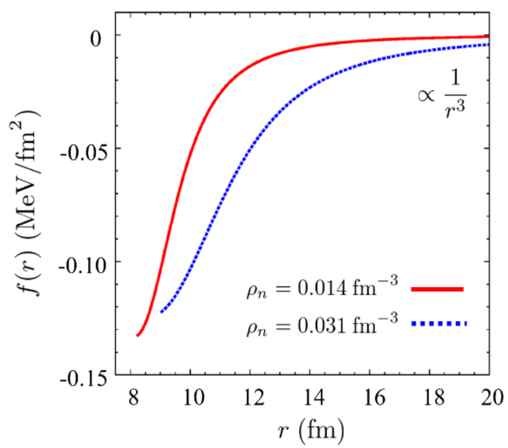
Minigap values

Magnetic field needed to polarize the core

D. Pęcak, N. Chamel, P.M., G. Wlazłowski, Phys. Rev. C**104**, 055801 (2021)

Vortex – impurity interaction (pinning force)





G. Wlazłowski, K. Sekizawa, P. Magierski, A. Bulgac, M.M. Forbes, Phys. Rev. Lett. 117, 232701(2016)

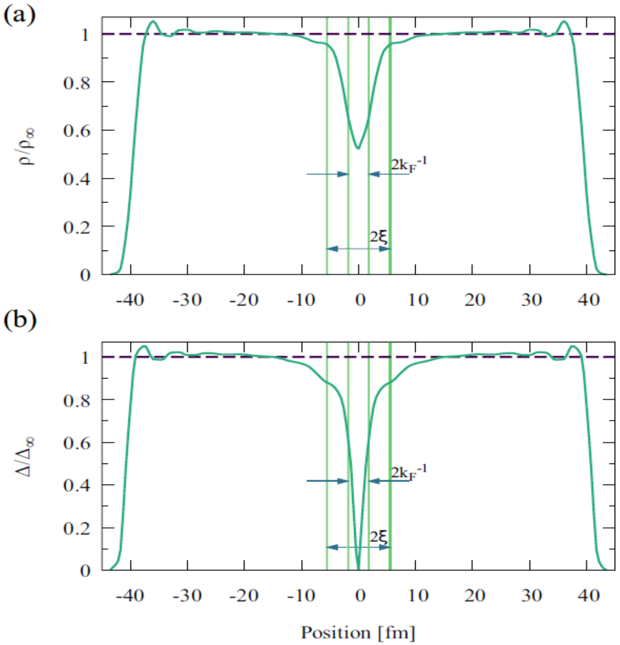
Is neutron star a turbulent system?

- What are differences and similarities of turbulence and its decay in Fermi and Bose superfluids?

A. Bulgac, A. Luo, P. Magierski, K.Roche, Y. Yu, Science 332, 1288 (2011).
M. Tylutki, G. Wlazłowski, Phys. Rev. A103, 051302 (2021).
K.Hossain, K.Kobuszewski, M.M.Forbes, P. Magierski, K.Sekizawa, G.Wlazłowski Phys. Rev. A 105, 013304 (2022).
G. Wlazłowski, M.M. Forbes, S.R. Sarkar, A. Marek, M. Szpindler, PNAS Nexus 3, 160 (2024).

