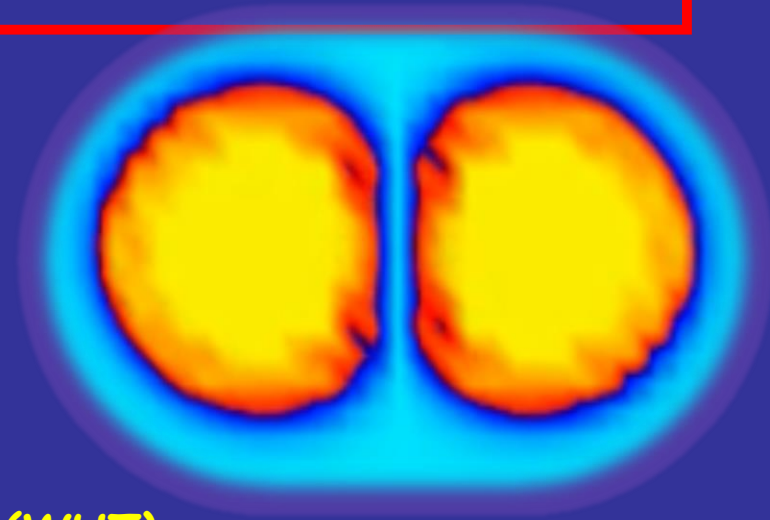
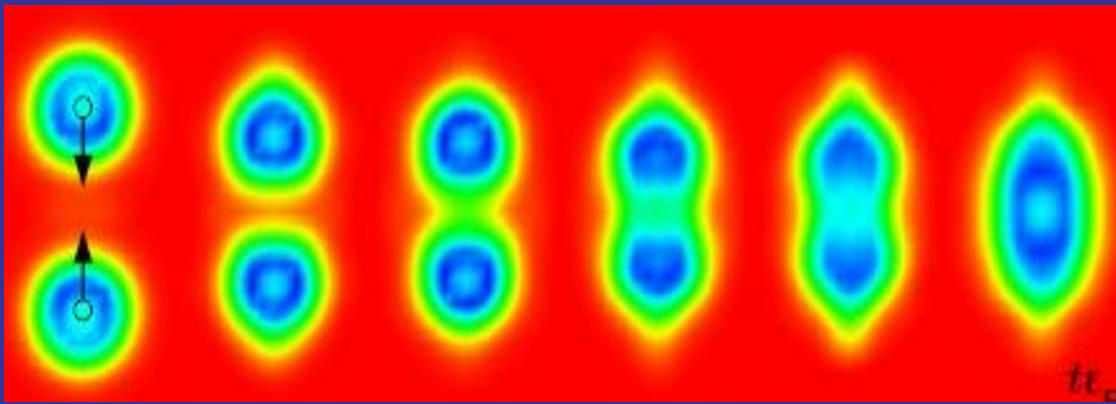


# *Solitonic excitations in heavy ion collision and spin imbalanced, ultracold atomic gases.*



*Piotr Magierski*  
*Warsaw University of Technology (WUT)*  
*University of Washington (UW)*

Collaborators:

Matthew Barton (WUT)  
Andrzej Makowski (WUT)  
Kazuyuki Sekizawa (Tokyo Inst. of Techn.)  
Buğra Tüzemen (WUT)  
Gabriel Wlazłowski (WUT)  
Tomasz Zawiślak (WUT)

CONNECTIONS BETWEEN COLD  
ATOMS AND NUCLEAR MATTER:  
FROM LOW TO HIGH ENERGIES.

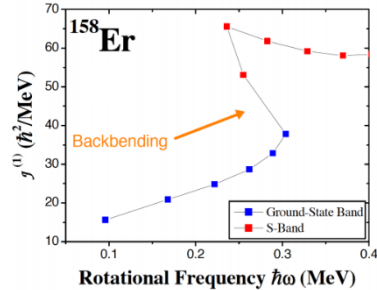


## OUTLINE:

- Solitonic excitations in nuclear collisions and ultracold Fermi gases.
- Metastable Larkin-Ovchinnikov droplets (*ferrons*) in ultracold Fermi gas.

# What do we know about pairing correlations in atomic nuclei?

Odd-even mass staggering gives us estimate of the pairing strength  $|\Delta| \approx \frac{12}{\sqrt{A}} \text{ MeV}$   
 (unfortunately obscured by polarization effects)

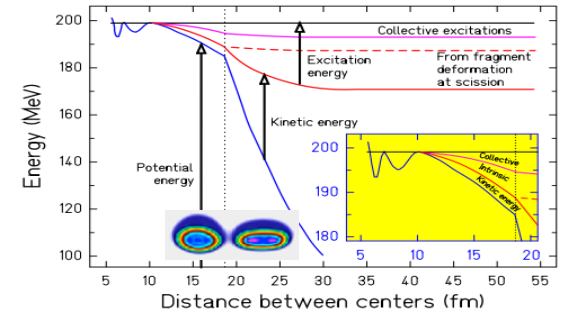


High spin experimental data: backbending of moments of inertia produced by the alignment of the correlated nucleon pair is a sensitive function of pairing correlations.

A. Johnson, H. Ryde, S.A. Hjorth,  
 Nucl. Phys. A179, 753 (1972)

Theoretical description of large amplitude nuclear motion require to include pairing correlations, e.g. Nuclear fission at low energies

A. Bulgac, P.M., K.J. Roche, I. Stetcu, Phys. Rev. Lett. 116, 122504 (2016)



From Schmidt & Jurado: Phys.Rev.C83:061601,2011

## Can we probe the pairing field phase in nuclei?

Nuclear Josephson junction: enhancement of neutron pair transfer in nuclear collision

V.I. Gol'danskii, A.I. Larkin JETP 53, 1032 (1967)

K. Dietrich, Phys.Lett. B32, 428 (1970)

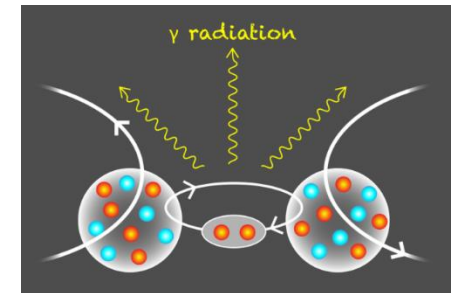
(Unfortunately experimental data are not conclusive)

Recent attempt: oscillatory pair transfer (AC Josephson junction)

C.Potel, F.Barranco, E.Vigezzi, R.A. Broglia, Phys.Rev. C103, L021601(2021)

surprising agreement of gamma spectra with experiment!

