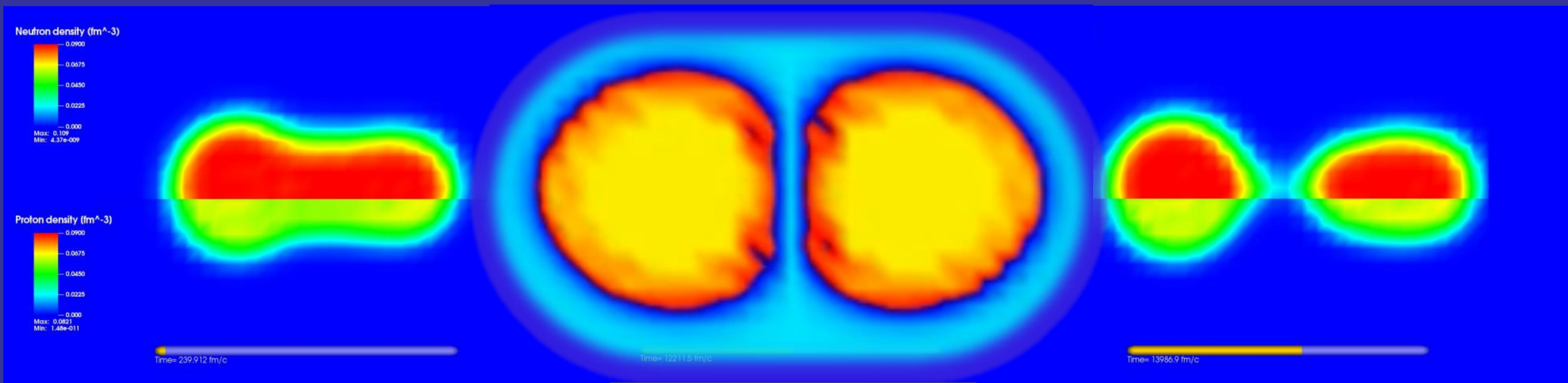
Selected aspects of pairing dynamics far from equilibrium



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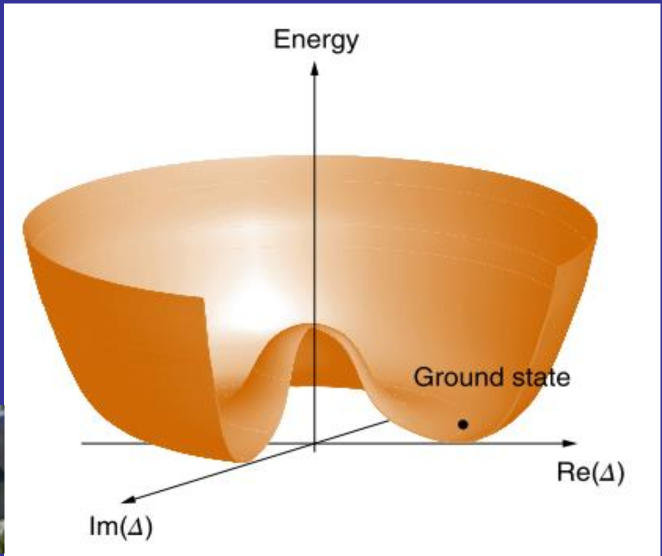
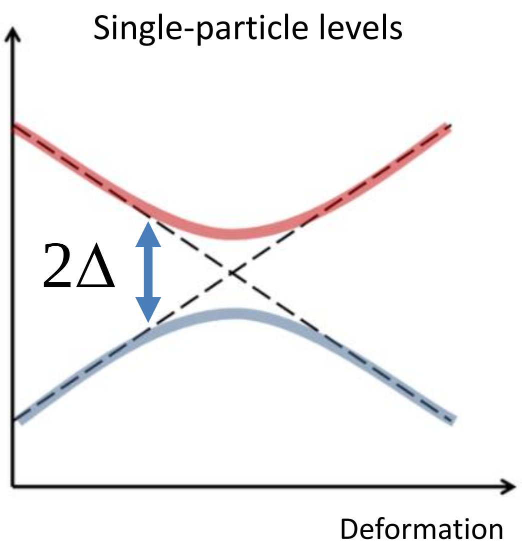
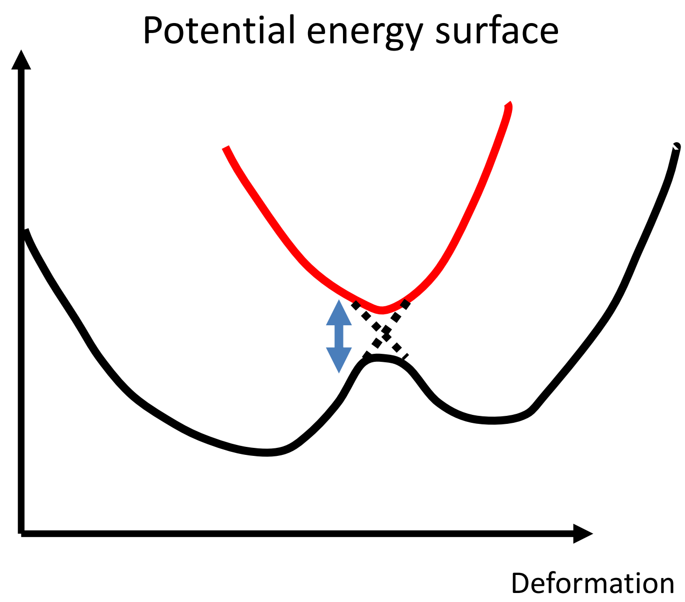


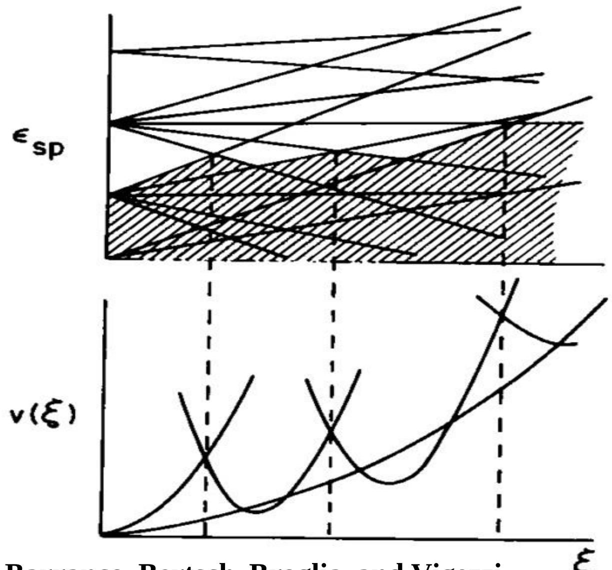
Pairing as an energy gap



Quasiparticle energy:

$$E_{qp} = \sqrt{(\varepsilon - \mu)^2 + |\Delta|^2}$$





From Barranco, Bertsch, Broglia, and Vigezzi
Nucl. Phys. A512, 253 (1990)

As a consequence of pairing correlations large amplitude nuclear motion becomes more adiabatic.

