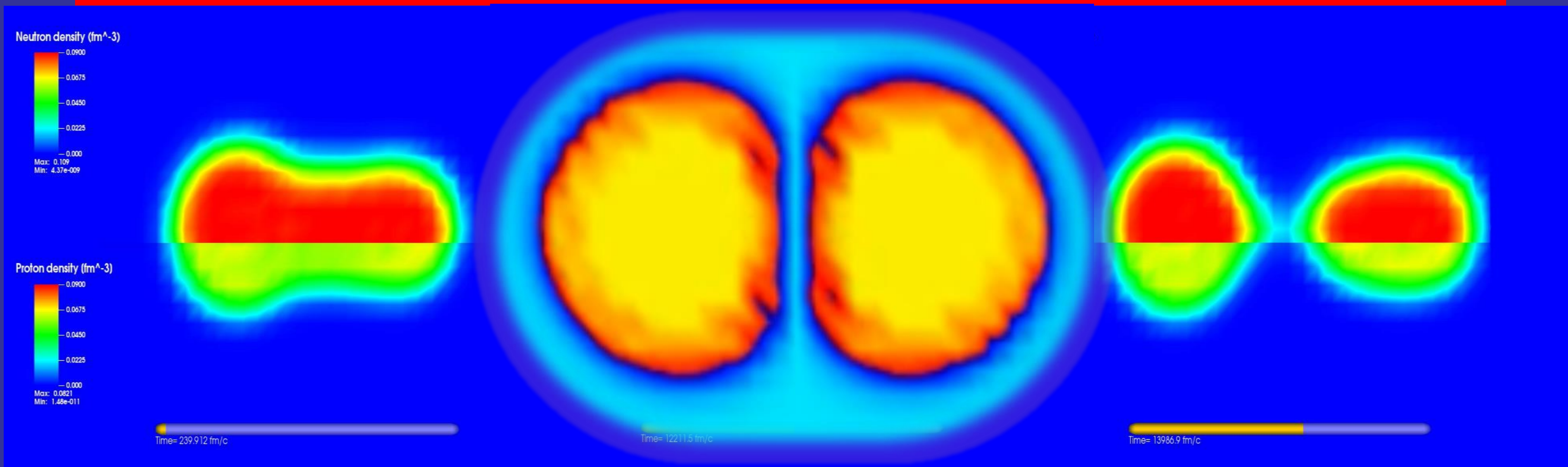
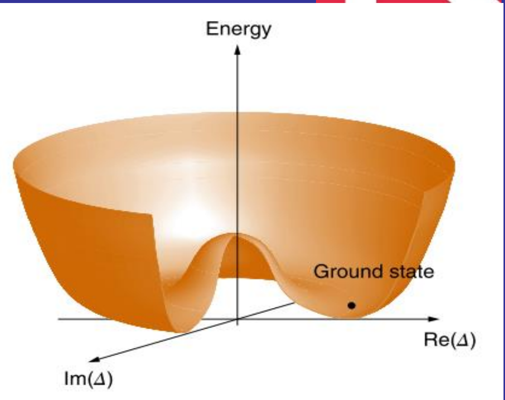
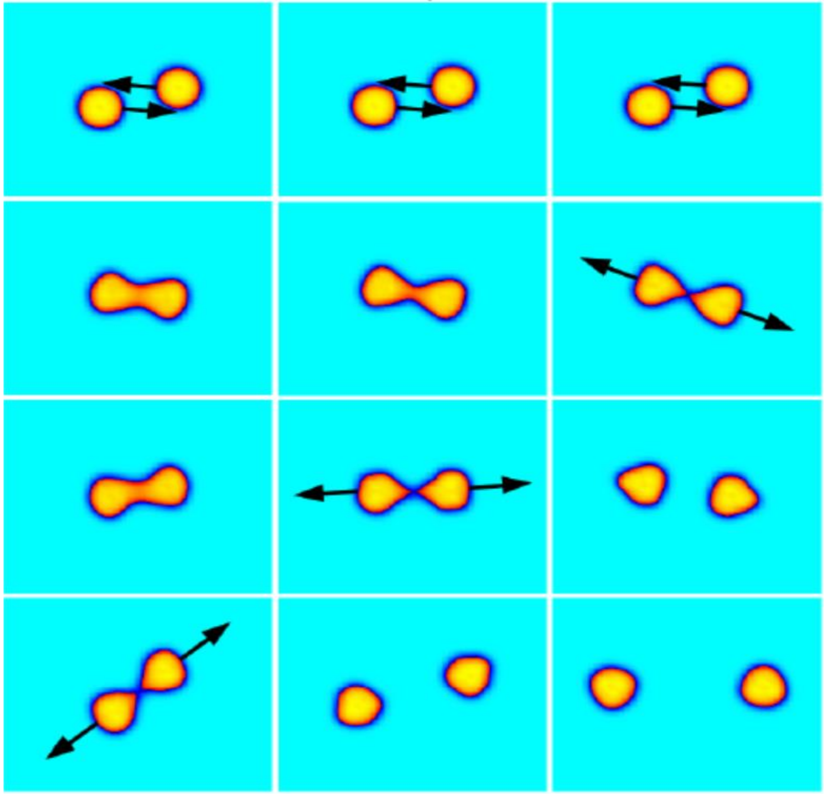
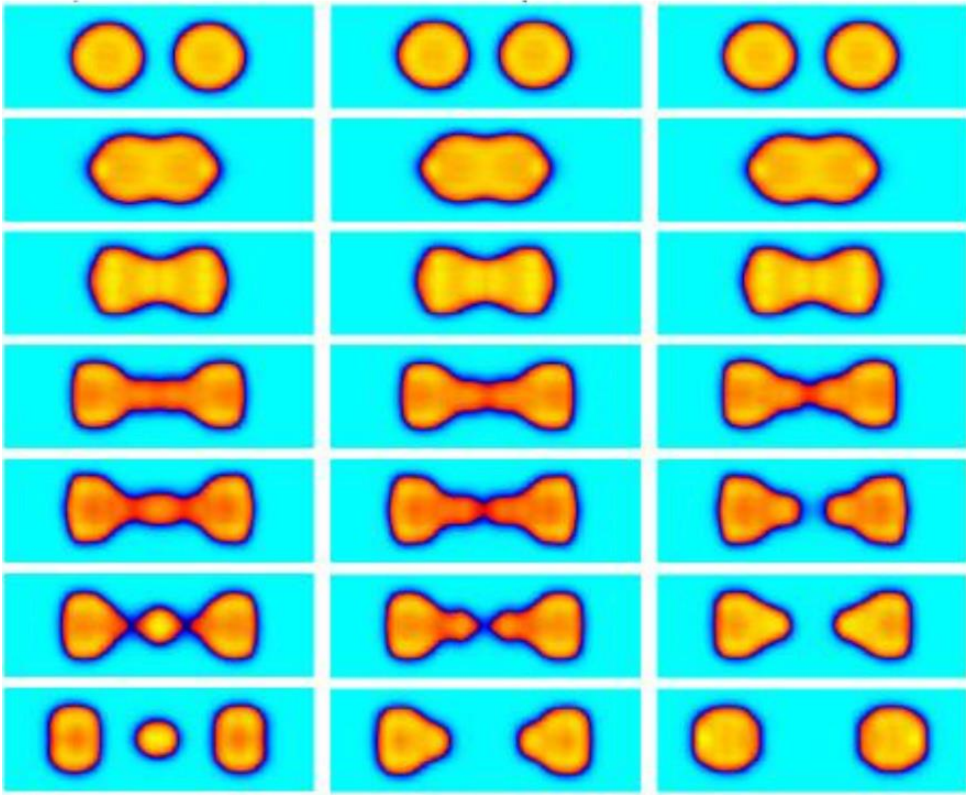
Nuclear dynamics in the framework of time-dependent density functional theory with pairing correlations.