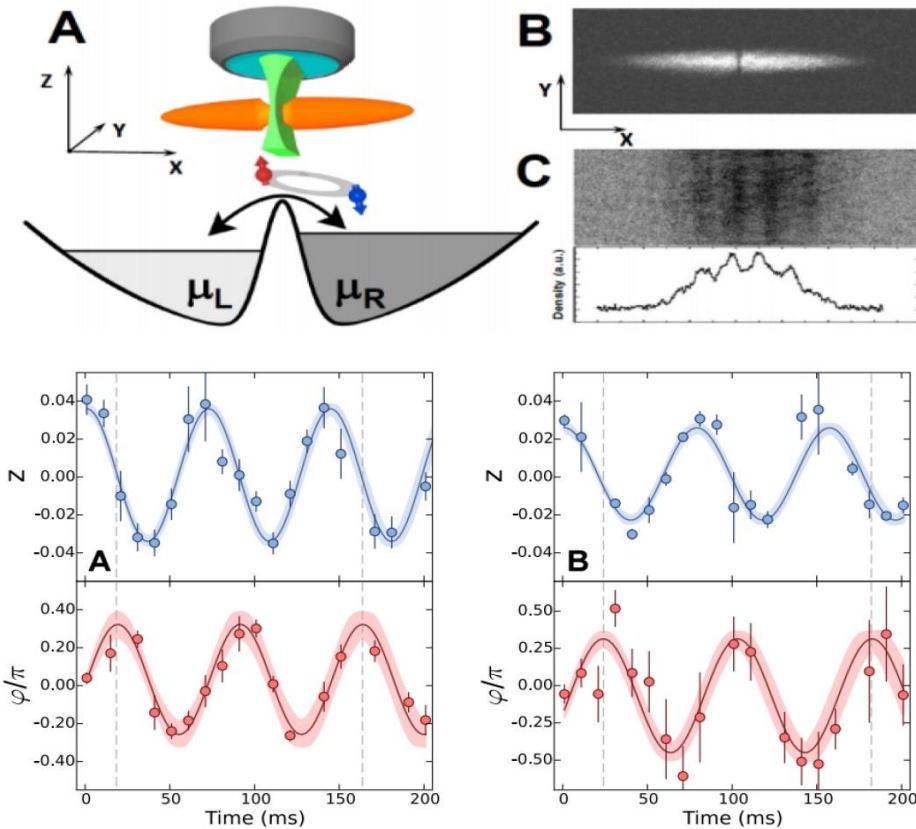
(Although just one reaction:  $^{116}\text{Sn} + ^{60}\text{Ni}$  has been studied)



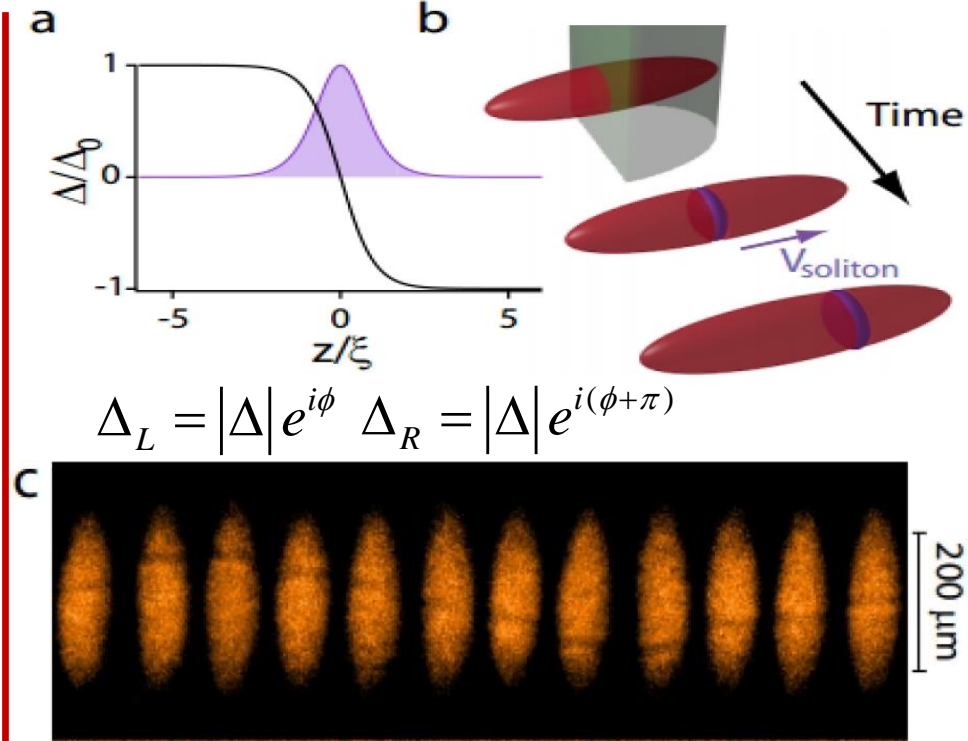
From P.M., Physics 14,27(2021)

# Ultracold atomic gases: two regimes of pairing dynamics induced by phase difference

Weak coupling (weak link)



Strong coupling



Creation of a „heavy soliton“ after merging two superfluid atomic clouds.

MIT experiments: Nature 499, 426 (2013).

PRL 113, 065301 (2014); PRL 116, 045304 (2016);

**Decay pattern:**

dark soliton  $\rightarrow$  Phi soliton  $\rightarrow$  vortex ring  $\rightarrow$  vortex line

**THEORY:**

G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. M., Phys. Rev. Lett. **120**, 253002 (2018)

Observation of **AC Josephson effect** between two  $^6\text{Li}$  atomic clouds.

It need not to be accompanied by creation of a topological excitation.

G. Valtolina et al., Science 350, 1505 (2015).

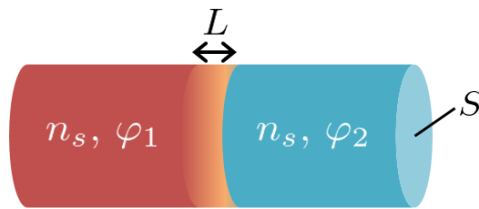
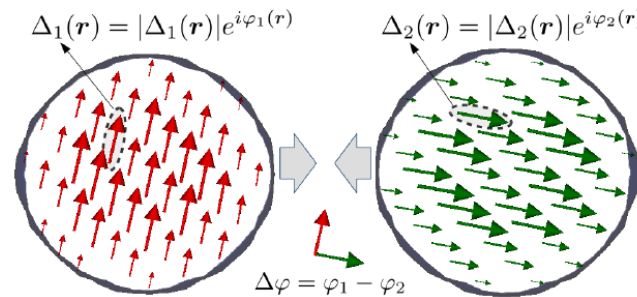
# Nuclear collisions: strong coupling regime

Collisions of superfluid nuclei having different phases of the pairing fields

The main questions are:

- how a possible solitonic structure can be manifested in nuclear system?
- what observable effect it may have on heavy ion reaction:  
kinetic energy distribution of fragments, capture cross section, etc.?

Clearly, we cannot control phases of the pairing field in nuclear experiments and the possible signal need to be extracted after averaging over the phase difference.



$$\Delta\varphi (\equiv \varphi_1 - \varphi_2)$$

From Ginzburg-Landau (G-L) approach:

$$E_j = \frac{S}{L} \frac{\hbar^2}{2m} n_s \sin^2 \frac{\Delta\varphi}{2}$$

For typical values characteristic for two medium nuclei:  $E_j \approx 30\text{MeV}$

# Solving time-dependent problem for superfluids...

The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations

$$h \sim f_1(n, \nu, \dots) \nabla^2 + \mathbf{f}_2(n, \nu, \dots) \cdot \nabla + f_3(n, \nu, \dots)$$

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_{n,a}(\mathbf{r}, t) \\ u_{n,b}(\mathbf{r}, t) \\ v_{n,a}(\mathbf{r}, t) \\ v_{n,b}(\mathbf{r}, t) \end{pmatrix} = \begin{pmatrix} h_a(\mathbf{r}, t) & 0 & 0 & \Delta(\mathbf{r}, t) \\ 0 & h_b(\mathbf{r}, t) & -\Delta(\mathbf{r}, t) & 0 \\ 0 & -\Delta^*(\mathbf{r}, t) & -h_a^*(\mathbf{r}, t) & 0 \\ \Delta^*(\mathbf{r}, t) & 0 & 0 & -h_b^*(\mathbf{r}, t) \end{pmatrix} \begin{pmatrix} u_{n,a}(\mathbf{r}, t) \\ u_{n,b}(\mathbf{r}, t) \\ v_{n,a}(\mathbf{r}, t) \\ v_{n,b}(\mathbf{r}, t) \end{pmatrix}$$