While a nucleus elongates its Fermi surface becomes oblate and its sphericity must be restored

Hill and Wheeler, PRC, 89, 1102 (1953)

Bertsch, PLB, 95, 157 (1980)

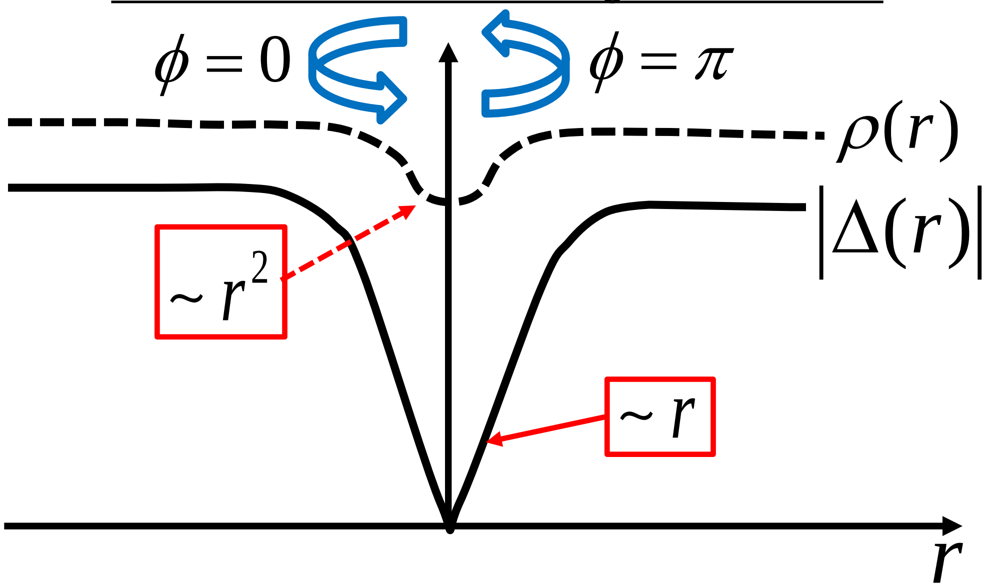
Pairing as a field

$$\Delta(\vec{r},t) = \left|\Delta(\vec{r},t)\right| e^{i\phi(\vec{r},t)}$$

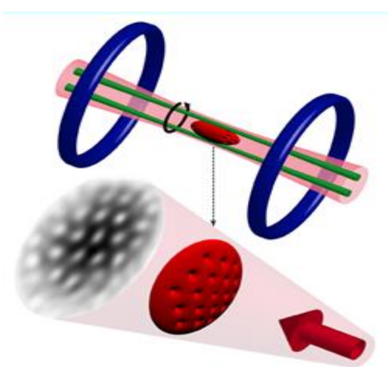
Both magnitude and phase may have a nontrivial spatial and time dependence.

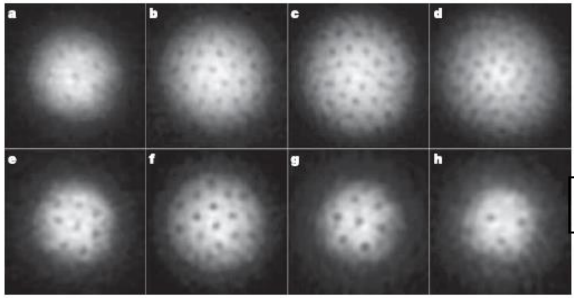
Example of a nontrivial spatial dependence: *quantum vortex*

Vortex structure – section through the vortex core



Example of a topological excitation: magnitude of the pairing gap vanishes in the vortex core.





bosons

fermions

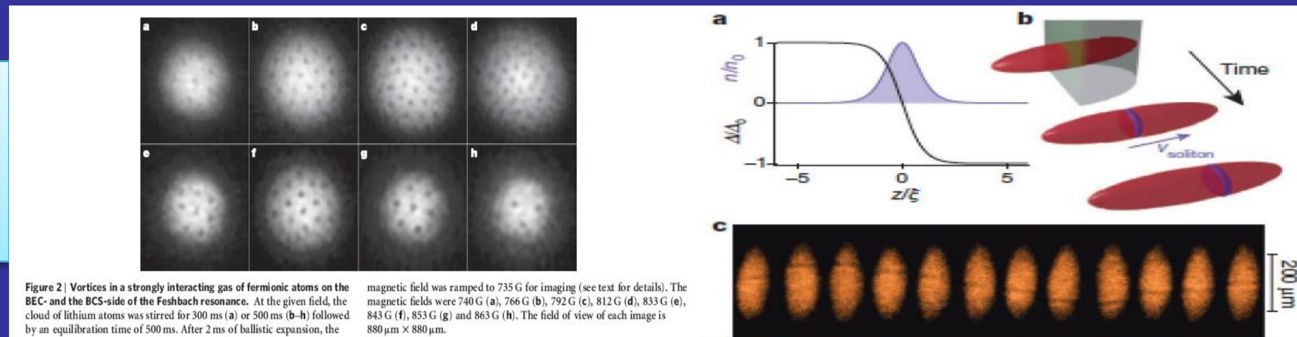
Experiments with ultracold Li-6 atoms: pictures of the vortex lattice.

Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b**–**h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**), 812 G (**d**), 833 G (**e**), 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of view of each image is 880 μm × 880 μm.

M.W. Zwierlein *et al.*, Nature, 435, 1047 (2005)

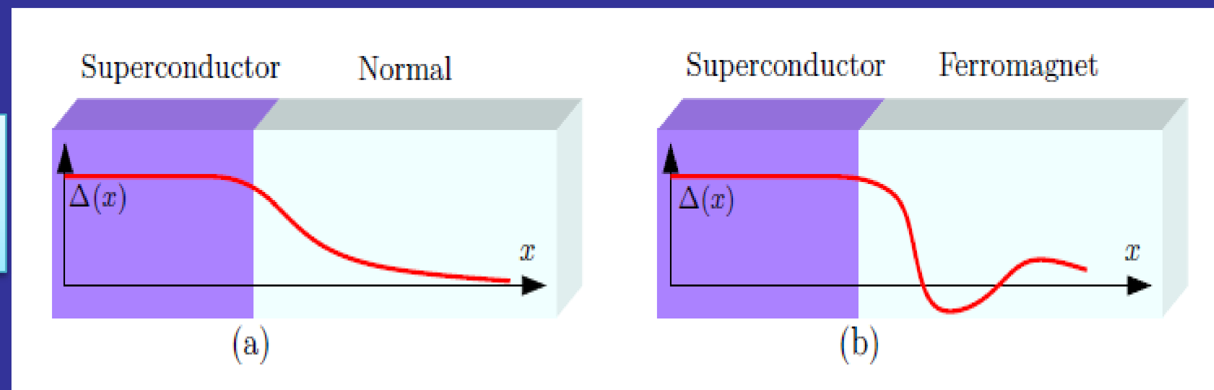
The well known effects in superconductors where the simplified BCS approach fails

1) Quantum vortices, solitonic excitations related to pairing field (e.g. domain walls)

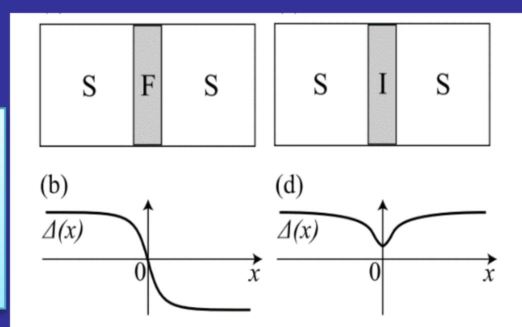


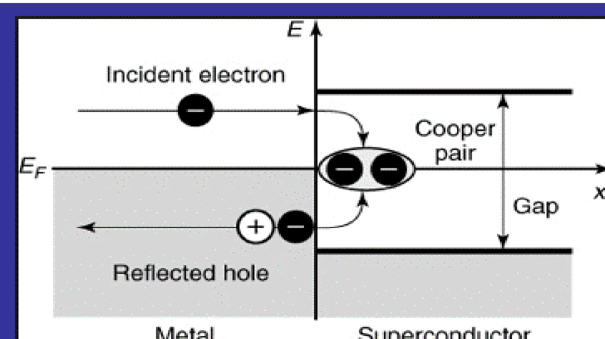
2) Bogoliubov – Anderson phonons

3) proximity effects: variations of the pairing field on the length scale of the coherence length.