Piotr Magierski
(Warsaw University of Technology)

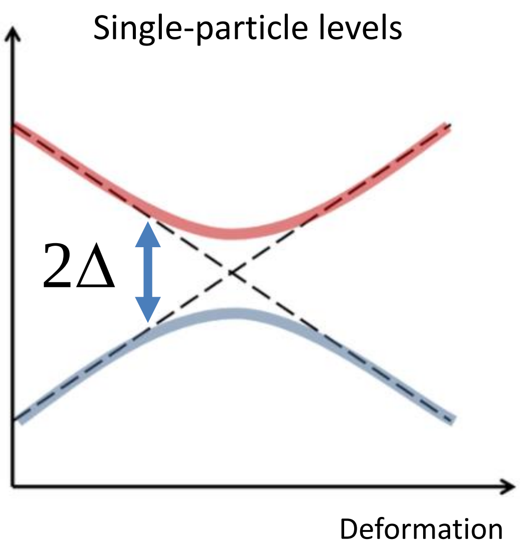






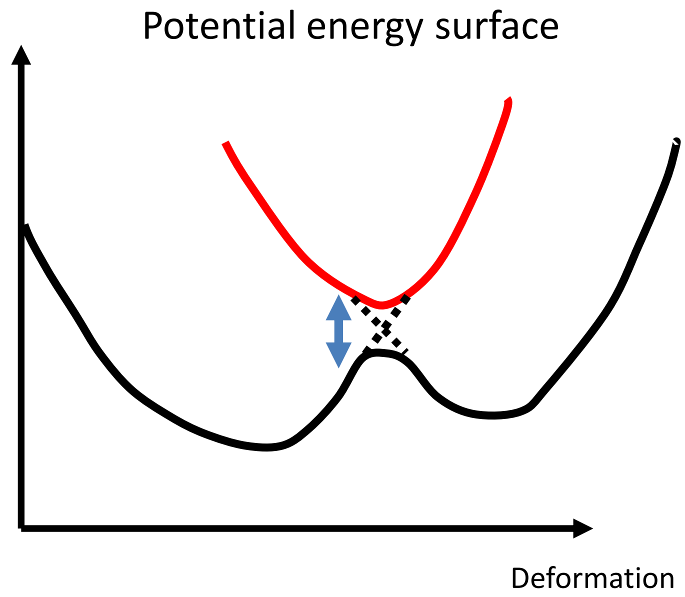
1. Introduction
2. Theoretical framework (Superfluid local density approximation)
3. Induced fission
4. Nucleus-nucleus collisions
5. Nucleus in a superfluid environment (neutron star crust)

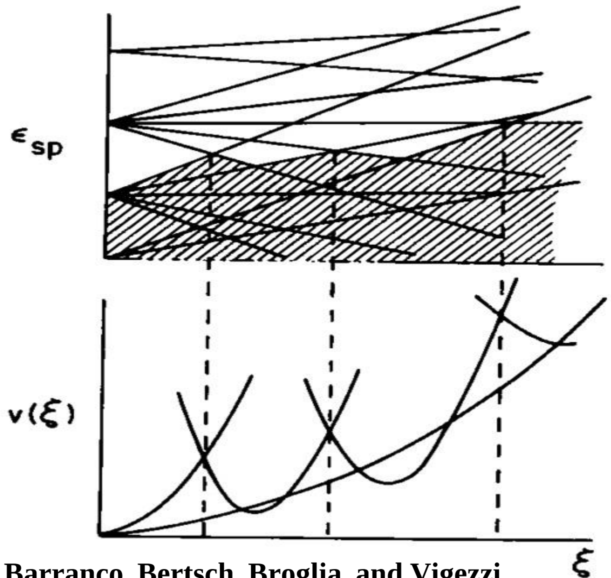
Pairing as an energy gap



Quasiparticle energy:

$$\boxed{E_{qp} = \sqrt{(\varepsilon - \mu)^2 + |\Delta|^2}}$$





From Barranco, Bertsch, Broglia, and Vigezzi
Nucl. Phys. A512, 253 (1990)

As a consequence of pairing correlations large amplitude nuclear motion becomes more adiabatic.

While a nucleus elongates its Fermi surface becomes oblate and its sphericity must be restored

Hill and Wheeler, PRC, 89, 1102 (1953)

Bertsch, PLB, 95, 157 (1980)

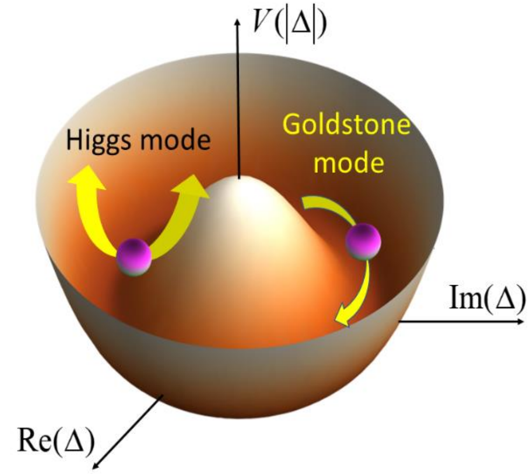
$$\Delta(\vec{r},t)=\left|\Delta(\vec{r},t)\right|e^{i\phi(\vec{r},t)}$$

Appearance of pairing field in Fermi systems is associated with U(1) symmetry breaking.

There are two characteristic modes associated with the field $\Delta(\vec{r},t)$

1) **Nambu-Goldstone mode** explores the degree of freedom associated with the phase: $\phi(\vec{r},t)$

2) **Higgs mode** explores the degree of freedom associated with the magnitude: $\left|\Delta(\vec{r},t)\right|$



<u>What's the difference between pairing correlations and existence of superfluid phase?</u>

- Superfluid phase exists if the *off-diagonal long range order* is present:

$$\lim_{|\boldsymbol{r}_1-\boldsymbol{r}_2|\to\infty}\langle\hat{\psi}^\dagger_\uparrow(\boldsymbol{r}_1)\,\hat{\psi}^\dagger_\downarrow(\boldsymbol{r}_1)\,\hat{\psi}_\downarrow(\boldsymbol{r}_2)\,\hat{\psi}_\uparrow(\boldsymbol{r}_2)\rangle\neq 0$$

C.N. Yang, Rev. Mod. Phys. 34, 694 (1962)

- This limit is unreachable in atomic nuclei due to their finite size. Therefore it is more convenient to look, instead, for the manifestations of the phase $\Delta(\vec{r},t)=\left|\Delta(\vec{r},t)\right|e^{i\phi(\vec{r},t)}$

The well known effects in superconductors where the simplified BCS approach fails

1) Quantum vortices, solitonic excitations related to pairing field (e.g. domain walls)