We explicitly track fermionic degrees of freedom!

where  $h$  and  $\Delta$  depends on “densities”:

$$n_\sigma(\mathbf{r}, t) = \sum_{E_n < E_c} |v_{n,\sigma}(\mathbf{r}, t)|^2, \quad \tau_\sigma(\mathbf{r}, t) = \sum_{E_n < E_c} |\nabla v_{n,\sigma}(\mathbf{r}, t)|^2,$$

$$\chi_c(\mathbf{r}, t) = \sum_{E_n < E_c} u_{n,\uparrow}(\mathbf{r}, t) v_{n,\downarrow}^*(\mathbf{r}, t), \quad \mathbf{j}_\sigma(\mathbf{r}, t) = \sum_{E_n < E_c} \text{Im}[v_{n,\sigma}^*(\mathbf{r}, t) \nabla v_{n,\sigma}(\mathbf{r}, t)],$$

$$\Delta(\mathbf{r}) = g_{eff}(\mathbf{r}) \chi_c(\mathbf{r})$$

$$\frac{1}{g_{eff}(\mathbf{r})} = \frac{1}{g(\mathbf{r})} - \frac{mk_c(\mathbf{r})}{2\pi^2 \hbar^2} \left( 1 - \frac{k_F(\mathbf{r})}{2k_c(\mathbf{r})} \ln \frac{k_c(\mathbf{r}) + k_F(\mathbf{r})}{k_c(\mathbf{r}) - k_F(\mathbf{r})} \right)$$

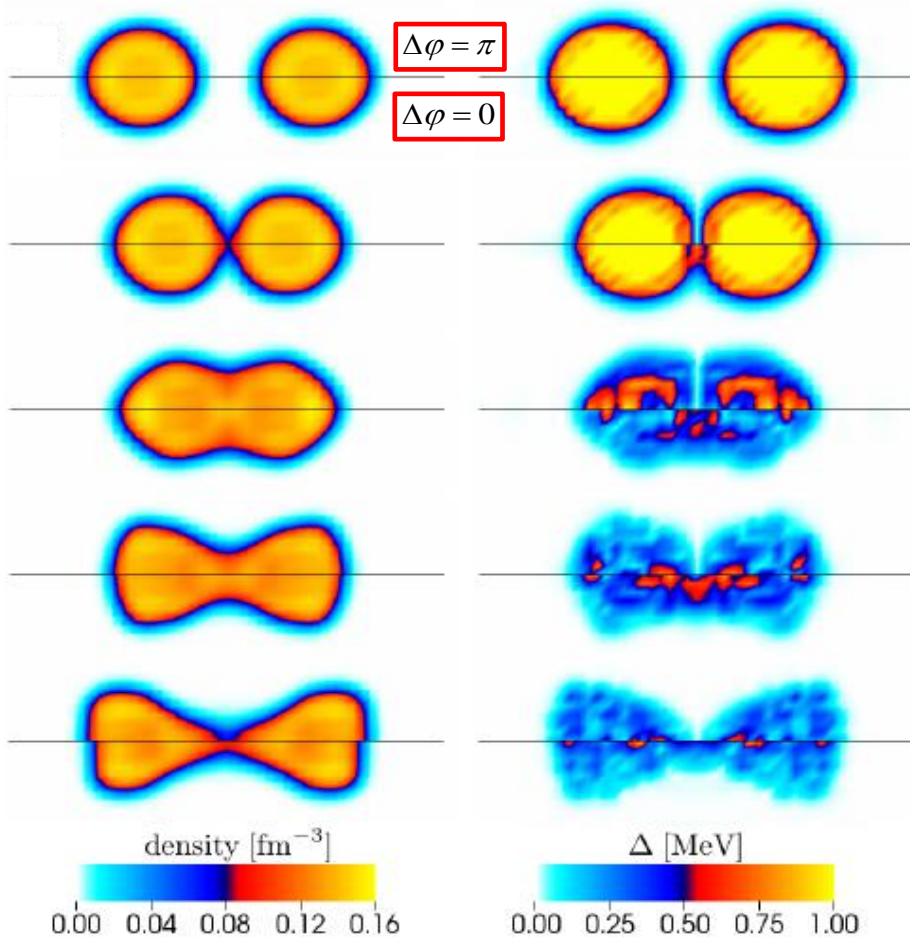
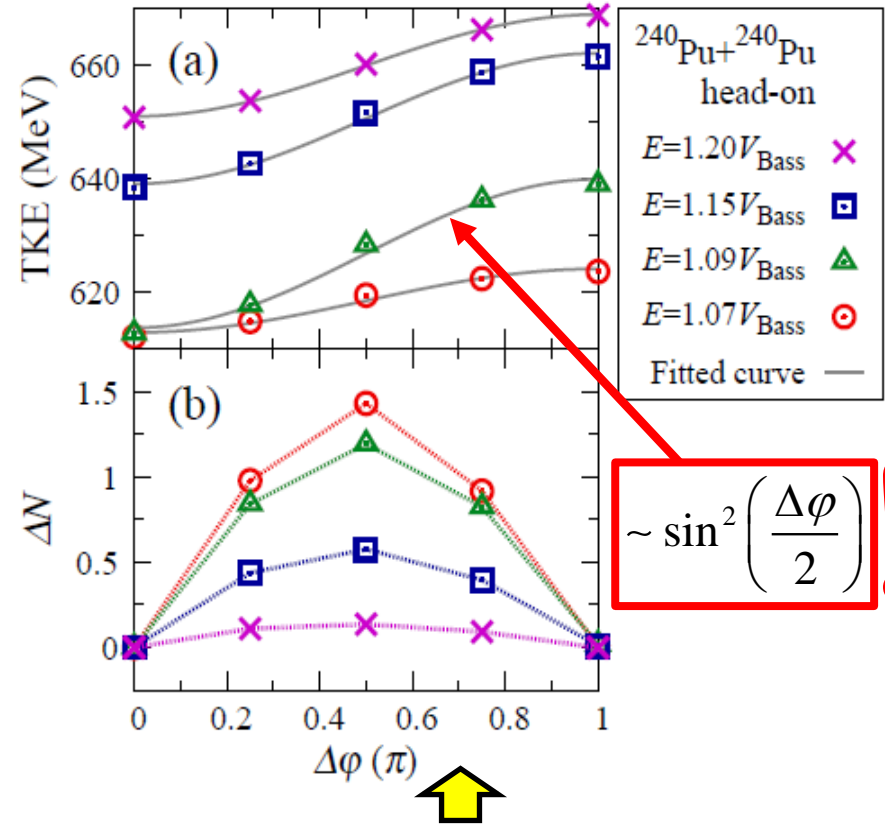
A. Bulgac, Y. Yu, Phys. Rev. Lett. 88 (2002) 042504  
A. Bulgac, Phys. Rev. C65 (2002) 051305

**huge number of nonlinear coupled 3D Partial Differential Equations**  
(in practice  $n=1,2,\dots, 10^5 - 10^6$ )

**Present computing capabilities:**

- ▶ full 3D (unconstrained) superfluid dynamics
  - ▶ spatial mesh up to  $100^3$
  - ▶ max. number of particles of the order of  $10^4$
  - ▶ up to  $10^6$  time steps
- (for cold atomic systems - time scale: a few ms  
for nuclei - time scale: 100 zs)