4) physics of Josephson junction (superfluid - normal metal), pi-Josephson junction (superfluid - ferromagnet)





5) Andreev reflection (particle-into-hole and hole-into-particle scattering) Andreev states cannot be obtained within BCS

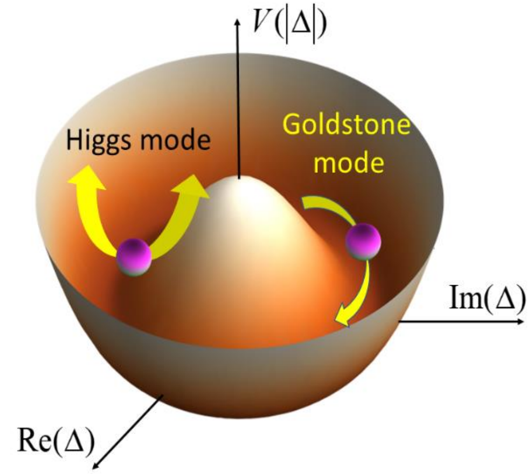
$$\Delta(\vec{r},t) = |\Delta(\vec{r},t)| e^{i\phi(\vec{r},t)}$$

Appearance of pairing field in Fermi systems is associated with U(1) symmetry breaking.

There are two characteristic modes associated with the field $\Delta(\vec{r},t)$

1) **Nambu-Goldstone mode** explores the degree of freedom associated with the phase: $\phi(\vec{r},t)$

2) **Higgs mode** explores the degree of freedom associated with the magnitude: $|\Delta(\vec{r},t)|$



What's the difference between pairing correlations and existence of superfluid phase?

- Superfluid phase exists if the *off-diagonal long range order* is present:

$$\lim_{|r-r'|\to\infty} \langle \psi^+_\uparrow(r)\psi^+_\downarrow(r')\psi_\downarrow(r')\psi_\uparrow(r)\rangle \neq 0$$

C.N. Yang, Rev. Mod. Phys. 34, 694 (1962)

- This limit is unreachable in atomic nuclei due to their finite size. Therefore it is more convenient to look instead for the manifestations of the phase: $\Delta(\vec{r},t) = |\Delta(\vec{r},t)| e^{i\phi(\vec{r},t)}$

Note: whenever I mean theory I mean: time dependent HFB (TDHFB) or time dependent Density Functional Theory (TDDFT) with local pairing field.

Pairing correlations in time-dependent DFT (with local pairing field)

$$S = \int_{t_0}^{t_1} \left(\left\langle 0(t) \middle| i\frac{d}{dt} \middle| 0(t) \right\rangle - E[\rho(t), \chi(t)] \right) dt$$

Stationarity requirement produces the set of equations (TDHFB eq.):

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} U_\mu(\mathbf{r},t) \\ V_\mu(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r},t) & \Delta(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & -h^*(\mathbf{r},t) \end{pmatrix} \begin{pmatrix} U_\mu(\mathbf{r},t) \\ V_\mu(\mathbf{r},t) \end{pmatrix},$$

$$B(t) = \begin{pmatrix} U(t) & V^*(t) \\ V(t) & U^*(t) \end{pmatrix} = \exp[iG(t)] \qquad G(t) = \begin{pmatrix} h(t) & \Delta(t) \\ \Delta^\dagger(t) & -h^*(t) \end{pmatrix}$$

Orthogonality and completeness has to be fulfilled: $B^\dagger(t)B(t) = B(t)B^\dagger(t) = I,$

In order to fulfill the completenes relation of Bogoliubov transform all states need to be evolved!
Otherwise Pauli principle is violated, i.e. the evolved densities do not describe a fermionic system (spurious bosonic effects are introduced).

Consequence: the computational cost increases considerably.

Solving time-dependent problem for superfluids...

The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations

$$h \sim f_1(n,\nu,\ldots)\nabla^2 + \boldsymbol{f}_2(n,\nu,\ldots)\cdot\nabla + f_3(n,\nu,\ldots)$$

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t)\\ u_{n,b}(\boldsymbol{r},t)\\ v_{n,a}(\boldsymbol{r},t)\\ v_{n,b}(\boldsymbol{r},t)\end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) & 0 & 0 & \Delta(\boldsymbol{r},t)\\ 0 & h_b(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0\\ 0 & -\Delta^*(\boldsymbol{r},t) & -h_a^*(\boldsymbol{r},t) & 0\\ \Delta^*(\boldsymbol{r},t) & 0 & 0 & -h_b^*(\boldsymbol{r},t)\end{pmatrix}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t)\\ u_{n,b}(\boldsymbol{r},t)\\ v_{n,a}(\boldsymbol{r},t)\\ v_{n,b}(\boldsymbol{r},t)\end{pmatrix}$$

We explicitly track fermionic degrees of freedom!

where *h* and Δ depends on “densities”:

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r},t)|^2, \qquad \tau_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r},t)|^2,$$

$$\chi_c(r,t) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r},t)v^*_{n,\downarrow}(\boldsymbol{r},t), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r},t)\nabla v_{n,\sigma}(\boldsymbol{r},t)],$$

$$\Delta(\mathbf{r}) = g_{eff}(\mathbf{r})\chi_c(\mathbf{r})$$

$$\frac{1}{g_{eff}(\mathbf{r})} = \frac{1}{g(\mathbf{r})} - \frac{mk_c(\mathbf{r})}{2\pi^2\hbar^2}\left(1 - \frac{k_F(\mathbf{r})}{2k_c(\mathbf{r})}\ln\frac{k_c(\mathbf{r})+k_F(\mathbf{r})}{k_c(\mathbf{r})-k_F(\mathbf{r})}\right)$$

A. Bulgac, Y. Yu, Phys. Rev. Lett. 88 (2002) 042504
A. Bulgac, Phys. Rev. C65 (2002) 051305

huge number of nonlinear coupled 3D Partial Differential Equations
(in practice n=1,2,…, 10^5 - 10^6)

- P. Magierski, *Nuclear Reactions and Superfluid Time Dependent Density Functional Theory*, Frontiers in Nuclear and Particle Physics, vol. 2, 57 (2019)
- A. Bulgac, *Time-Dependent Density Functional Theory and Real-Time Dynamics of Fermi Superfluids*, Ann. Rev. Nucl. Part. Sci. 63, 97 (2013)
- A. Bulgac, M.M. Forbes, P. Magierski, Lecture Notes in Physics, Vol. 836, Chap. 9, p.305-373 (2012)

Present computing capabilities:

- full 3D (unconstrained) superfluid dynamics
- spatial mesh up to 100^3
- max. number of particles of the order of 10^4
- up to 10^6 time steps

(for cold atomic systems – time scale: a few ms
for nuclei – time scale: 100 zs)

Advantages of TDDFT for nuclear reactions

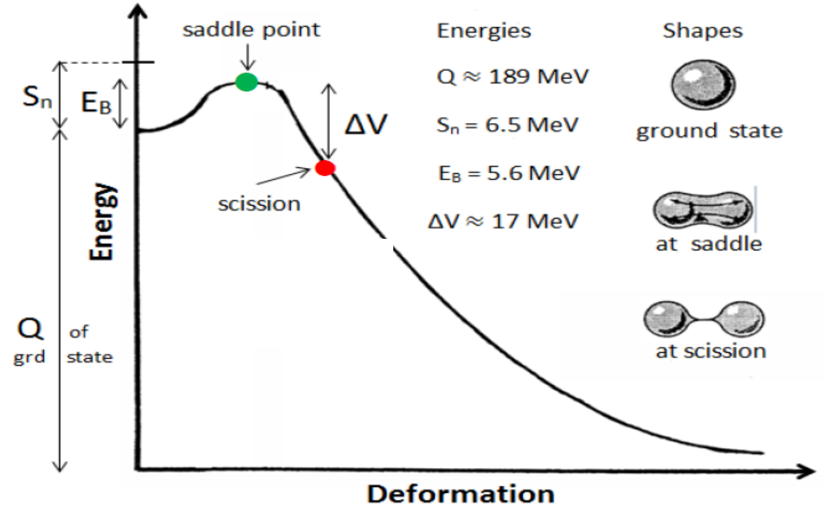
- The same framework describes various limits: eg. linear and highly nonlinear regimes, adiabatic and nonadiabatic (dynamics far from equilibrium).
- Interaction with basically any external probe (weak or strong) easy to implement.
- TDDFT does not require introduction of hard-to-define collective degrees of freedom and there are no ambiguities arising from defining potential energy surfaces and inertias.
- One-body dissipation, the window and wall dissipation mechanisms are automatically incorporated into the theoretical framework.
- All shapes are allowed and the nucleus chooses dynamically the path in the shape space, the forces acting on nucleons are determined by the nucleon distributions and velocities, and the nuclear system naturally and smoothly evolves into separated fission fragments.
- There is no need to introduce such unnatural quantum mechanical concepts as "rupture" and there is no worry about how to define the scission configuration.