Example: vortices across the neutron star crust

Section through the vortex core

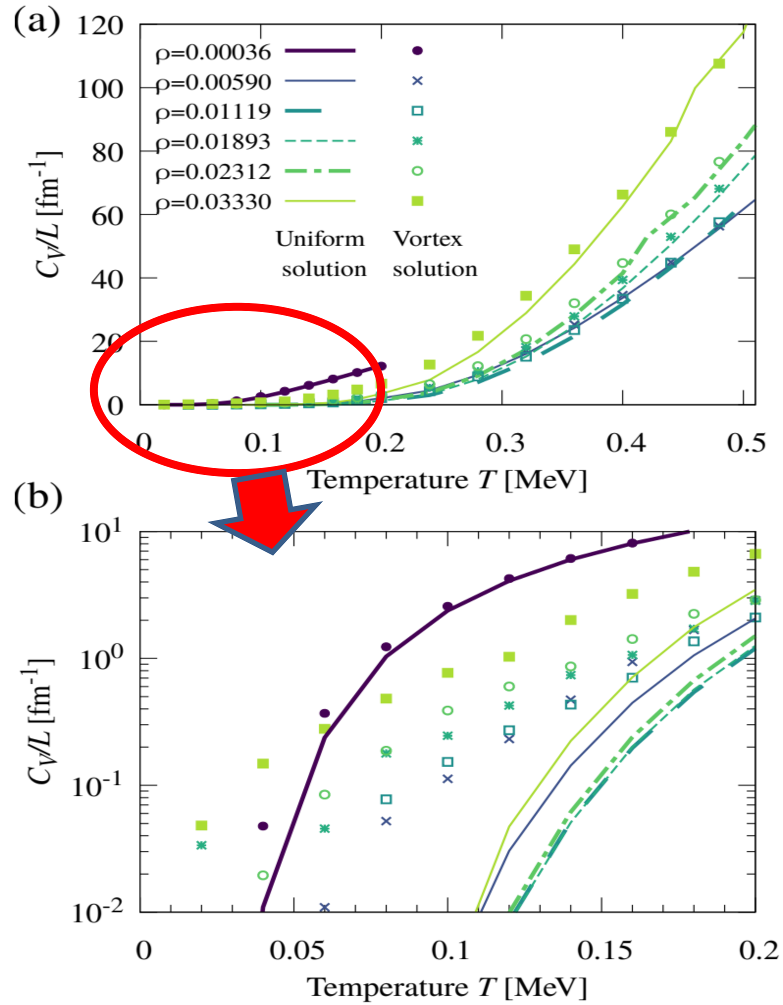


Normal density

Pairing field

Note two different length scales inside the core as explained by:
Sensarma, Randeria, Ho, Phys. Rev. Lett. 96, 090403 (2006)

Specific heat contribution vs uniform matter



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Minigap values

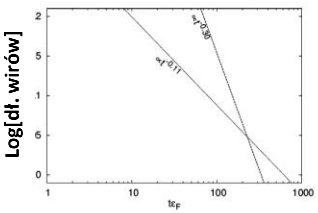
Magnetic field needed to polarize the core

D. Pęcak, N. Chamel, P.M., G. Wlazłowski, Phys. Rev. C**104**, 055801 (2021)

Superfluid turbulence (quantum turbulence):
disordered set of quantized vortices (vortex tangle).

Interesting questions:

- What are differences and similarities of turbulence in Fermi and Bose superfluids?
- Characteristics of turbulence in spin imbalanced systems?



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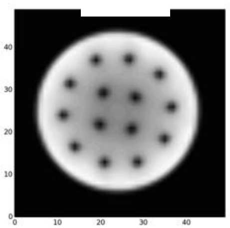
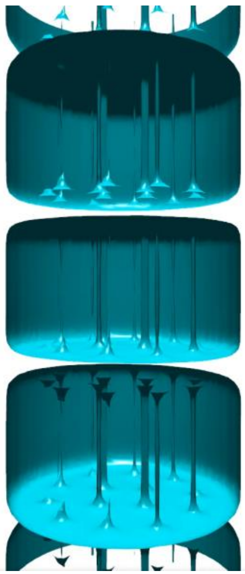




time = 0 $[\varepsilon_F^{-1}]$

Creation and evolution of disordered vortex tangle – microscopic simulation (TDDFT)
K.Hossain, K.Kobuszewski, M.M.Forbes, PM, K.Sekizawa, G.Wlazłowski
Phys. Rev. A 105, 013304 (2022)
Vortex reconnections, Kelvin waves and one body dissipation are crucial for decay of turbulent state.

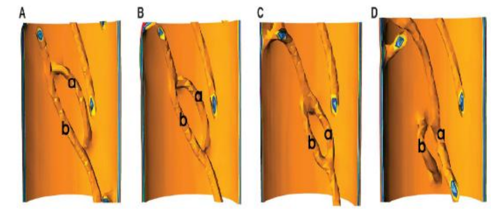
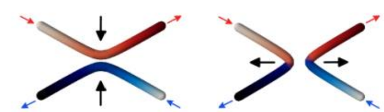
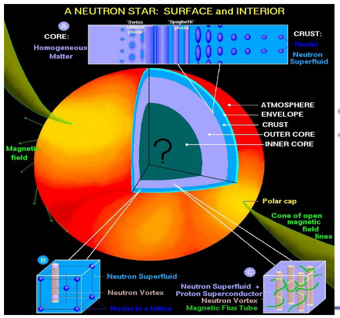


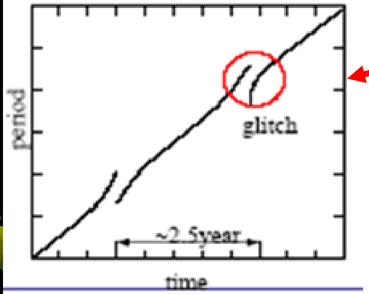
Fig. 3. (A to D) Two vortex lines approach each other, connect at two points, form a ring and exchange between them a portion of the vortex line, and subsequently separate. Segment (a), which initially belonged to the vortex line attached to the wall, is transferred to the long vortex line (b) after reconnection and vice versa.