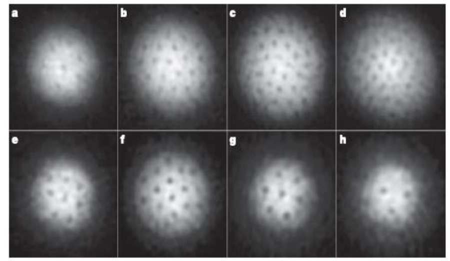
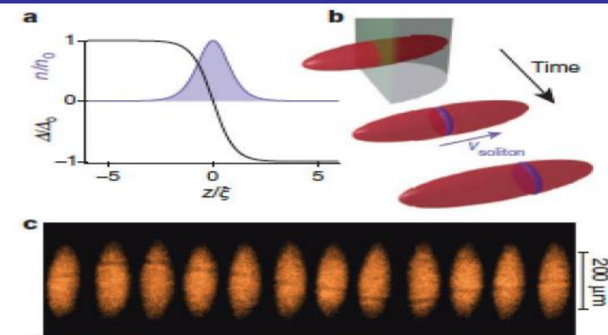
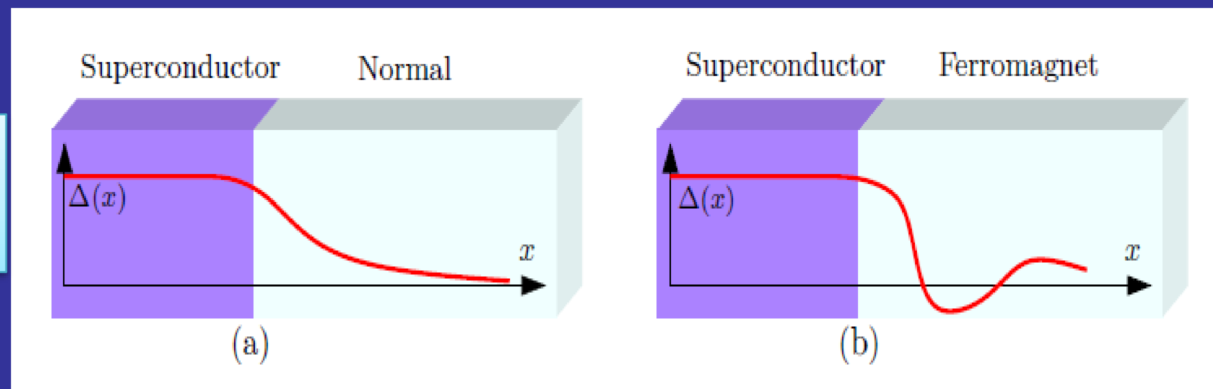


Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b**–**h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**), 812 G (**d**), 833 G (**e**), 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of view of each image is 880 μm × 880 μm.

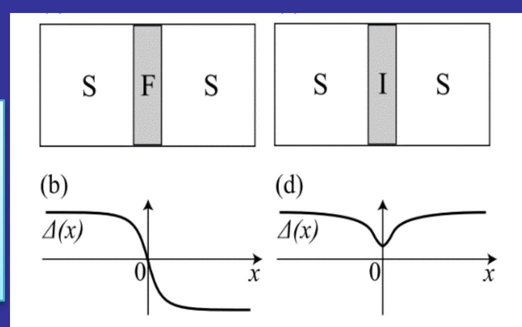


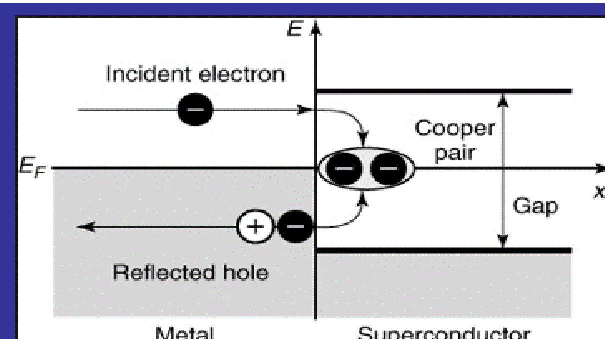
2) Bogoliubov – Anderson phonons

3) Proximity effects: variations of the pairing field on the length scale of the coherence length.



4) Physics of Josephson junction (superfluid - normal metal), pi-Josephson junction (superfluid - ferromagnet)



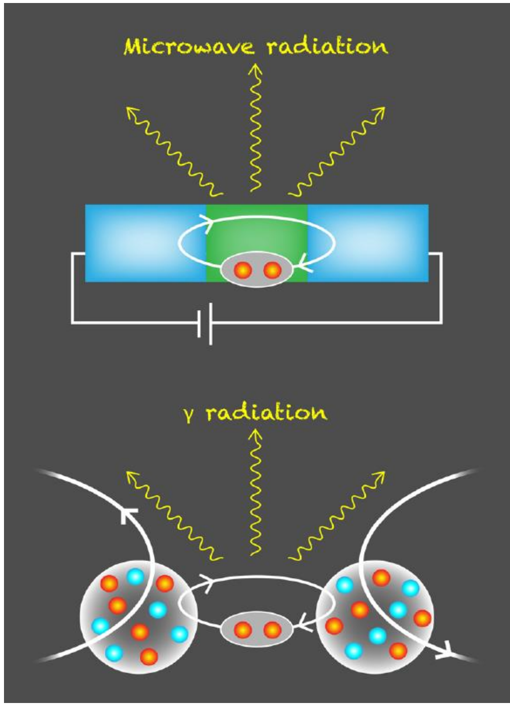


5) Andreev reflection (particle-into-hole and hole-into-particle scattering) Andreev states cannot be obtained within BCS

Nuclear systems

Some evidence for a nuclear **DC Josephson effect** has been gathered over the years, following ideas presented in papers:
V.I. Gol'danskii, A.I. Larkin, JETP 26, 617 (1968), K. Dietrich, Phys. Lett. 32B 428 (1970)

Experimental evidence of enhanced nucleon pair transfer reported eg. in:
M.C. Mermaz, Phys. Rev. C36 1192, (1987), M.C. Mermaz, M. Girod, Phys. Rev. C53 1819 (1996)



From P. Magierski, *Physics* 14 (2021) 27.

Surprisingly evidence for AC Josephson effect has also been found

G.Potel, F.Barranco, E.Vigezzi, R.A. Broglia, "*Quantum entanglement in nuclear Cooper-pair tunneling with gamma rays*," Phys.Rev. C103, L021601 (2021)
R. Broglia, F. Barranco, G. Potel, E. Vigezzi
„*Transient Weak Links between Superconducting Nuclei: Coherence Length*"
Nuclear Physics News 31, 25 (2021)

(see talk by Gregory Potel on Wednesday)

GOAL:

Unified description of nuclear dynamics involving medium and heavy nuclei based on microscopic theoretical framework including pairing correlations.

Microscopic framework = explicit treatment of fermionic degrees of freedom.

In most cases we are interested in extracting one-body observables.

This leads to considering Energy Density Functional (EDF) expressed in terms of local densities.

Kohn-Sham prescription in Time Dependent Density Functional Theory (TDDFT):
replacing the interacting many-body system with the selfconsistent eqs. representing the equivalent noninteracting system.

Equivalence: one-body densities representing both systems are the same.

Runge E. and Gross E. K. U., *Phys. Rev. Lett.*, **52** (1984) 997.