Induced nuclear fission dynamics – example of slow and nonadiabatic motion

Estimation of characteristic time scales for low energy fission (<10MeV):

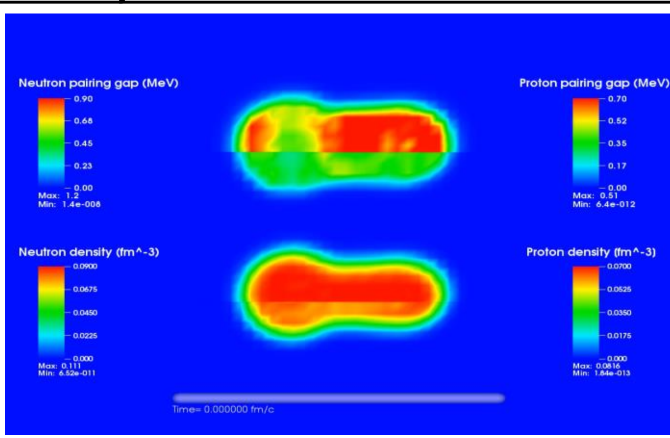
Ground state to saddle	**-**	**1 000 000 zs**
Saddle to scission	**-**	**10-100 zs**
Acceleration of fission fragments to 90% of their final velocity	**-**	**10 zs**
Neutron evaporation	**-**	**1 000 zs**

Potential energy versus deformation



From F. Gonnenwein FIESTA2014

<u>Fission dynamics of ^{240}*Pu within TDSLDA*</u>



Total kinetic energy of the fragments

E^* (MeV)	E_n (MeV)	TKE_{TDSLDA} (MeV)	TKE_{syst} (MeV)	err (%)	Z_L	N_L
8.08	1.542	173	177.26	1.95	40.825	62.246
9.60	3.063	174	176.73	1.13	40.500	61.536
10.10	3.560	179	176.56	1.43	41.625	62.783
10.57	4.032	173	176.39	1.55	40.092	61.256
10.58	4.043	173	176.39	1.70	40.146	61.388
10.58	4.047	175	176.39	0.72	40.313	61.475
10.60	4.065	174	176.38	0.92	40.904	62.611
11.07	4.534	176	176.22	0.14	41.495	63.134
11.56	5.024	175	176.05	0.51	40.565	61.894
12.05	5.515	176	175.88	0.49	40.412	61.809
12.15	5.610	176	175.84	0.29	40.355	61.695
12.16	5.626	176	175.84	0.15	41.386	62.764

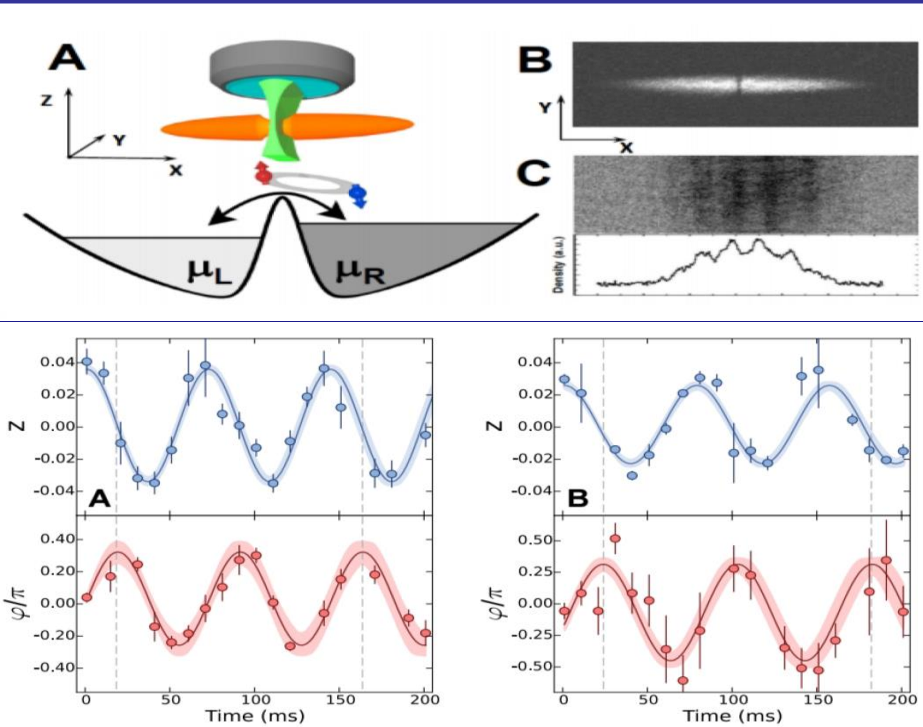
Calculated TKEs reproduce experimental data with accuracy < 2%

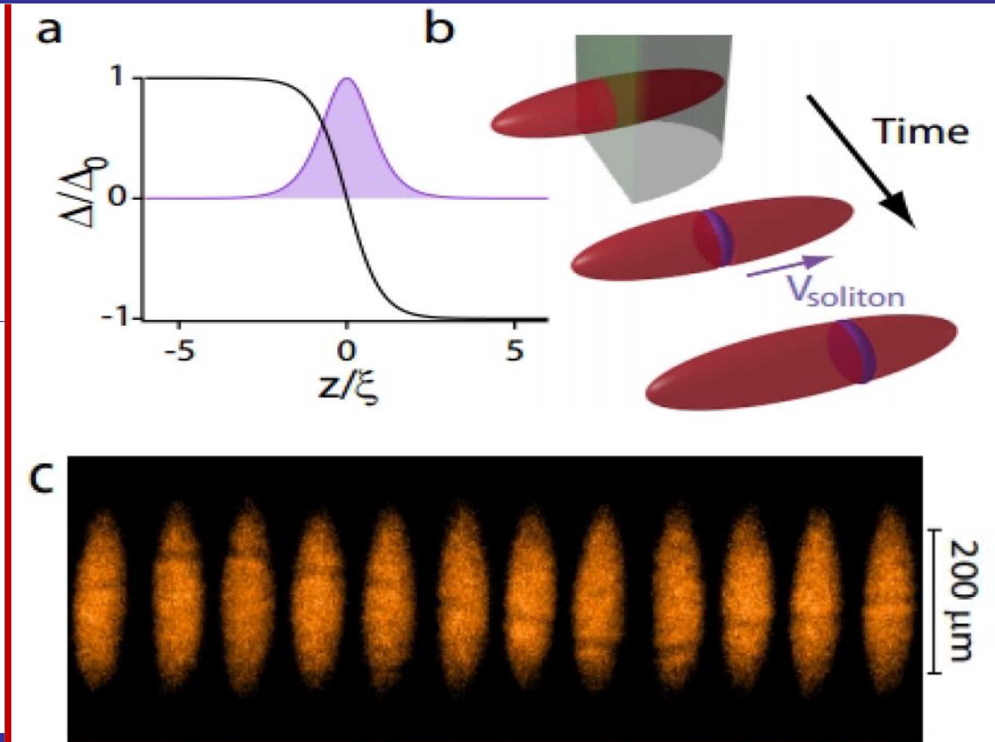
A. Bulgac, P.Magierski, K.J. Roche, and I. Stetcu, Phys. Rev. Lett. 116, 122504 (2016)

Two regimes for phase-induced effects in fermionic superfluids

Weak coupling (weak link)

Strong coupling





Observation of ***AC Josephson effect*** between two 6Li atomic clouds.