Bulgac, Luo, Magierski, Roche, Yu, Science 332, 1288 (2011)

Is neutron star a turbulent system?

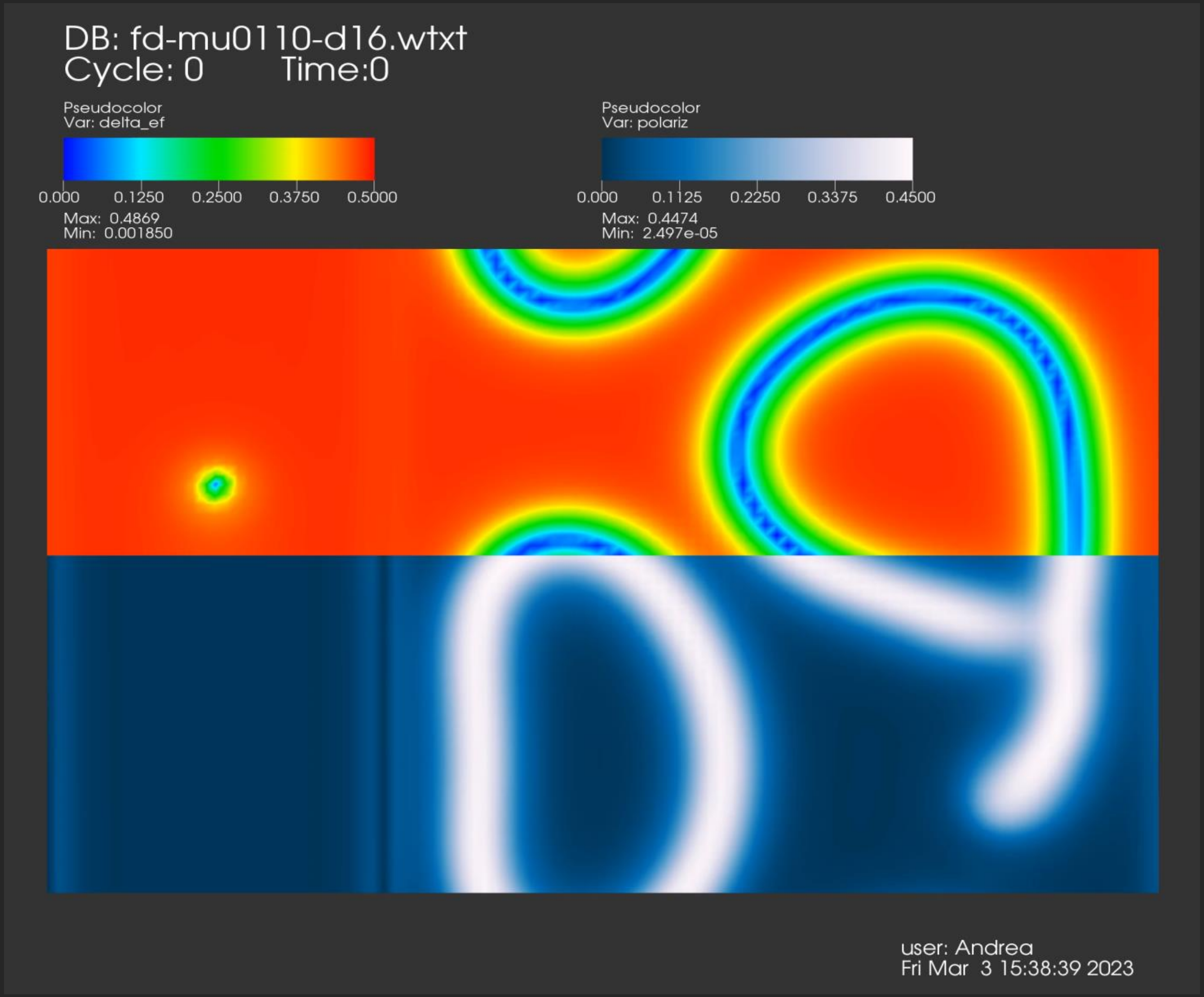




Periodic increase of rotational frequency of neutron star is observed (glitch phenomenon)

Since 70's the effect is associated with rapid rearrangement of quantum vortices inside neutron star caused by its inhomogeneous structure.
To date there is no theory which would explain the effect quantitatively.