- P. M., *Nuclear Reactions and Superfluid Time Dependent Density Functional Theory*, Frontiers in Nuclear and Particle Physics, vol. 2, 57 (2019)
- A. Bulgac, *Time-Dependent Density Functional Theory and Real-Time Dynamics of Fermi Superfluids*, Ann. Rev. Nucl. Part. Sci. 63, 97 (2013)
- A. Bulgac, M.M. Forbes, P. M., Lecture Notes in Physics, Vol. 836, Chap. 9, p.305-373 (2012)

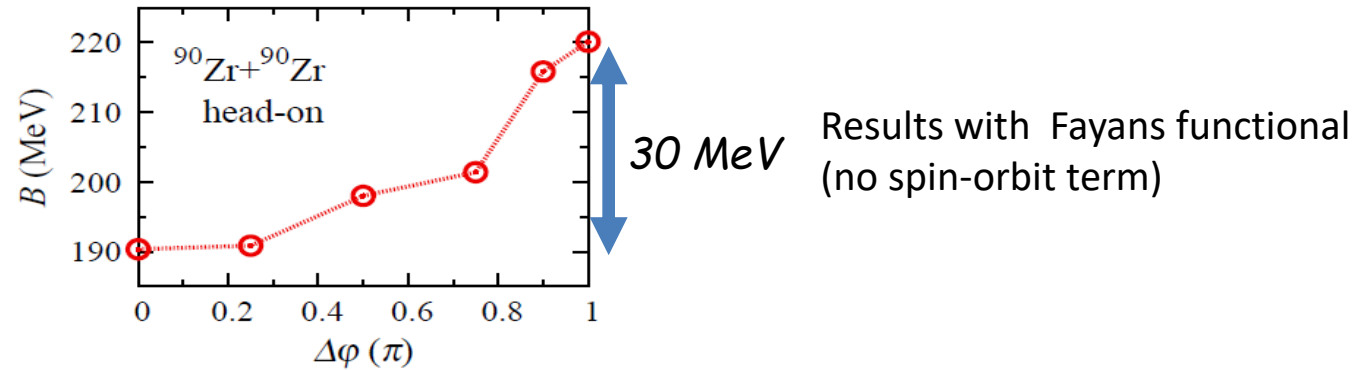
$^{240}\text{Pu}+^{240}\text{Pu}$ Total kinetic energy of the fragments (TKE)

Average particle transfer between fragments.

Creation of the solitonic structure between colliding nuclei prevents energy transfer to internal degrees of freedom and consequently enhances the kinetic energy of outgoing fragments.

Surprisingly, the gauge angle dependence from the G-L approach is perfectly well reproduced in the kinetic energies of outgoing fragments!

# Effective barrier height for fusion as a function of the phase difference



What is an average extra energy needed for the capture?

$$E_{extra} = \frac{1}{\pi} \int_0^{\pi} (B(\Delta\varphi) - V_{Bass}) d(\Delta\varphi) \approx 10 \text{ MeV}$$

The effect is found (within TDDFT) to be of the order of 30 MeV for medium nuclei and occur for energies up to 20-30% of the barrier height.

P. M., K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119 042501 (2017)

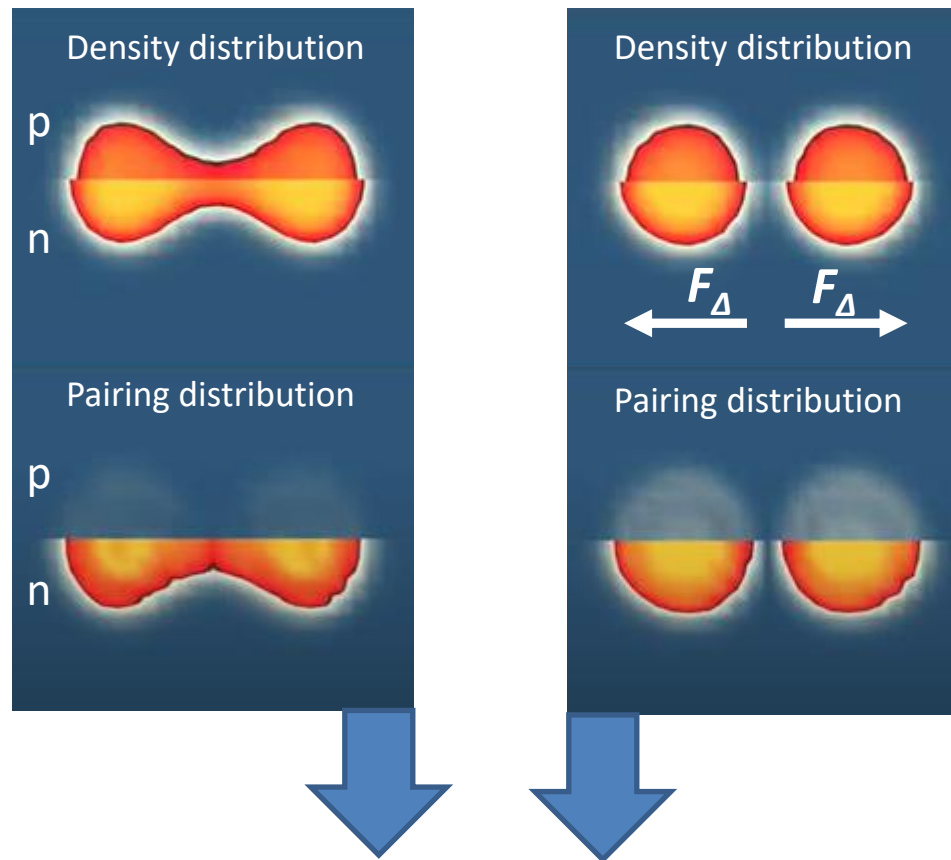
G. Scamps, Phys. Rev. C 97, 044611 (2018): **barrier fluctuations extracted from experimental Data indicate that the effect although is weaker than predicted by TDDFT**

	$\bar{\Delta}_q$ (MeV)	$E_{\text{thresh}}(0)$ (MeV)	$E_{\text{thresh}}(\pi)$ (MeV)	$\Delta E_s$
$^{90}\text{Zr}$	$\bar{\Delta}_n = 0.00$	184	184	0
	$\bar{\Delta}_p = 0.09$			
$^{96}\text{Zr}$	$\bar{\Delta}_n = 1.98$	179	185	6
	$\bar{\Delta}_p = 0.32$			
	$\bar{\Delta}_n = 2.44$	178	187	9
	$\bar{\Delta}_p = 0.33$			
$\bar{\Delta}_n = 2.94$	178	187	9	
$\bar{\Delta}_p = 0.34$				

Recent results with full SkM\* functional:  
Minimum energy needed for capture.