G. Valtolina et al., Science 350, 1505 (2015).

Creation of a „**heavy soliton**" after merging two superfluid atomic clouds.

T. Yefsah et al., Nature 499, 426 (2013);

M.J.H. Ku et al., Phys. Rev. Lett. 116, 045304 (2016)

Superflow is accompanied with creation of topological excitations (vortices) leading to energy dissipation.

G. Wlazłowski, K. Xhani, M. Tylutki, N.P. Proukakis, P. Magierski, Phys. Rev. Lett. **130**, 023003 (2023)

„Heavy soliton" decays through the unique sequence of topological excitations.

G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. Magierski, Phys. Rev. Lett. **120**, 253002 (2018)

Nuclear systems

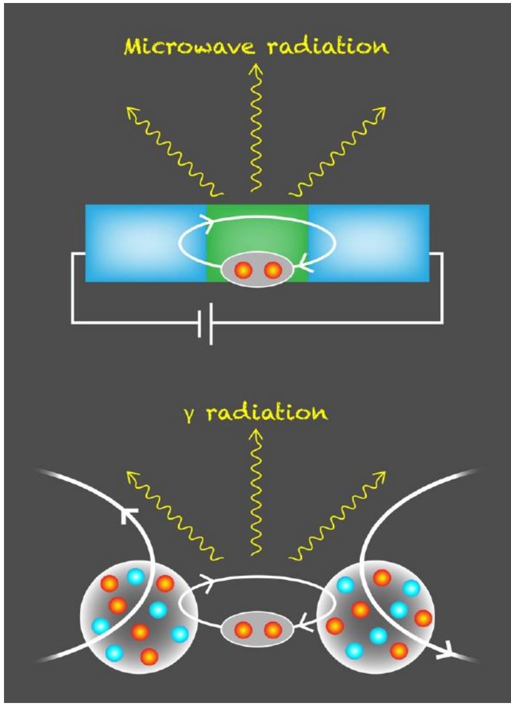
Some evidence for a nuclear **DC Josephson effect** has been gathered over the years, following ideas presented in papers:
V.I. Gol'danskii, A.I. Larkin, JETP 26, 617 (1968), K. Dietrich, Phys. Lett. 32B 428 (1970)

Experimental evidence of enhanced nucleon pair transfer reported eg. in:
M.C. Mermaz, Phys. Rev. C36 1192, (1987), M.C. Mermaz, M. Girod, Phys. Rev. C53 1819 (1996)

Surprisingly evidence for AC Josephson effect has also been found in collision: $^{116}Sn+^{60}Ni$ **at 140.60 MeV < Ecm < 167.95 MeV**

G.Potel, F.Barranco, E.Vigezzi, R.A. Broglia, "*Quantum entanglement in nuclear Cooper-pair tunneling with gamma rays*," Phys.Rev. C103, L021601 (2021)
R. Broglia, F. Barranco, G. Potel, E. Vigezzi
„*Transient Weak Links between Superconducting Nuclei: Coherence Length*"
Nuclear Physics News 31, 25 (2021)

Need for other examples if the effect really exists.



From P. Magierski, *Physics* 14 (2021) 27.

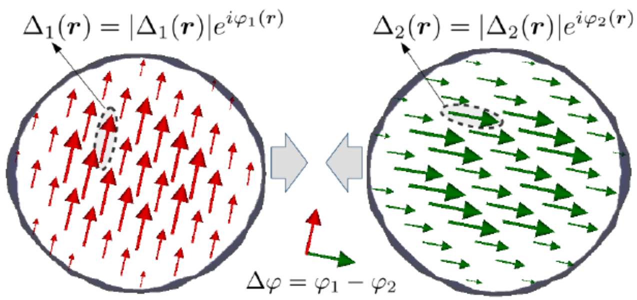
„Heavy soliton" creation in nuclear collision

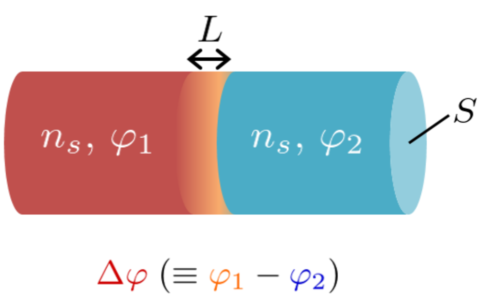
Collisions of superfluid nuclei having <u>different phases</u> of the <u>pairing fields</u>

<u>The main questions are:</u>

-how a possible solitonic structure can be manifested in nuclear system?
-what observable effect it may have on heavy ion reaction:
kinetic energy distribution of fragments, capture cross section, etc.?

Clearly, we cannot control phases of the pairing field in nuclear experiments and the possible signal need to be extracted after averaging over the phase difference.



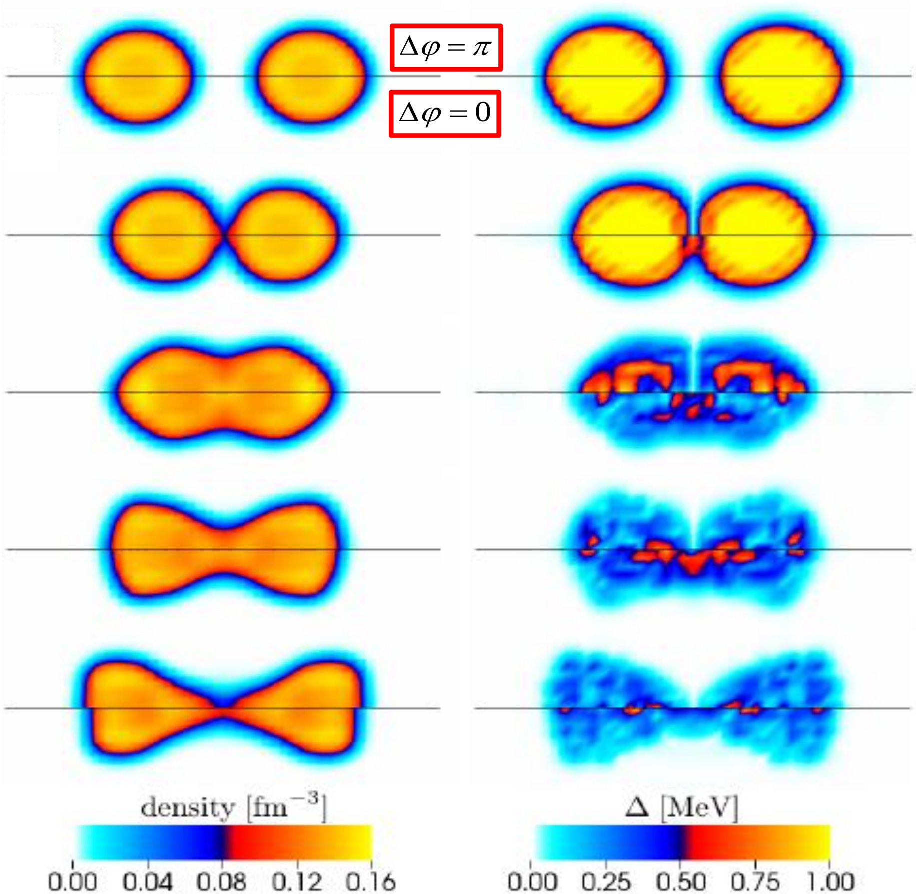


From Ginzburg-Landau (G-L) approach:

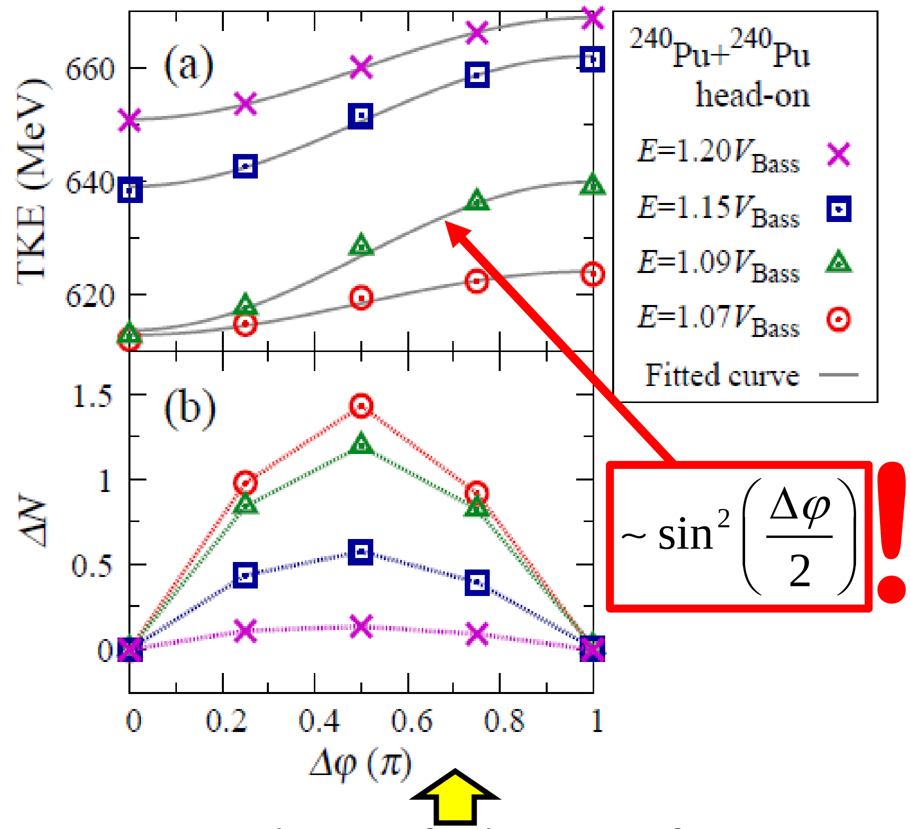
$$E_j = \frac{S}{L}\frac{\hbar^2}{2m} n_s \sin^2\frac{\Delta\varphi}{2}$$

For typical values characteristic for two medium nuclei: $E_j \approx 30 MeV$

^{240}Pu+^{240}Pu



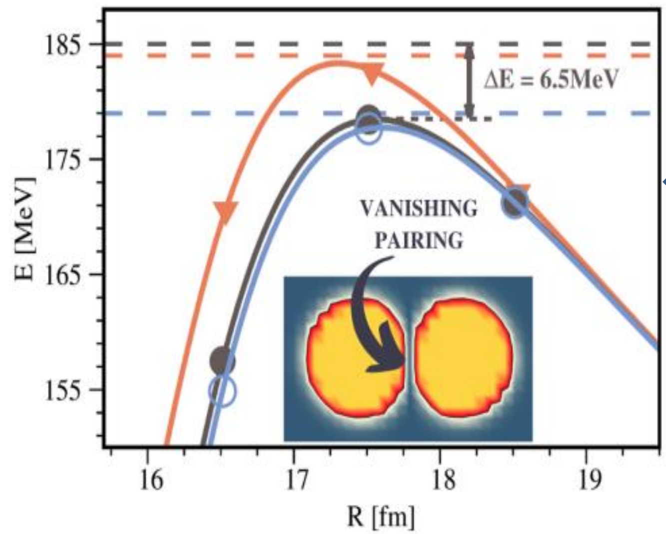
Total kinetic energy of the fragments (TKE)



Average particle transfer between fragments.

Creation of the solitonic structure between colliding nuclei prevents energy transfer to internal degrees of freedom and consequently enhances the kinetic energy of outgoing fragments.
Surprisingly, the gauge angle dependence from the G-L approach is perfectly well reproduced in the kinetic energies of outgoing fragments!

P. Magierski, K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119 042501 (2017)



Dynamic nature of the effect:

Solid lines: static barrier between two nuclei (with pairing included):
90Zr+90Zr - brown
96Zr+96Zr - black (0-phase diff.) and
blue (Pi-phase diff.)

Static barriers are practically insensitive to the phase difference of pairing fields.

Dashed lines: Actual threshold for capture obtained in dynamic calculations.
Hence ΔE measures the additional energy which has to be added to the system to merge nuclei.

TABLE I: The minimum energies needed for capture in ^{90}Zr+^{90}Zr and ^{96}Zr+^{96}Zr for the case of $\Delta\phi = 0$ [$E_{\text{thresh}}(0)$] and $\Delta\phi = \pi$ [$E_{\text{thresh}}(\pi)$]. The energy difference between the two cases is shown in the last column. The average pairing gap $\overline{\Delta}_i$ is defined by Eq. (4).

	$\overline{\Delta}_q$ (MeV)	$E_{\text{thresh}}(0)$ (MeV)	$E_{\text{thresh}}(\pi)$ (MeV)	ΔE_s
^{90}Zr	$\overline{\Delta}_n = 0.00$ $\overline{\Delta}_p = 0.09$	184	184	0
^{96}Zr	$\overline{\Delta}_n = 1.98$ $\overline{\Delta}_p = 0.32$	179	185	6
	$\overline{\Delta}_n = 2.44$ $\overline{\Delta}_p = 0.33$	178	187	9
	$\overline{\Delta}_n = 2.94$ $\overline{\Delta}_p = 0.34$	178	187	9

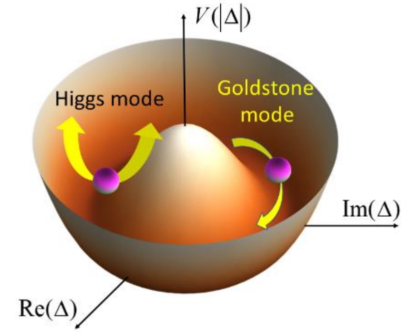
Dependence of the additional energy on pairing gap in colliding nuclei

P. Magierski, A. Makowski, M. Barton, K. Sekizawa, G. Wlazłowski, Phys. Rev. C 105, 064602, (2022)

G. Scamps, Phys. Rev. C 97, 044611 (2018): ***barrier fluctuations extracted from experimental data provide evidence that the effect exists.***

For details concerning dynamics and shape evolution see today's presentation of A. Makowski and poster

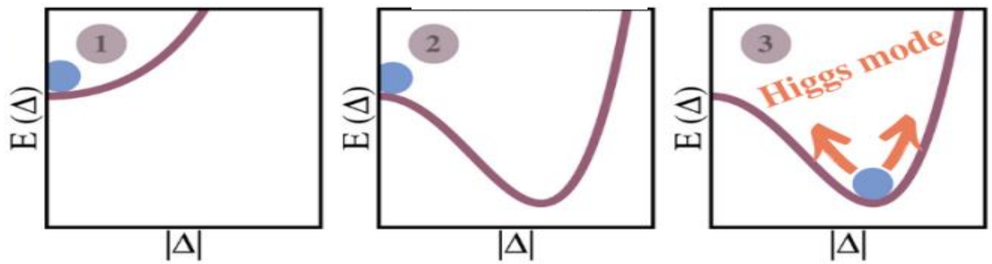
Pairing Higgs mode



Let's consider Fermi gas with schematic pairing interaction and coupling constant depending on time:

$$\hat{H} = \sum_k \varepsilon_k \hat{\psi}_k^+ \hat{\psi}_k - g(t) \sum_{k,l>0} \hat{\psi}_k^+ \hat{\psi}_{\bar{k}}^+ \hat{\psi}_{\bar{l}} \hat{\psi}_l$$

$g(t) = g_0 \theta(t)$ coupling constant is switched on withing time scale much shorter than $\hbar / \varepsilon_F$



As a result pairing becomes unstable and increases exponentially $\Delta(t) \propto e^{-i\zeta t} = e^{-i\omega t} e^{\gamma t}$