Complex dynamics (strongly damped) of vortices in the spin imbalanced environment



Thanks to A. Barresi *et al.*

THANK YOU

Effective mass of a nucleus in superfluid neutron environment

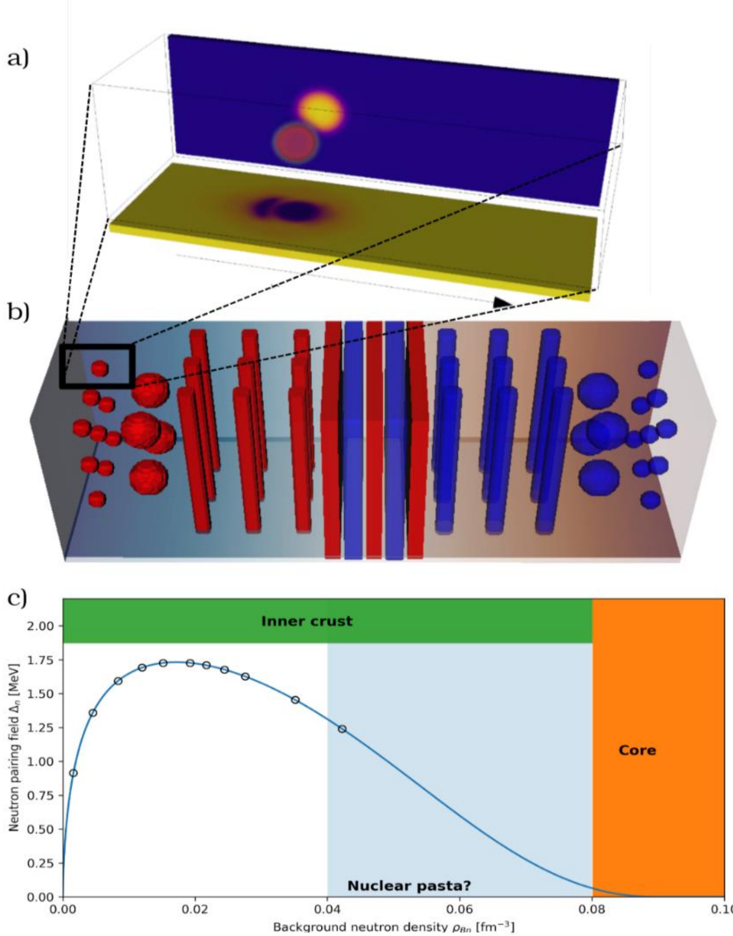
Suppose we would like to evaluate an effective mass of a heavy particle immersed in a Fermi bath.
Can one come up with the effective (classical) equation of motion of the type:

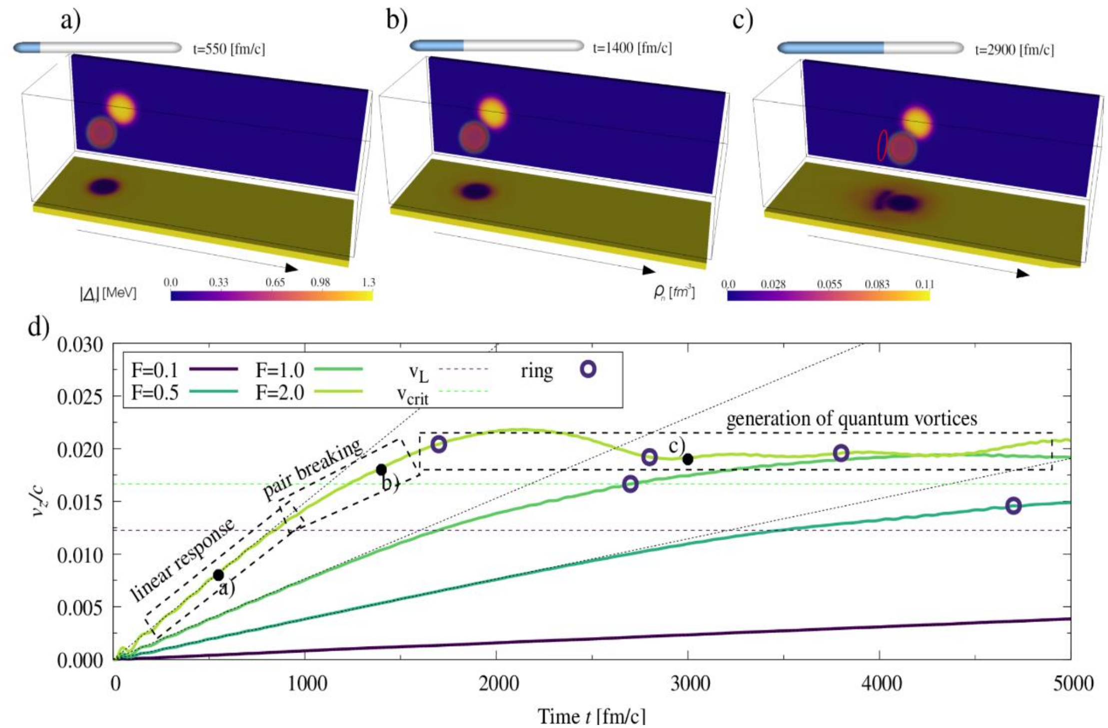
$$M\frac{d^2q}{dt^2} - F_D\left(\frac{dq}{dt},\ldots\right) + \frac{dE}{dq} = 0 \quad ?$$