P.M., A. Makowski, M. Barton, K. Sekizawa, G. Wlazłowski,  
Phys. Rev. C 105, 064602 (2022)





	$\bar{\Delta}_q$ (MeV)	$E_{c.m.}$ (MeV)	TXE (MeV)	
			$\Delta\phi = 0$	$\Delta\phi = \pi$
$^{96}\text{Zr}$	$\bar{\Delta}_n = 1.98$ $\bar{\Delta}_p = 0.32$	178	37	25
	$\bar{\Delta}_n = 2.44$ $\bar{\Delta}_p = 0.33$	177	34	10
	$\bar{\Delta}_n = 2.94$ $\bar{\Delta}_p = 0.34$	177	34	8

Total excitation energy of the fragments

# Inhomogeneous systems: Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase

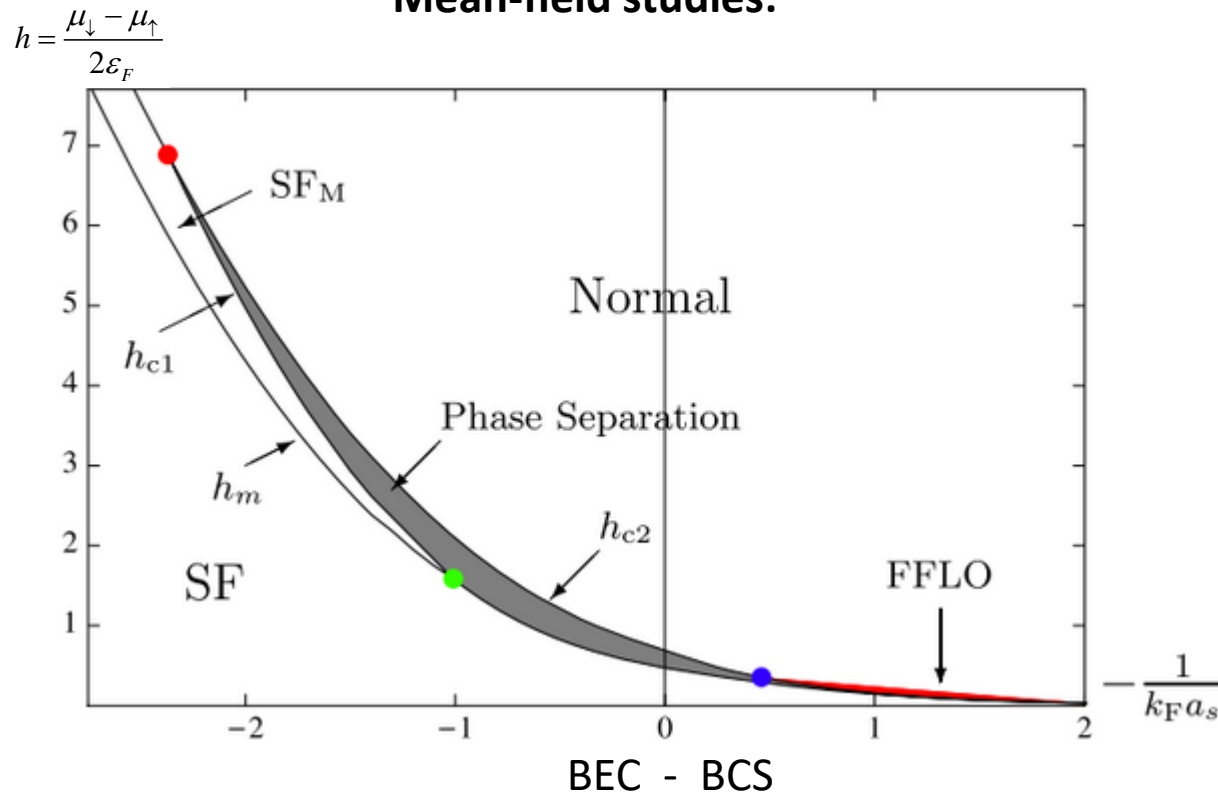
$$\text{Larkin-Ovchinnikov: } \Delta(r) \sim \cos(qr)$$

$$\text{Fulde-Ferrell: } \Delta(r) \sim \exp(iqr)$$

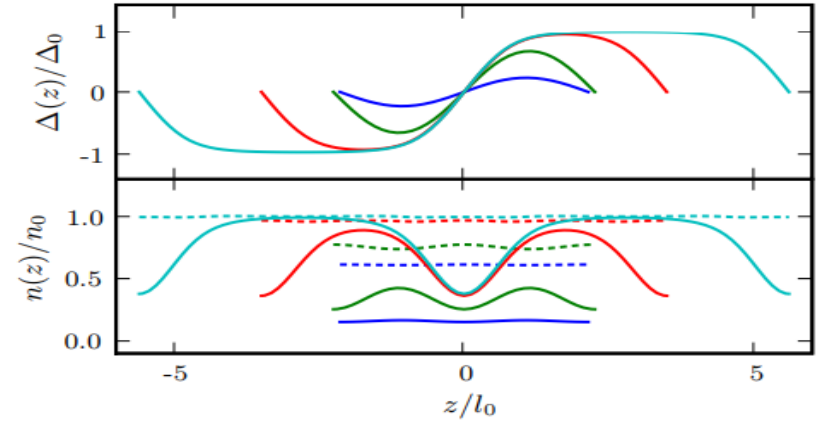
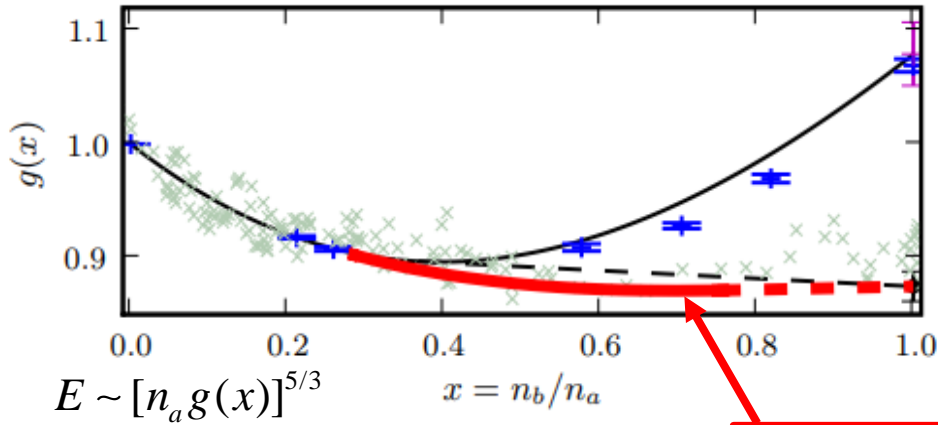
A.I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP 20, 762 (1965)

P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964)

## Mean-field studies:



Bulgac & Forbes have shown, within DFT, that Larkin-Ovchinnikov (LO) phase may exist in the unitary Fermi gas (UFG)



LO configuration – supersolid state

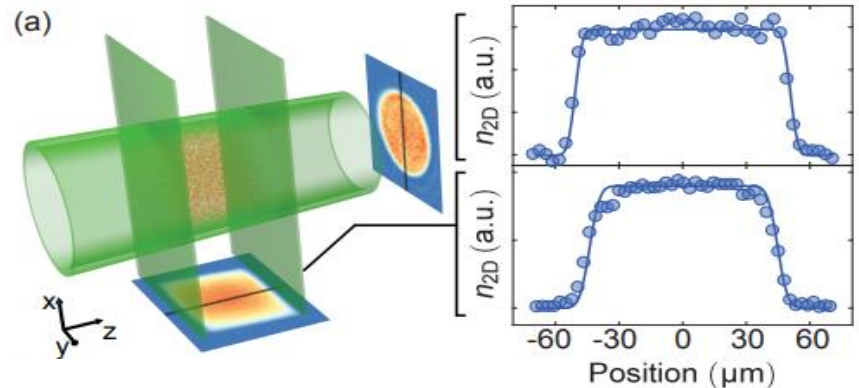
A. Bulgac, M.M.Forbes, PRL101,215301 (2008)