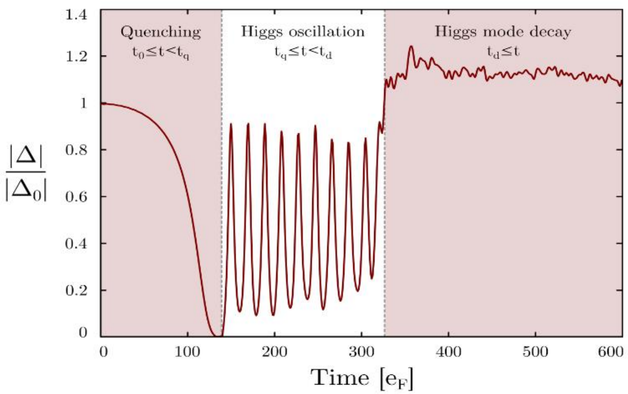
$$\frac{1}{g_0} = \sum_{k>0,\varepsilon_k>\mu} \frac{\tanh\left(\frac{\beta|\varepsilon_k-\mu|}{2}\right)}{2|\varepsilon_k-\mu|+\zeta} + \sum_{k>0,\varepsilon_k<\mu} \frac{\tanh\left(\frac{\beta|\varepsilon_k-\mu|}{2}\right)}{2|\varepsilon_k-\mu|-\zeta}$$

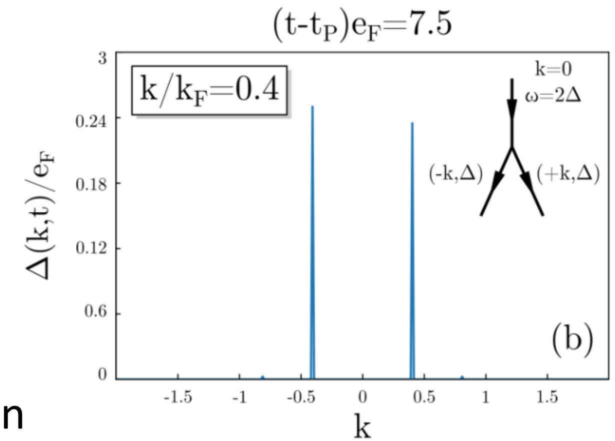
Time scale of growth and the period of subsequent oscillation is related to static value of pairing Δ_0 :

$$\tau = \frac{1}{\gamma} \approx \frac{\hbar}{\Delta_0}$$

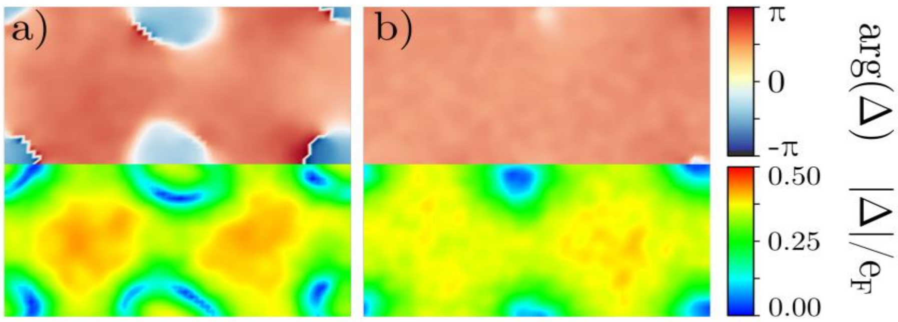
Contrary to low-energy Goldstone modes Higgs modes are unstable and decay.

M. Dzero, E. A. Yuzbashyan, and B. L. Altshuler, EPL 85, 20004 (2009)





Leading eventually to inhomogenous pairing field distribution

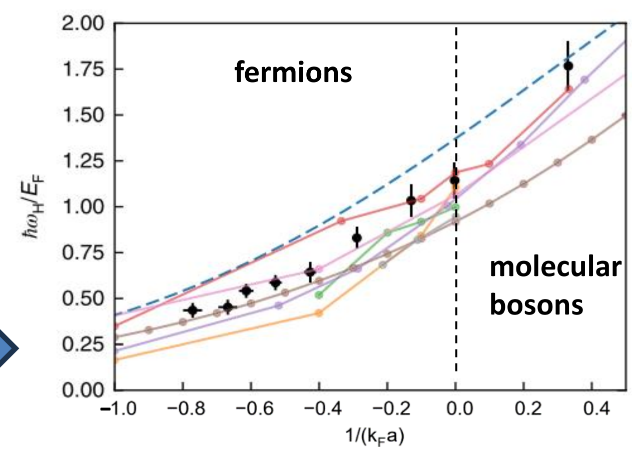


A. Barresi, A. Boulet, G. Wlazłowski, P. Magierski, Sci. Rep. 13, 11285 (2023)

In ultracold atomic gases one can induce Higgs mode by varying coupling constant.

A. Behrle et al.
Higgs mode in a strongly interacting fermionic Superfluid, Nature Physics **14**, 781 (2018).
Li-6 atoms in harmonic trap

Measured peak position of the energy absorption spectra (black dots) and theory predictions for Higgs mode.



Pairing instability in nuclear reaction

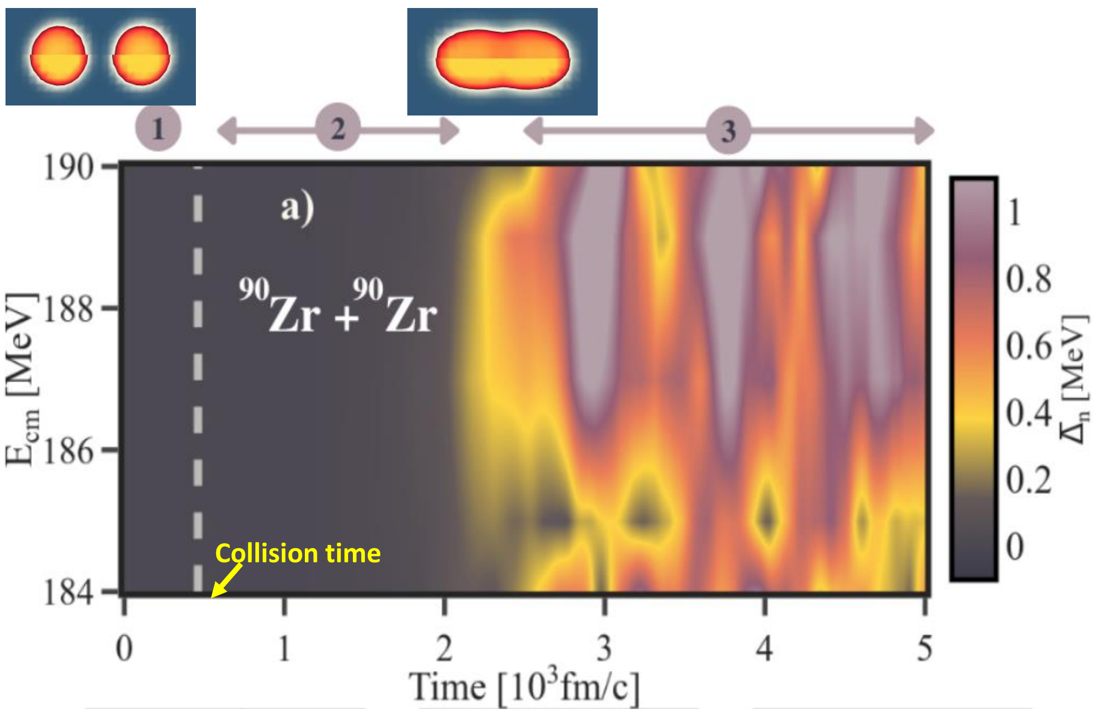
$$\Delta = \frac{8}{e^2}\varepsilon_F \exp\left(\frac{-2}{gN(\varepsilon_F)}\right)$$ - BCS formula – weak coupling limit