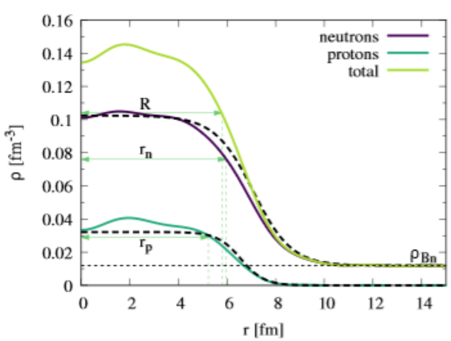
In general it is a complicated task as the first and the second term may not be unambiguously separated.

However for the superfluid system it can be done as for sufficiently slow motion (below the critical velocity) the second term may be neglected due to the presence of the pairing gap.

Dynamics of nuclear impurity in the neutron star crust: effective mass and energy dissipation



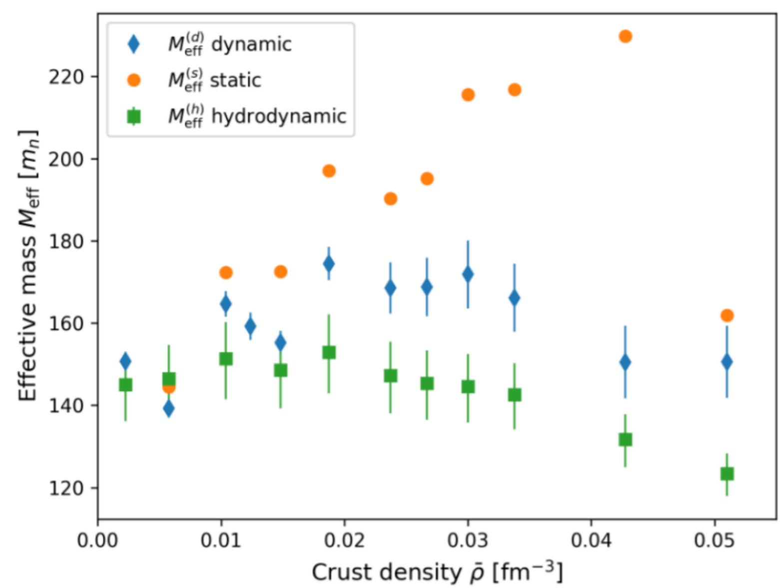




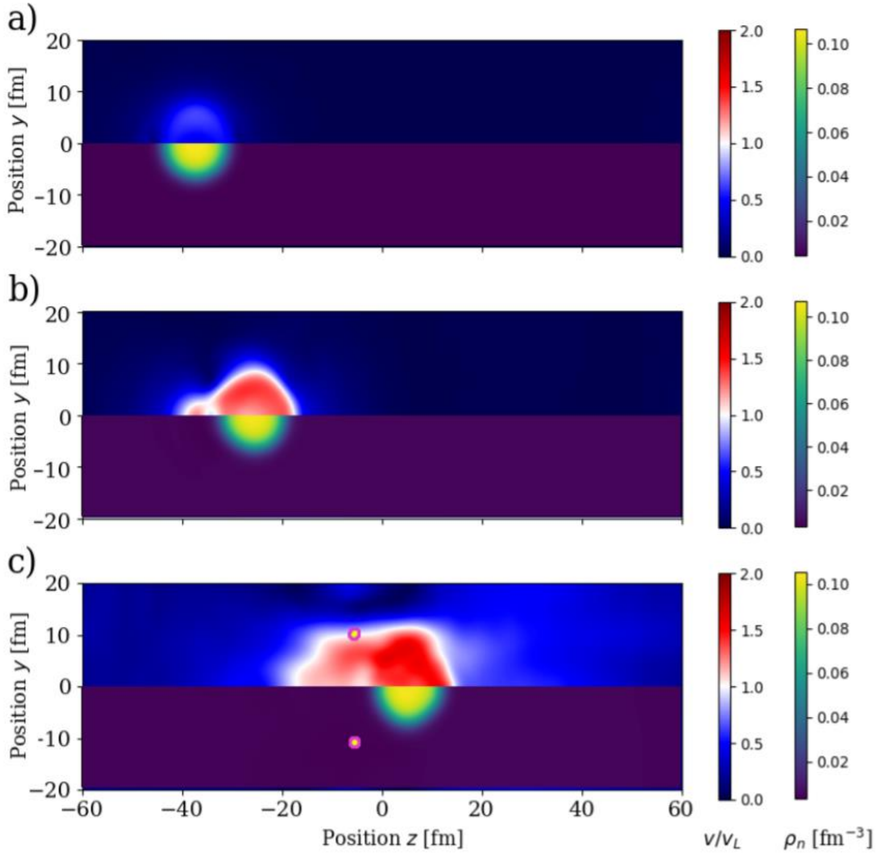
$$M_{\mathrm{eff}}^{(d)} = a_z/F$$

$$M_{\mathrm{eff}}^{(s)} = Zm_p + N_{\mathrm{bound}}m_n,$$

$$M_{\mathrm{eff}}^{(h)} = \frac{4}{3}\pi R_h^3 m_n \frac{(\rho_{\mathrm{in}} - \rho_{\mathrm{out}})^2}{\rho_{\mathrm{in}} + 2\rho_{\mathrm{out}}}.$$



D. Pęcak, A.Zdanowicz, N. Chamel, P. Magierski, G. Wlazłowski, arXiv:2403.17499



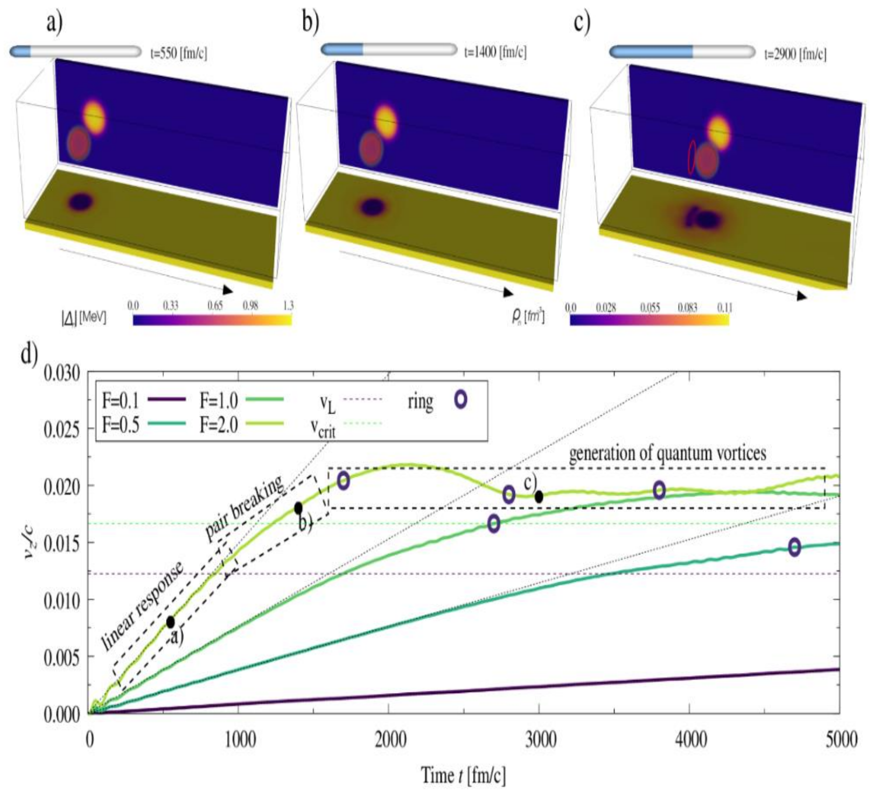


FIG. 6. Each panel presents the neutron density cross section through $x = 0$ (lower part), and local velocity in units of bulk Landau velocity (upper part). The consecutive panels are taken at times 550, 1400, and 2900 fm/c, which correspond to Fig. 3a)–c). a) in the linear response regime mainly the impurity is moving. b) in the breaking pair regime the free neutrons in the vicinity of impurity are affected. c) in the turbulent regime a large volume of neutrons is affected. Two points shown behind the impurity (at $z \approx -5\,\mathrm{fm}$) are the cross section of the vortex ring generated in this regime.

$$v_{\mathrm{L}} = \frac{\Delta_n}{\hbar k_{\mathrm{F}n}},$$

- Quasiparticle exc. energy is zero (gapless regime)

$$v_{\mathrm{crit}} = \frac{e}{2} v_{\mathrm{L}} \approx 1.4 v_{\mathrm{L}}$$

- Pairing disappears

Pairing correlations in time-dependent superfluid local density approximation (TDSLDA)

$$S = \int_{t_0}^{t_1} \left(\left\langle 0(t) \left| i\frac{d}{dt} \right| 0(t) \right\rangle - E[\rho(t), \chi(t)] \right) dt$$

Stationarity requirement produces the set of equations:

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} U_\mu(\mathbf{r},t) \\ V_\mu(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r},t) & \Delta(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & -h^*(\mathbf{r},t) \end{pmatrix}\begin{pmatrix} U_\mu(\mathbf{r},t) \\ V_\mu(\mathbf{r},t) \end{pmatrix},$$

$$B(t) = \begin{pmatrix} U(t) & V^*(t) \\ V(t) & U^*(t) \end{pmatrix} = \exp[iG(t)] \qquad G(t) = \begin{pmatrix} h(t) & \Delta(t) \\ \Delta^\dagger(t) & -h^*(t) \end{pmatrix}$$

Orthogonality and completeness has to be fulfilled: $B^\dagger(t)B(t) = B(t)B^\dagger(t) = I,$

In order to fulfill the completeness relation of Bogoliubov transform all states need to be evolved!
Otherwise Pauli principle is violated, i.e. the evolved densities do not describe a fermionic system (spurious bosonic effects are introduced).

Consequence: the computational cost increases considerably.

P. Magierski, *Nuclear Reactions and Superfluid Time Dependent Density Functional Theory*, Frontiers in Nuclear and Particle Physics vol. 2, 57 (2019)
A. Bulgac, *Time-Dependent Density Functional Theory and Real-Time Dynamics of Fermi Superfluids*, Ann. Rev. Nucl. Part. Sci. 63, 97 (2013)

Solving time-dependent problem for superfluids...

The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations

$$h \sim f_1(n,\nu,\ldots)\boldsymbol{\nabla}^2 + \boldsymbol{f}_2(n,\nu,\ldots)\cdot\boldsymbol{\nabla} + f_3(n,\nu,\ldots)$$

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t)\end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) & 0 & 0 & \Delta(\boldsymbol{r},t) \\ 0 & h_b(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h_a^*(\boldsymbol{r},t) & 0 \\ \Delta^*(\boldsymbol{r},t) & 0 & 0 & -h_b^*(\boldsymbol{r},t)\end{pmatrix}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t)\end{pmatrix}$$

We explicitly track fermionic degrees of freedom!

where *h* and Δ depends on "densities":

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r},t)|^2, \qquad \tau_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r},t)|^2,$$

$$\chi_c(r,t) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r},t)v^*_{n,\downarrow}(\boldsymbol{r},t), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r},t)\nabla v_{n,\sigma}(\boldsymbol{r},t)],$$

$$\Delta(\mathbf{r}) = g_{eff}(\mathbf{r})\chi_c(\mathbf{r})$$

$$\frac{1}{g_{eff}(\mathbf{r})} = \frac{1}{g(\mathbf{r})} - \frac{mk_c(\mathbf{r})}{2\pi^2\hbar^2}\left(1 - \frac{k_F(\mathbf{r})}{2k_c(\mathbf{r})}\ln\frac{k_c(\mathbf{r})+k_F(\mathbf{r})}{k_c(\mathbf{r})-k_F(\mathbf{r})}\right)$$

A. Bulgac, Y. Yu, Phys. Rev. Lett. 88 (2002) 042504
A. Bulgac, Phys. Rev. C65 (2002) 051305

huge number of nonlinear coupled 3D Partial Differential Equations
(in practice n=1,2,…, 10^5 - 10^6)

Present computing capabilities:
- full 3D (unconstrained) superfluid dynamics
- spatial mesh up to 100^3
- max. number of particles of the order of 10^4
- up to 10^6 time steps

(for cold atomic systems – time scale: a few ms
for nuclei – time scale: 100 zs)

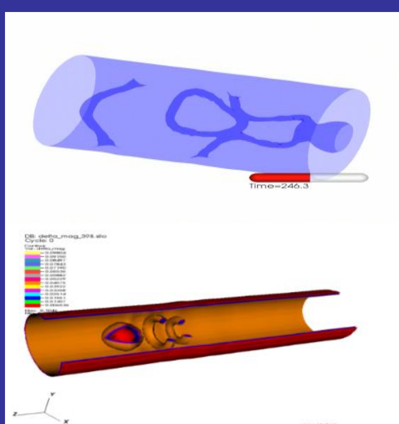
- P. Magierski, *Nuclear Reactions and Superfluid Time Dependent Density Functional Theory*, Frontiers in Nuclear and Particle Physics, vol. 2, 57 (2019)
- A. Bulgac, *Time-Dependent Density Functional Theory and Real-Time Dynamics of Fermi Superfluids*, Ann. Rev. Nucl. Part. Sci. 63, 97 (2013)
- A. Bulgac, M.M. Forbes, P. Magierski, Lecture Notes in Physics, Vol. 836, Chap. 9, p.305-373 (2012)

Superconducting systems of interest

$$\frac{\Delta}{\varepsilon_F} \leq 0.5$$

Ultracold atomic (fermionic) gases. Unitary regime.