**The problem:**

In the trap the volume where LOFF phase may be created is relatively small .

unless →

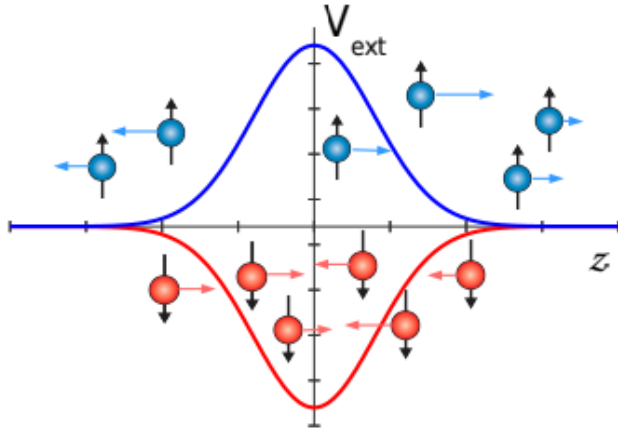
Ultracold atoms in a uniform potential



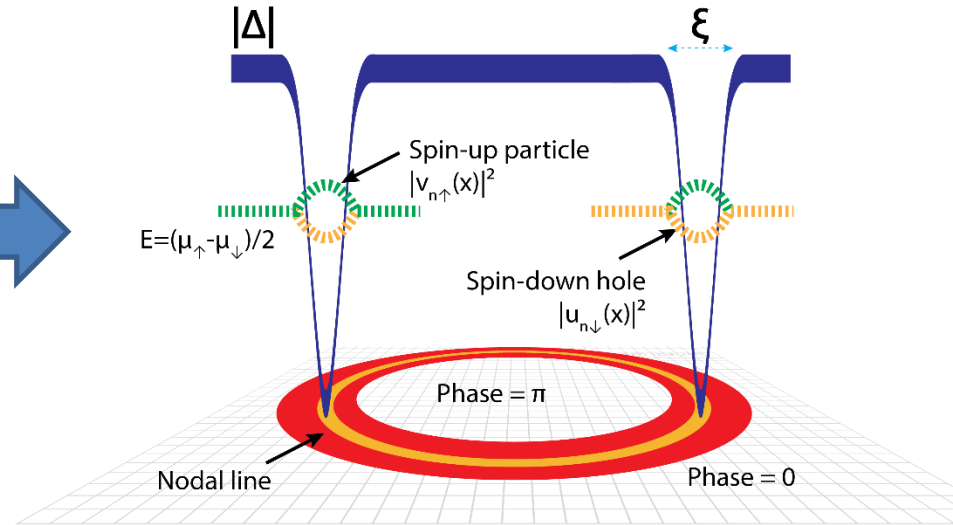
B. Mukherjee et al. Phys. Rev. Lett. 118, 123401 (2017)

# Creating Larkin-Ovchinnikov droplet (ferron) dynamically in unitary Fermi gas

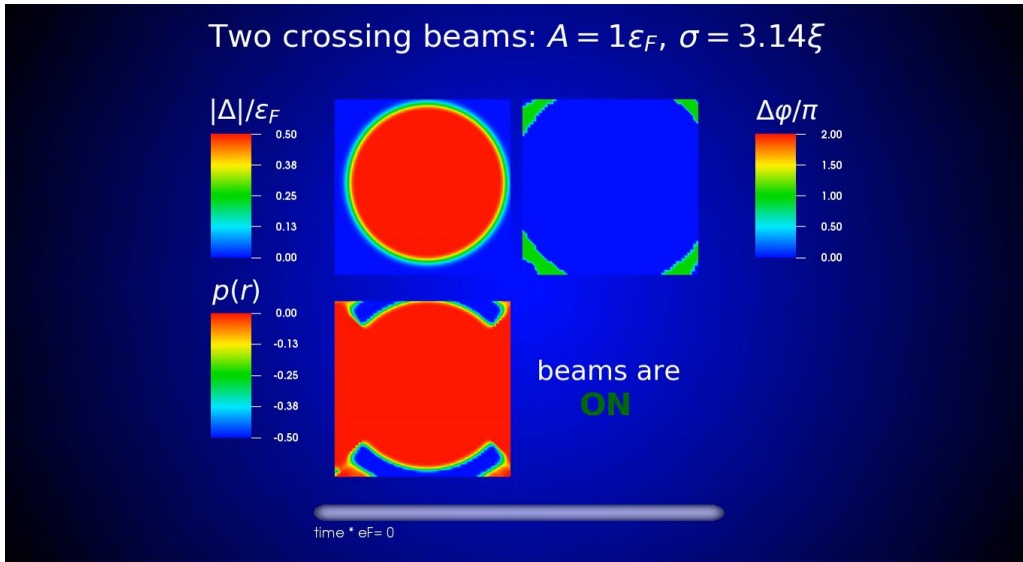
Spin-selective potential applied locally leads to Cooper pair breaking



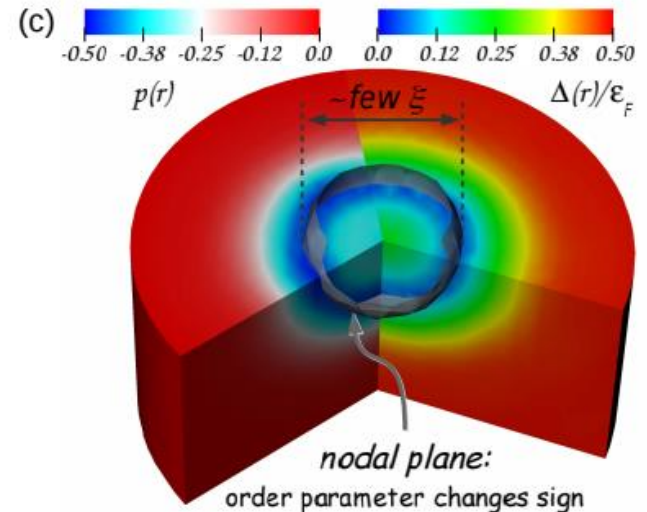
Pairing field nodal structure



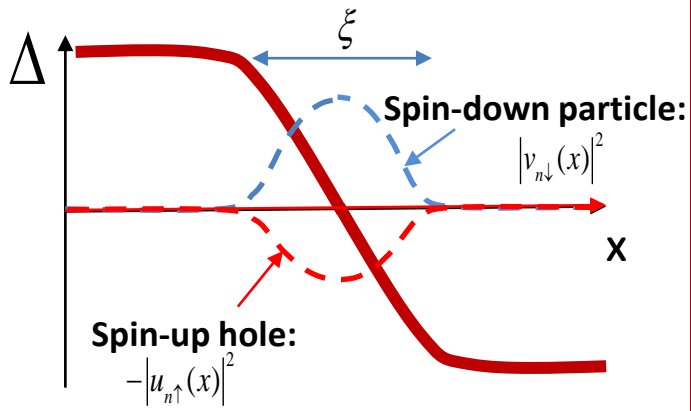
Generation of *ferron* in the unitary regime