ε_F - Fermi energy

g - Pairing coupling constant

$N(\varepsilon_F)$ - Density of states at the Fermi level

Although one cannot change coupling constant in atomic nuclei one may affect ***density of states at the Fermi surface and consequently trigger pairing instability.***



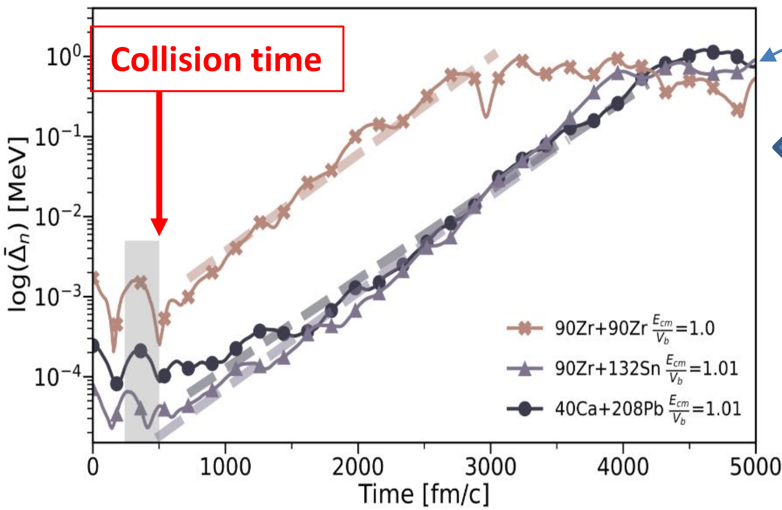
Collision of two neutron magic systems creates an elongated di-nuclear system.

Within 1500 fm/c pairing is enhanced in the system and reveals oscillations with frequency:

$$\Delta < \hbar\omega < 2\Delta$$

P. Magierski, A. Makowski, M. Barton, K. Sekizawa, G. Wlazłowski, Phys. Rev. C 105, 064602, (2022)

Interestingly the effect is generic and occurs for various collisions of magic nuclei.



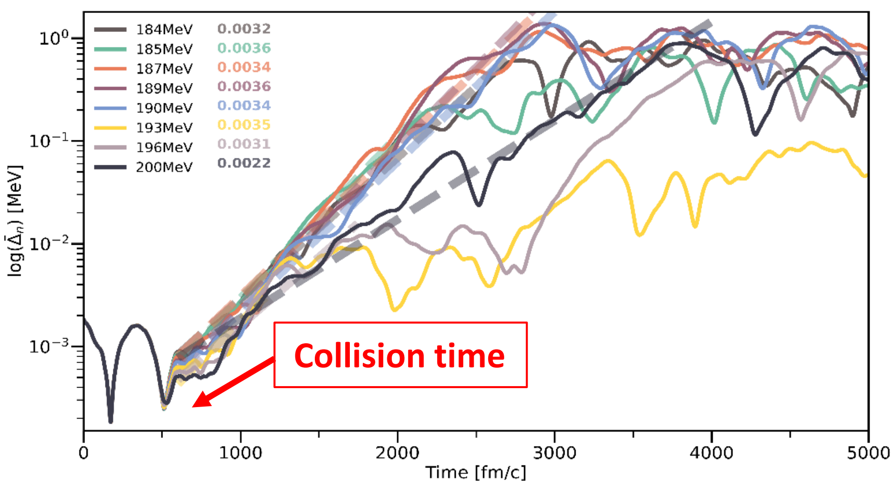
Exponential increase of pairing gap after collision indicating **pairing instability** in di-nuclear system.

Time scale of pairing enhancement:

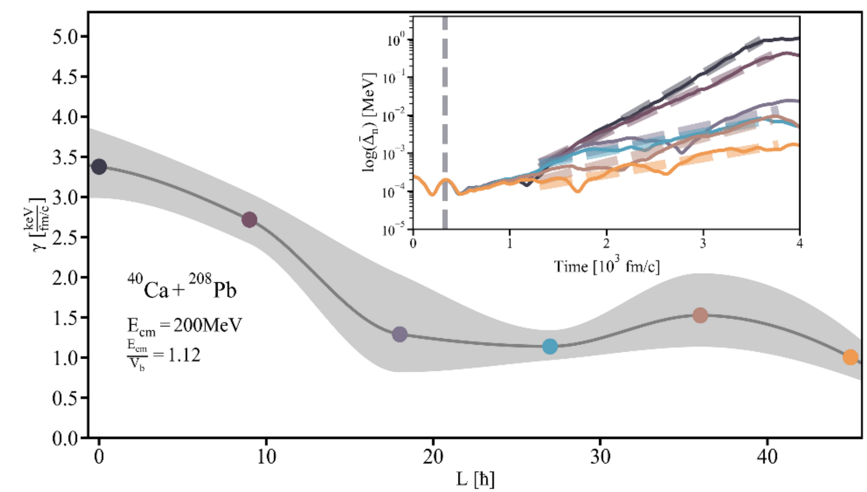
$$\tau \gg \frac{\hbar}{\Delta_0}$$

It occurs up to relatively high collision energies

$^{90}Zr + {}^{90}Zr$ head-on collision as a function of Ecm



Non-central collision



The excitation energy of a compound system after merging exceeds **20-30 MeV**.

It corresponds to temperatures **close to critical temperature for superfluid-to-normal transition.**

Therefore it is unlikely that the system develops superfluid phase and it is rather nonequilibrium enhancement of pairing correlations.

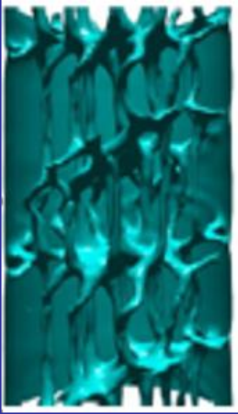
Summary and open questions

- TDHFB provides evidence for nontrivial behavior of **pairing correlations in highly nonequilibrium conditions** which includes **solitonic excitations** (dynamic barrier modification for capture) and **pairing enhancement** as a result of collision**.**

- There is certain experimental evidence for **solitonic excitations**, although not easy to extract (G. Scamps, Phys. Rev. C C 97, 044611 (2018)).

- **Pairing enhancement** in collision of magic nuclei is **a generic feature of TDHFB** appearing in collisions of magic nuclei at energies close to the Coulomb barrier.

- Impact of **pairing enhancement** on dynamics is unknown and requires more theoretical effort: investigation of noncentral collisions, considerations of pairing correlations during subsequent stages of compound nucleus formation.



Quantum turbulence

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Vortex dynamics in neutron star crust

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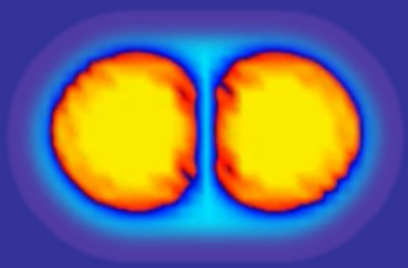

Nuclear collisions

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Josephson junction in atomic Fermi gases - dissipative effects

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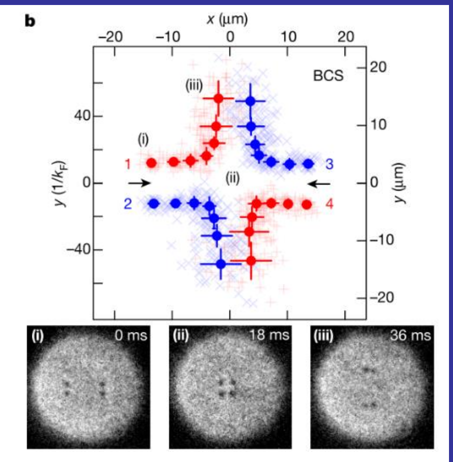
Nonequilibrium superfluidity in Fermi systems



Collisions of vortex-antivortex pairs

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Spin-imbalanced Fermi gases

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