For normal (nonsuperfluid) systems:

$$\boxed{\begin{aligned} \left(-\frac{\hbar^2}{2m}\nabla^2 + V_0(\boldsymbol{r},t)\right)\phi_i(\boldsymbol{r},t) = h(\boldsymbol{r},t)\phi_i(\boldsymbol{r},t) = i\hbar\frac{\partial}{\partial t}\phi_i\boldsymbol{r},t), \\ \rho(\boldsymbol{r},t) = \sum_{i=1}^{N}|\phi_i(\boldsymbol{r},t)|^2, \\ h(\boldsymbol{r},t) = \frac{\delta E[\rho]}{\delta\rho}(\boldsymbol{r},t) + V_{ext}(\boldsymbol{r},t). \end{aligned}}$$

TDHF eqs.

See eg. talks of C. Simenel and R. Gumbel

For superfluid systems:

$$\begin{pmatrix} h(\boldsymbol{r},t) & \Delta_0(\boldsymbol{r},t) \\ \Delta_0^*(\boldsymbol{r},t) & -h^*(\boldsymbol{r},t) \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix} = i\hbar\frac{\partial}{\partial t} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix}$$

$$\rho(\boldsymbol{r},t) = \sum_n |v_n(\boldsymbol{r},t)|^2,$$

$$\chi(\boldsymbol{r},t) = \sum_n v_n^*(\boldsymbol{r},t)u_n(\boldsymbol{r},t),$$

$$h(\boldsymbol{r},t) = \frac{\delta E[\rho,\chi,t]}{\delta\rho} + V_{ext}(\boldsymbol{r},t),$$

$$\Delta_0(\boldsymbol{r},t) = -\frac{\delta E[\rho,\chi,t]}{\delta\chi^*} + \Delta_{ext}(\boldsymbol{r},t)$$

L. N. Oliveira, E. K. U. Gross, and W. Kohn, Phys. Rev. Lett. 60 2430 (1988).

O.-J. Wacker, R. Kümmel, E.K.U. Gross, Phys. Rev. Lett. 73, 2915 (1994).

Kohn, W., Gross, E.K.U., Oliveira, L.N. (1989): J. de Physique (Paris) **50**, 2601

S. Kurth, M. Marques, M. Lüders, E.K.U. Gross, Phys. Rev. Lett. 83 2628 (1999).

J.F. Dobson, M.J. Brunner, E.K.U. Gross, Phys. Rev. Lett. 79 1905 (1997).

G. Vignale, C. A. Ullrich, S. Conti, Phys. Rev. Lett. 79 4878 (1997).

TDHFB eqs. (TDSLDA)

Note that now:

$$\lim_{|\boldsymbol{r}_1-\boldsymbol{r}_2|\to\infty} \langle \hat{\psi}^\dagger_\uparrow(\boldsymbol{r}_1)\,\hat{\psi}^\dagger_\downarrow(\boldsymbol{r}_1)\,\hat{\psi}_\downarrow(\boldsymbol{r}_2)\,\hat{\psi}_\uparrow(\boldsymbol{r}_2)\rangle = \chi^*_{\uparrow\downarrow}(\boldsymbol{r}_1)\chi_{\uparrow\downarrow}(\boldsymbol{r}_2)$$

Solving time-dependent problem for superfluids within TDSLDA

The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations

$$h \sim f_1(n,\nu,\ldots)\nabla^2 + \boldsymbol{f}_2(n,\nu,\ldots)\cdot\nabla + f_3(n,\nu,\ldots)$$

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t)\end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) & 0 & 0 & \Delta(\boldsymbol{r},t) \\ 0 & h_b(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h_a^*(\boldsymbol{r},t) & 0 \\ \Delta^*(\boldsymbol{r},t) & 0 & 0 & -h_b^*(\boldsymbol{r},t)\end{pmatrix}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t)\end{pmatrix}$$

where h and Δ depends on "densities":

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r},t)|^2, \qquad \tau_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r},t)|^2,$$

$$\chi_c(r,t) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r},t)v^*_{n,\downarrow}(\boldsymbol{r},t), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r},t)\nabla v_{n,\sigma}(\boldsymbol{r},t)],$$

$$\begin{aligned}\Delta(\mathbf{r}) &= g_{eff}(\mathbf{r})\chi_c(\mathbf{r}) \\ \frac{1}{g_{eff}(\mathbf{r})} &= \frac{1}{g(\mathbf{r})} - \frac{mk_c(\mathbf{r})}{2\pi^2\hbar^2}\left(1 - \frac{k_F(\mathbf{r})}{2k_c(\mathbf{r})}\ln\frac{k_c(\mathbf{r})+k_F(\mathbf{r})}{k_c(\mathbf{r})-k_F(\mathbf{r})}\right)\end{aligned}$$

A. Bulgac, Y. Yu, Phys. Rev. Lett. 88 (2002) 042504
A. Bulgac, Phys. Rev. C65 (2002) 051305

huge number of nonlinear coupled 3D Partial Differential Equations
(in practice n=1,2,…, 10^5 - 10^6)

- P. Magierski, *Nuclear Reactions and Superfluid Time Dependent Density Functional Theory*, Frontiers in Nuclear and Particle Physics, vol. 2, 57 (2019)
- A. Bulgac, *Time-Dependent Density Functional Theory and Real-Time Dynamics of Fermi Superfluids*, Ann. Rev. Nucl. Part. Sci. 63, 97 (2013)
- A. Bulgac, M.M. Forbes, P. Magierski, Lecture Notes in Physics, Vol. 836, Chap. 9, p.305-373 (2012)

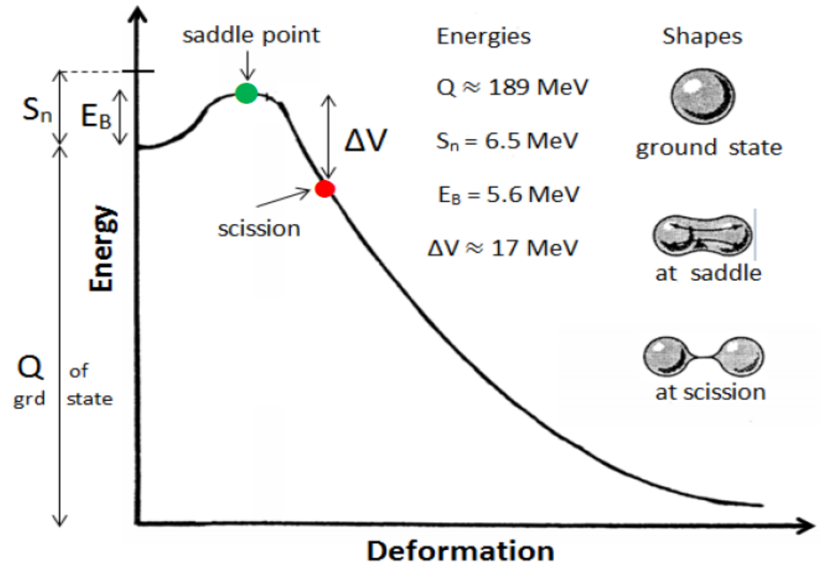
Present computing capabilities:

- full 3D (unconstrained) superfluid dynamics
- spatial mesh up to 100^3
- max. number of particles of the order of 10^4
- up to 10^6 time steps

(for cold atomic systems – time scale: a few ms
for nuclei – time scale: 100 zs)