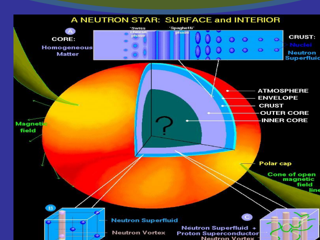
Dynamics of quantum vortices, solitonic excitations, quantum turbulence



$$\frac{\Delta}{\varepsilon_F} \leq 0.1-0.2$$

Astrophysical applications.

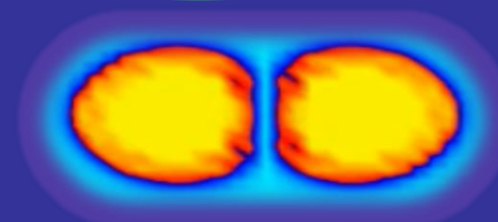
Modelling of neutron star interior (glitches): vortex dynamics, dynamics of inhomogeneous nuclear matter.

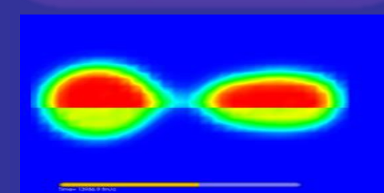


$$\frac{\Delta}{\varepsilon_F} \leq 0.03$$

Nuclear physics.

Induced nuclear fission, fusion, collisions.





$$\frac{\Delta}{\varepsilon_F}$$ **- Pairing gap to Fermi energy ratio**

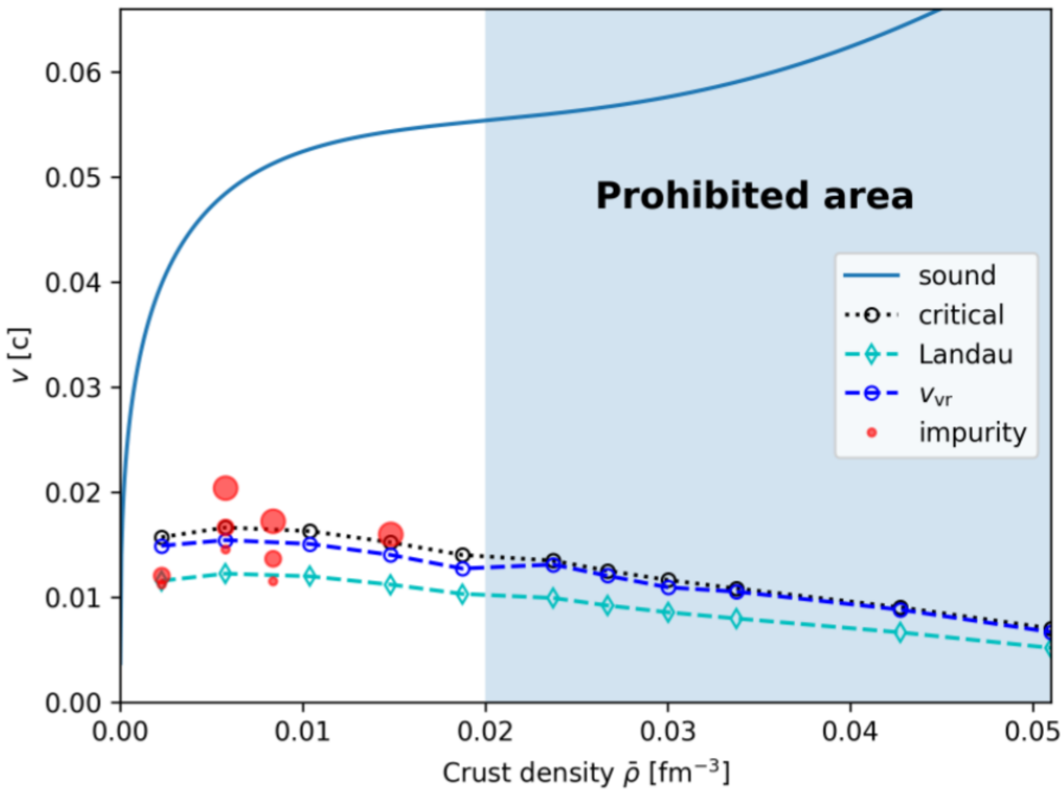


FIG. 7. The velocity scales in the system. The speed of sound (solid line) of the system is a few times larger than the Landau velocity v_L (black and green dashed lines). We plot the hydrodynamic velocity of a vortex ring v_{vr} with the dashed blue line. By red dots, we denote the nucleus velocity at which the vortex ring is created of three different sizes for different forces (MeV/fm): $F = 0.5$, 1, and 2, respectively. The vortex rings are not produced in the blue area on the right side.