*Ferron* structure



# Andreev states and stability of pairing nodal points

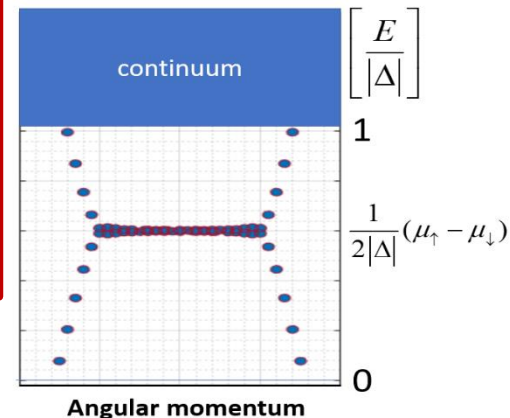


Due to quasiparticle scattering the localized Andreev states appear at the nodal point. These states induce local spin-polarization

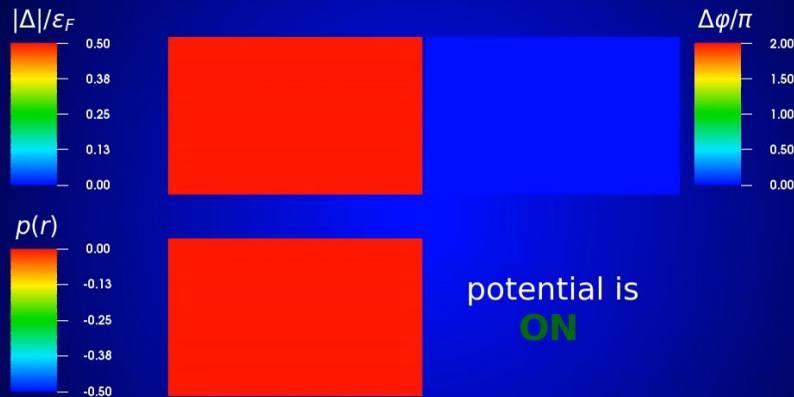
BdG in the Andreev approx. ( $\Delta \ll k_F^2$ )

$$\begin{bmatrix} -2ik_F \frac{d}{dx} & \Delta(x) \\ \Delta^*(x) & 2ik_F \frac{d}{dx} \end{bmatrix} \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix} = E_n \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix}$$

Schematic spectrum of subgap states for ferron



## Non-central collision of two impurities



time \* eF = 0

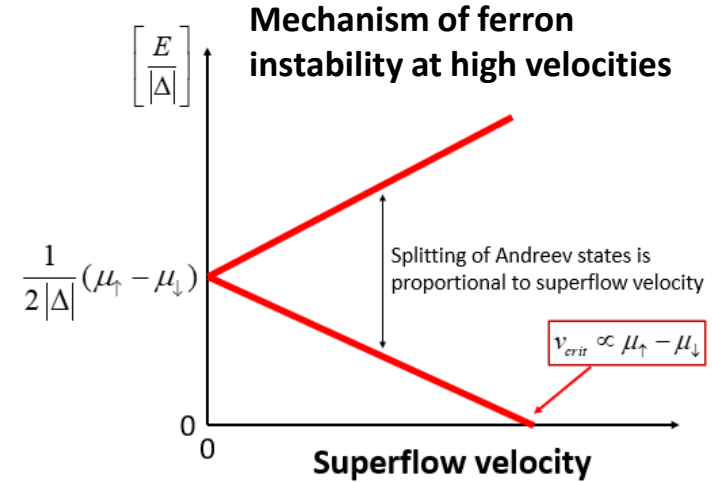
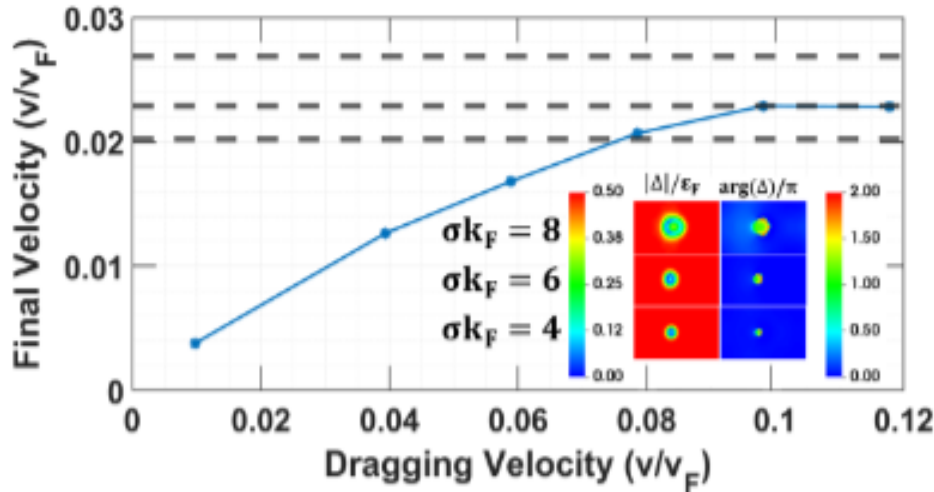
Surprisingly, the nodal structure remains stable even during collisions

P. M., B. Tüzemen, G. Wlazłowski,  
Phys. Rev. A100, 033613 (2019)

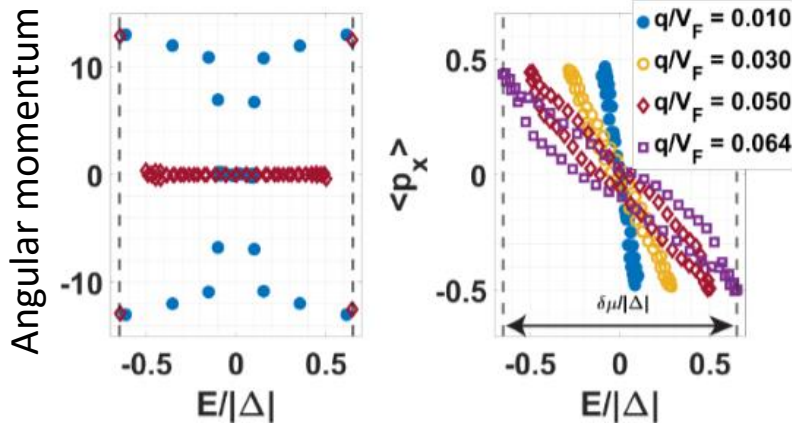
The velocities of impurities are about 30% of the velocity of sound.

# Ferron critical velocity

Peculiarity of ferron dynamics: there is a limiting velocity proportional to its polarization.

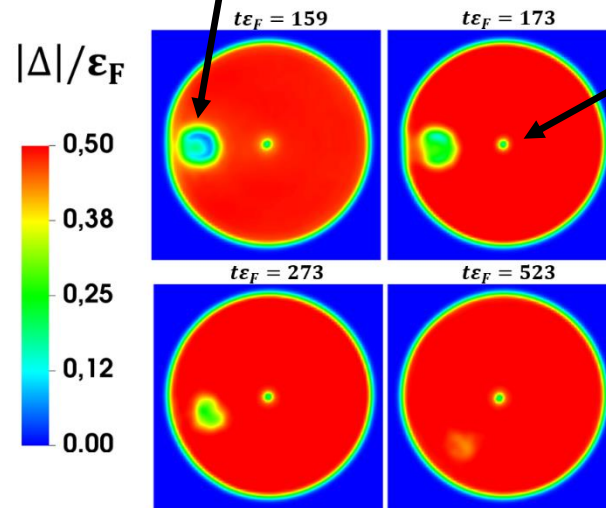


Andreev states for ferron BdG calculations in 2D



Consequence:

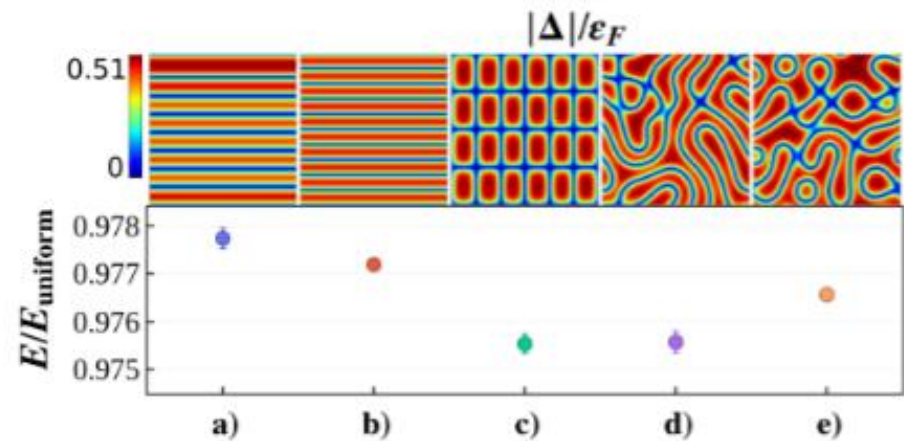
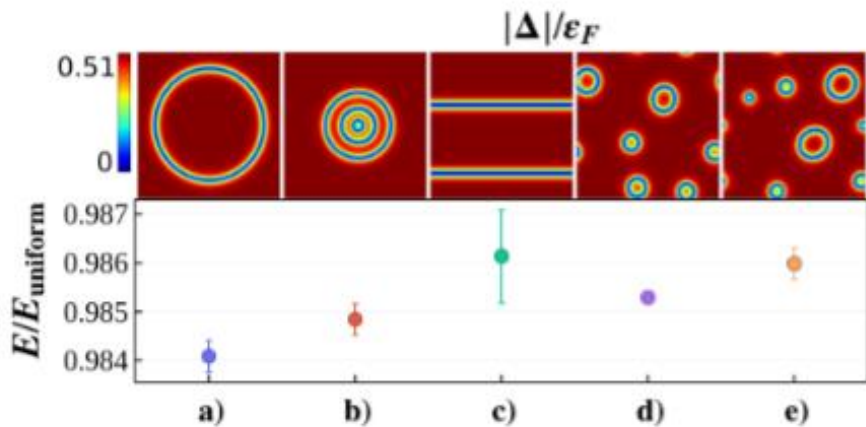
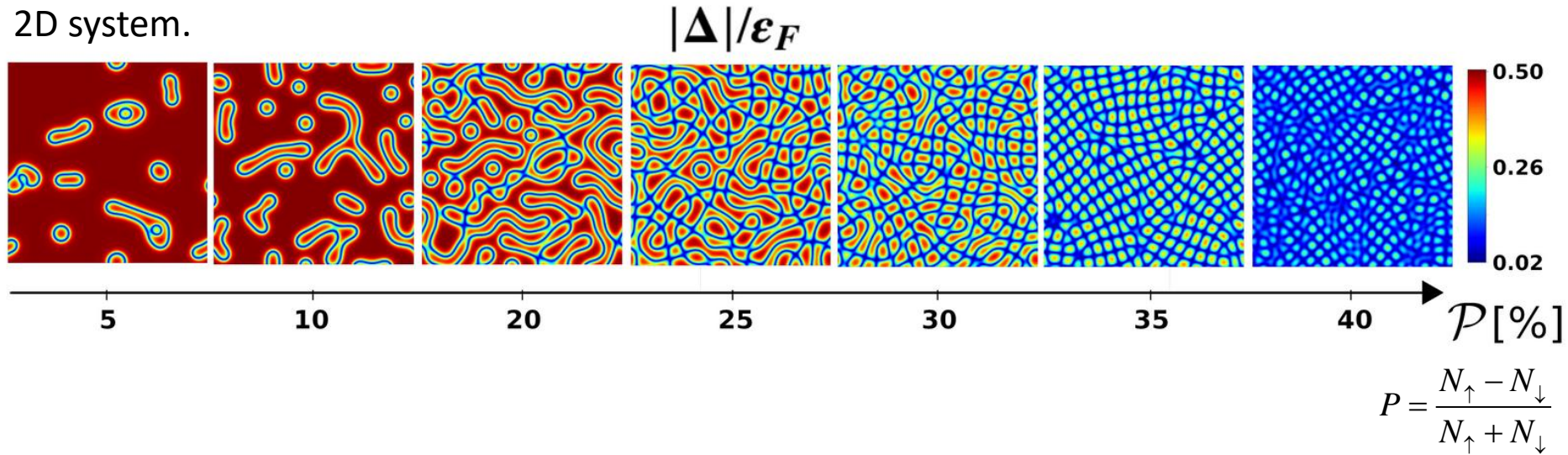
Instability of **ferron** in the vicinity of **quantum vortex**

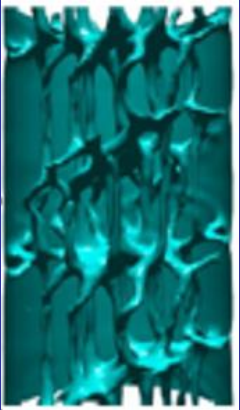


P. M., B. Tüzemen,  
G. Wlazłowski,  
Phys. Rev. A 104, 033304  
(2021)

# In search of LOFF phase: Supersolid or liquid crystal?

2D system.





## Quantum turbulence

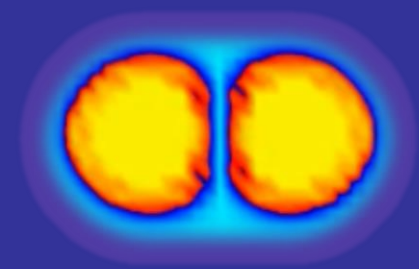
K. Hossain (WSU)  
M.M. Forbes (WSU)  
K. Kobuszewski (WUT)  
S. Sarkar (WSU)  
G. Wlazłowski (WUT)

K. Hossain et al.  
Phys. Rev. A 105, 013304 (2022)

## Vortex dynamics in neutron star crust

N. Chamel (ULB)  
D. Pęcak (WUT)  
G. Wlazłowski (WUT)

D. Pęcak et al.  
Phys. Rev. C 104, 055801 (2021)



## Nuclear collisions

M. Barton (WUT)  
A. Boulet (WUT)  
A. Makowski (WUT)  
K. Sekizawa (Tokyo I.)  
G. Wlazłowski (WUT)

P.M. et al. Phys. Rev. Lett 119  
042501 (2017)  
P.M. et al. Phys. Rev. C **105**,  
064602 (2022)



**Nonequilibrium  
superfluidity in  
Fermi systems**

## Josephson junction in atomic Fermi gases - dissipative effects

N. Proukakis (NU)  
M. Tylutki (WUT)  
G. Wlazłowski (WUT)  
K. Xhani (LENS & NU)

K. Xhani et al. (in preparation)

## Collisions of vortex-antivortex pairs

A. Barresi (WUT)  
A. Boulet (WUT)  
G. Wlazłowski (WUT)  
and LENS exp. Group

A. Boulet et al. arxiv:2201.07626  
A. Barresi et al. (in preparation)

## Spin-imbalanced Fermi gases

B. Tuzemen (WUT)  
G. Wlazłowski (WUT)  
T. Zawiślak (WUT)

P.M. et al. Phys. Rev. A100, 033613 (2019),  
Phys. Rev. A104, 033304 (2021),  
B. Tuzemen et al. (in preparation)