Nuclear fission dynamics

Potential energy versus deformation

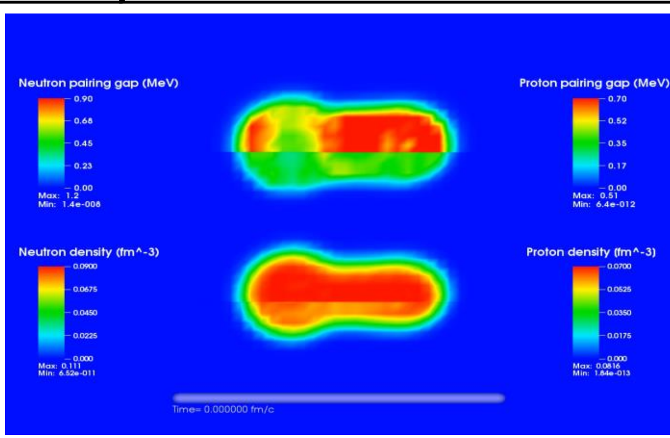


From F. Gonnenwein FIESTA2014

Estimation of characteristic time scales for low energy fission (<10MeV):

Ground state to saddle - 1 000 000 zs
Saddle to scission - 10-100 zs
Acceleration of fission fragments to 90% of their final velocity - 10 zs
Neutron evaporation - 1 000 zs
1 zs = 10^{-21} s

Fission dynamics of *^{240}Pu within TDSLDA*

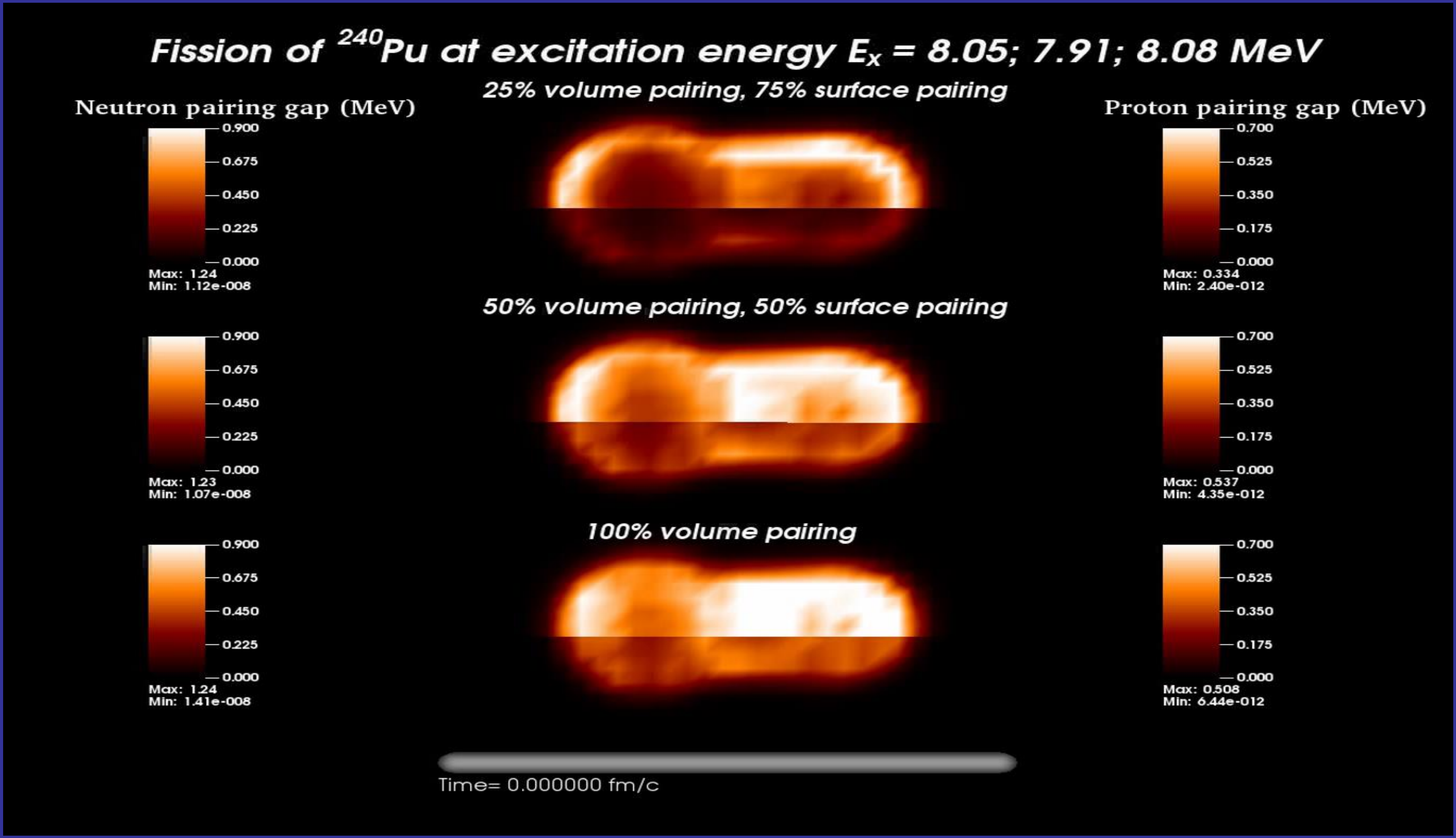


Total kinetic energy of the fragments

E^* (MeV)	E_n (MeV)	TKE_{TDSLDA} (MeV)	TKE_{syst} (MeV)	err (%)	Z_L	N_L
8.08	1.542	173	177.26	1.95	40.825	62.246
9.60	3.063	174	176.73	1.13	40.500	61.536
10.10	3.560	179	176.56	1.43	41.625	62.783
10.57	4.032	173	176.39	1.55	40.092	61.256
10.58	4.043	173	176.39	1.70	40.146	61.388
10.58	4.047	175	176.39	0.72	40.313	61.475
10.60	4.065	174	176.38	0.92	40.904	62.611
11.07	4.534	176	176.22	0.14	41.495	63.134
11.56	5.024	175	176.05	0.51	40.565	61.894
12.05	5.515	176	175.88	0.49	40.412	61.809
12.15	5.610	176	175.84	0.29	40.355	61.695
12.16	5.626	176	175.84	0.15	41.386	62.764

Calculated TKEs reproduce experimental data with accuracy < 2%

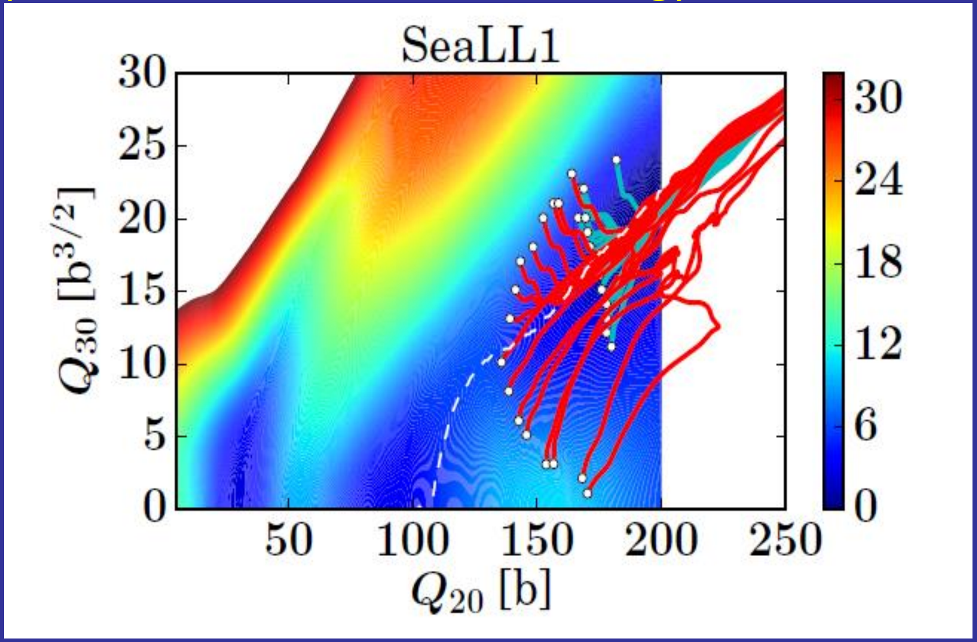
A. Bulgac, P.Magierski, K.J. Roche, and I. Stetcu, Phys. Rev. Lett. 116, 122504 (2016)



$1 \text{ zs} = 10^{-21} \text{ sec.} = 300 \text{ fm/c}$

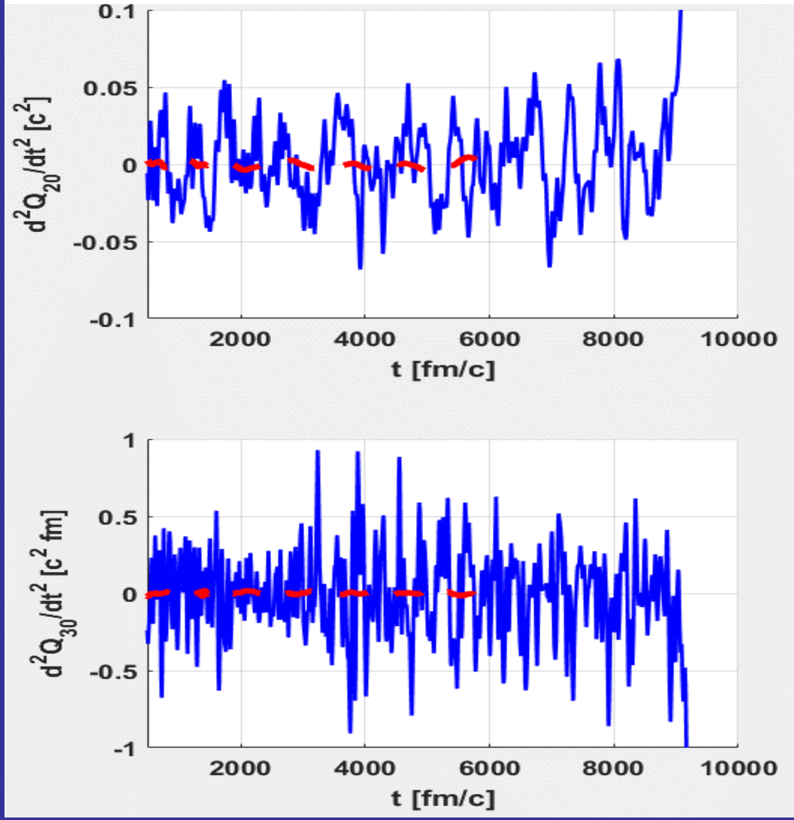
Fission dynamics of ^{240}Pu

Trajectories of fissioning ^{240}Pu in the collective space at excitation excitation energy of *E=8-9 MeV*:



A. Bulgac, et al. Phys. Rev. C 100, 034615 (2019)

Accelerations in quadrupole and octupole moments along the fission path



Note that despite the fact that nucleus is already <u>beyond the saddle point</u> the collective motion on the time scale of 1000 fm/c and larger is characterized by <u>the constant velocity</u> (*see red dashed line for an average acceleration*) till the very last moment before splitting. On times scales, of the order of 300 fm/c and shorter, the collective motion is a subject to random-like kicks indicating strong coupling to internal d.o.f

„Heavy soliton" creation in nuclear collision

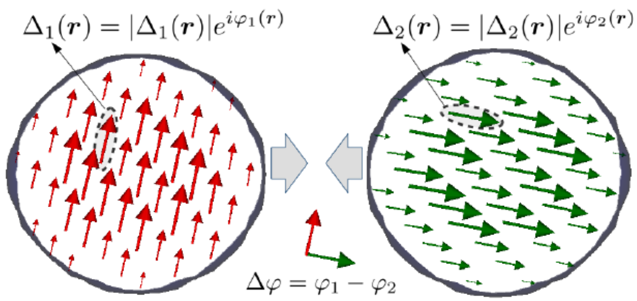
Collisions of superfluid nuclei having <u>different phases</u> of the <u>pairing fields</u>

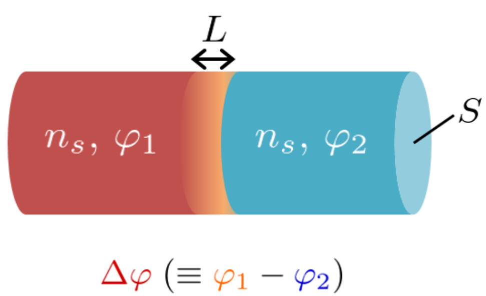
<u>The main questions are:</u>

-how a possible solitonic structure can be manifested in nuclear system?

-what observable effect it may have on heavy ion reaction:
kinetic energy distribution of fragments, capture cross section, etc.?

Clearly, we cannot control phases of the pairing field in nuclear experiments and the possible signal need to be extracted after averaging over the phase difference.



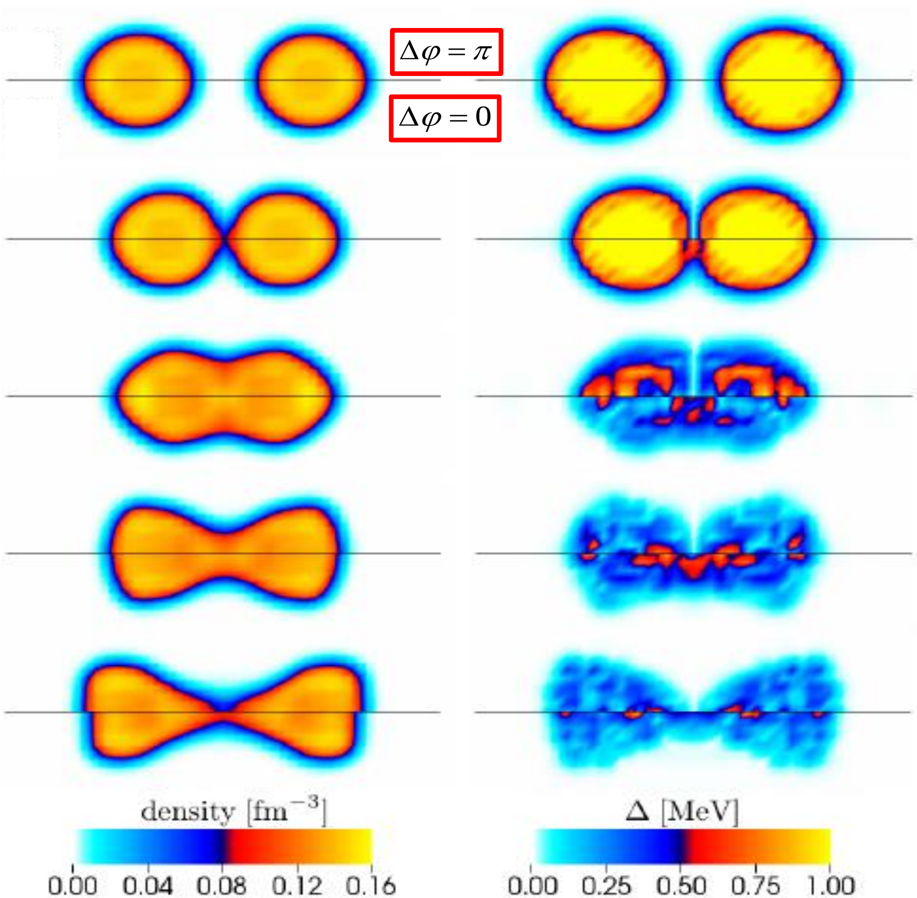


From Ginzburg-Landau (G-L) approach:

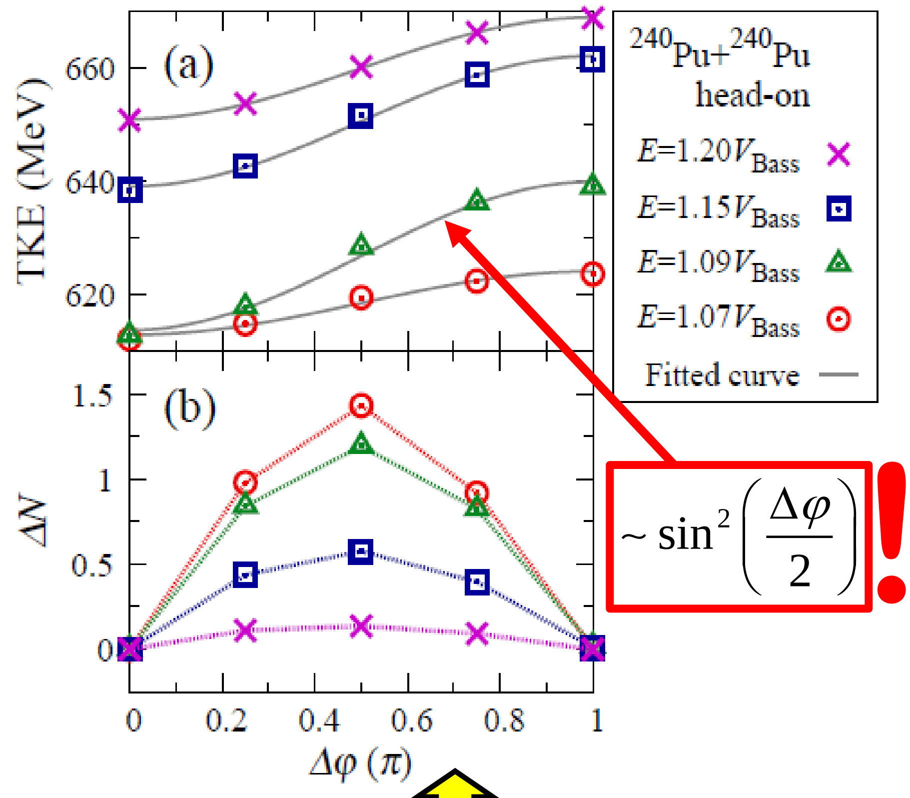
$$\boxed{E_j = \frac{S}{L}\frac{\hbar^2}{2m} n_s \sin^2\frac{\Delta\varphi}{2}}$$

For typical values characteristic for two medium nuclei: $E_j \approx 30 MeV$

$^{240}Pu+^{240}Pu$



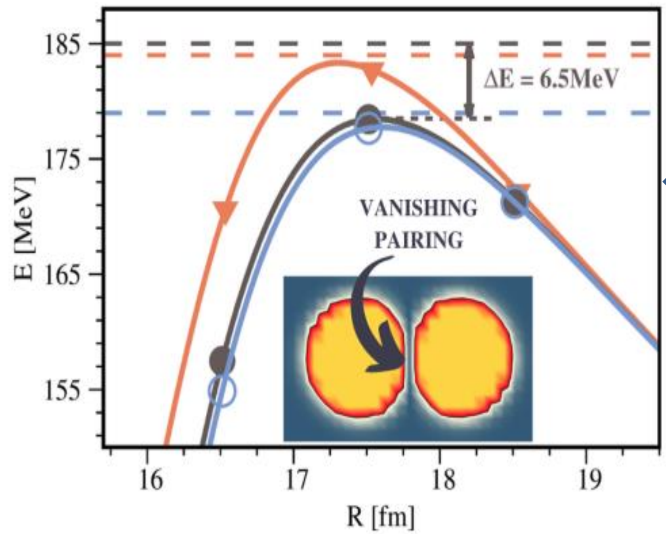
Total kinetic energy of the fragments (TKE)



Average particle transfer between fragments.

Creation of the solitonic structure between colliding nuclei prevents energy transfer to internal degrees of freedom and consequently enhances the kinetic energy of outgoing fragments.
Surprisingly, the gauge angle dependence from the G-L approach is perfectly well reproduced in the kinetic energies of outgoing fragments!

P. Magierski, K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119 042501 (2017)



Dynamic nature of the effect:

Solid lines: static barrier between two nuclei (with pairing included):
90Zr+90Zr - brown
96Zr+96Zr - black (0-phase diff.) and
blue (Pi-phase diff.)
Static barriers are practically insensitive to the phase difference of pairing fields.

Dashed lines: Actual threshold for capture obtained in dynamic calculations.
Hence ΔE measures the additional energy which has to be added to the system to merge nuclei.

TABLE I: The minimum energies needed for capture in ^{90}Zr+^{90}Zr and ^{96}Zr+^{96}Zr for the case of $\Delta\phi = 0$ [$E_{\rm thresh}(0)$] and $\Delta\phi = \pi$ [$E_{\rm thresh}(\pi)$]. The energy difference between the two cases is shown in the last column. The average pairing gap $\overline{\Delta}_i$ is defined by Eq. (4).

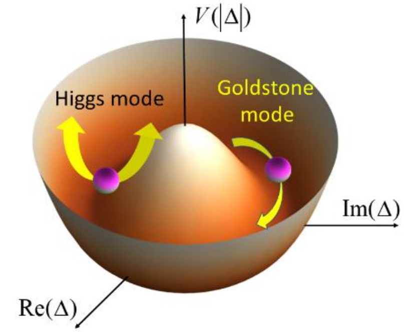
	$\overline{\Delta}_q$ (MeV)	$E_{\rm thresh}(0)$ (MeV)	$E_{\rm thresh}(\pi)$ (MeV)	ΔE_s
^{90}Zr	$\overline{\Delta}_n = 0.00$ $\overline{\Delta}_p = 0.09$	184	184	0
^{96}Zr	$\overline{\Delta}_n = 1.98$ $\overline{\Delta}_p = 0.32$	179	185	6
	$\overline{\Delta}_n = 2.44$ $\overline{\Delta}_p = 0.33$	178	187	9
	$\overline{\Delta}_n = 2.94$ $\overline{\Delta}_p = 0.34$	178	187	9

Dependence of the additional energy on pairing gap in colliding nuclei

P. Magierski, A. Makowski, M. Barton, K. Sekizawa, G. Wlazłowski, Phys. Rev. C 105, 064602, (2022)

G. Scamps, Phys. Rev. C 97, 044611 (2018): ***barrier fluctuations extracted from experimental data provide evidence that the effect exists.***

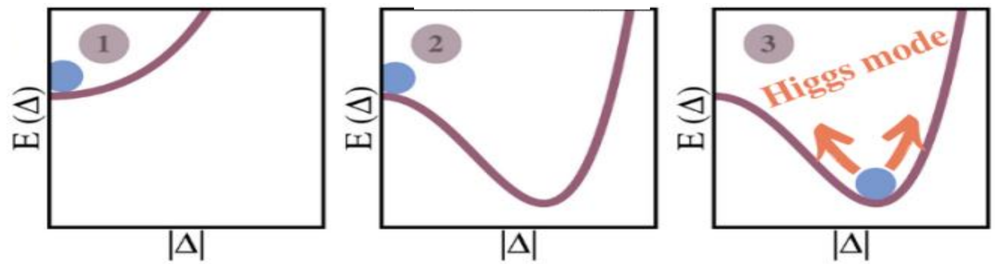
Pairing Higgs mode



Let's consider Fermi gas with schematic pairing interaction and coupling constant depending on time:

$$\hat{H} = \sum_k \varepsilon_k \hat{\psi}_k^+ \hat{\psi}_k - g(t) \sum_{k,l>0} \hat{\psi}_k^+ \hat{\psi}_{\bar{k}}^+ \hat{\psi}_{\bar{l}} \hat{\psi}_l$$

$g(t) = g_0 \theta(t)$ coupling constant is switched on withing time scale much shorter than $\hbar/\varepsilon_F$



As a result pairing becomes unstable and increases exponentially $\Delta(t) \propto e^{-i\zeta t} = e^{-i\omega t} e^{\gamma t}$

$$\frac{1}{g_0} = \sum_{k>0, \varepsilon_k > \mu} \frac{\tanh\left(\frac{\beta|\varepsilon_k - \mu|}{2}\right)}{2|\varepsilon_k - \mu| + \zeta} + \sum_{k>0, \varepsilon_k < \mu} \frac{\tanh\left(\frac{\beta|\varepsilon_k - \mu|}{2}\right)}{2|\varepsilon_k - \mu| - \zeta}$$

Time scale of growth and the period of subsequent oscillation is related to static value of pairing Δ_0 :

$$\tau = \frac{1}{\gamma} \approx \frac{\hbar}{\Delta_0}$$

Pairing instability in nuclear reaction

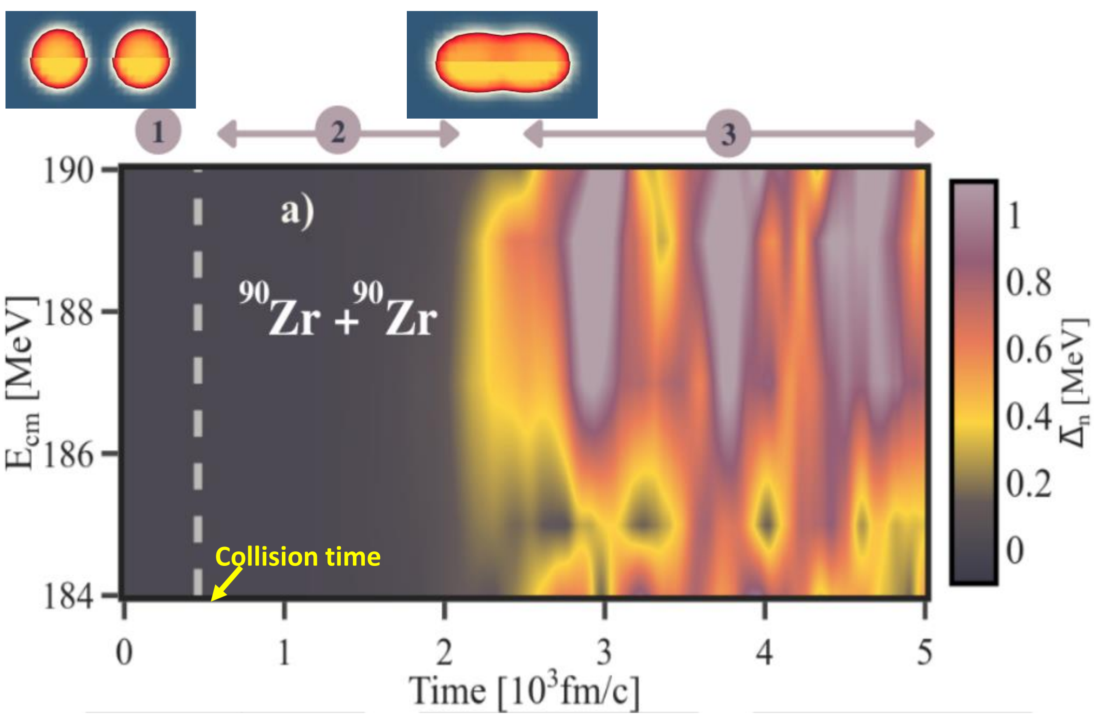
$$\boxed{\Delta = \frac{8}{e^2}\varepsilon_F \exp\left(\frac{-2}{gN(\varepsilon_F)}\right)}$$ - BCS formula – weak coupling limit

ε_F - Fermi energy

g - Pairing coupling constant

$N(\varepsilon_F)$ - Density of states at the Fermi level

Although one cannot change coupling constant in atomic nuclei one may affect ***density of states at the Fermi surface and consequently trigger pairing instability.***



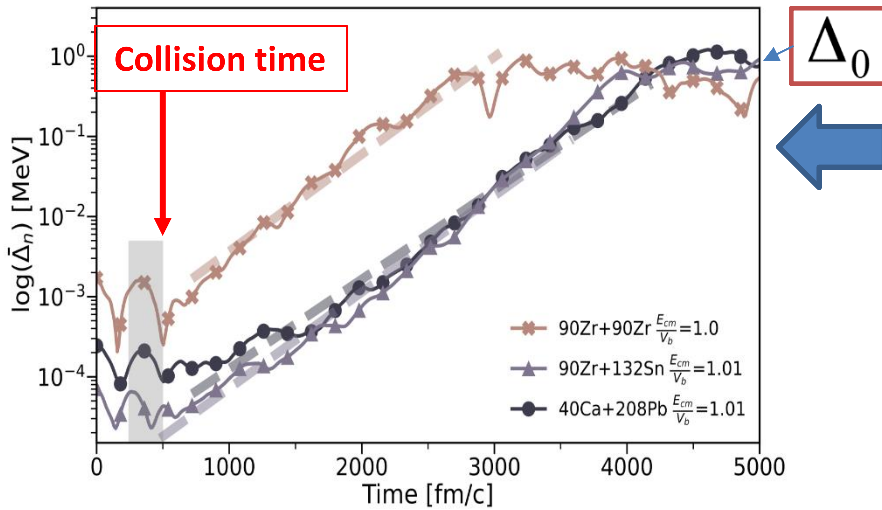
Collision of two neutron magic systems creates an elongated di-nuclear system.

Within 1500 fm/c pairing is enhanced in the system and reveals oscillations with frequency:

$$\Delta < \hbar\omega < 2\Delta$$

P.Magierski, A. Makowski, M. Barton, K. Sekizawa, G. Wlazłowski, Phys. Rev. C 105, 064602, (2022)

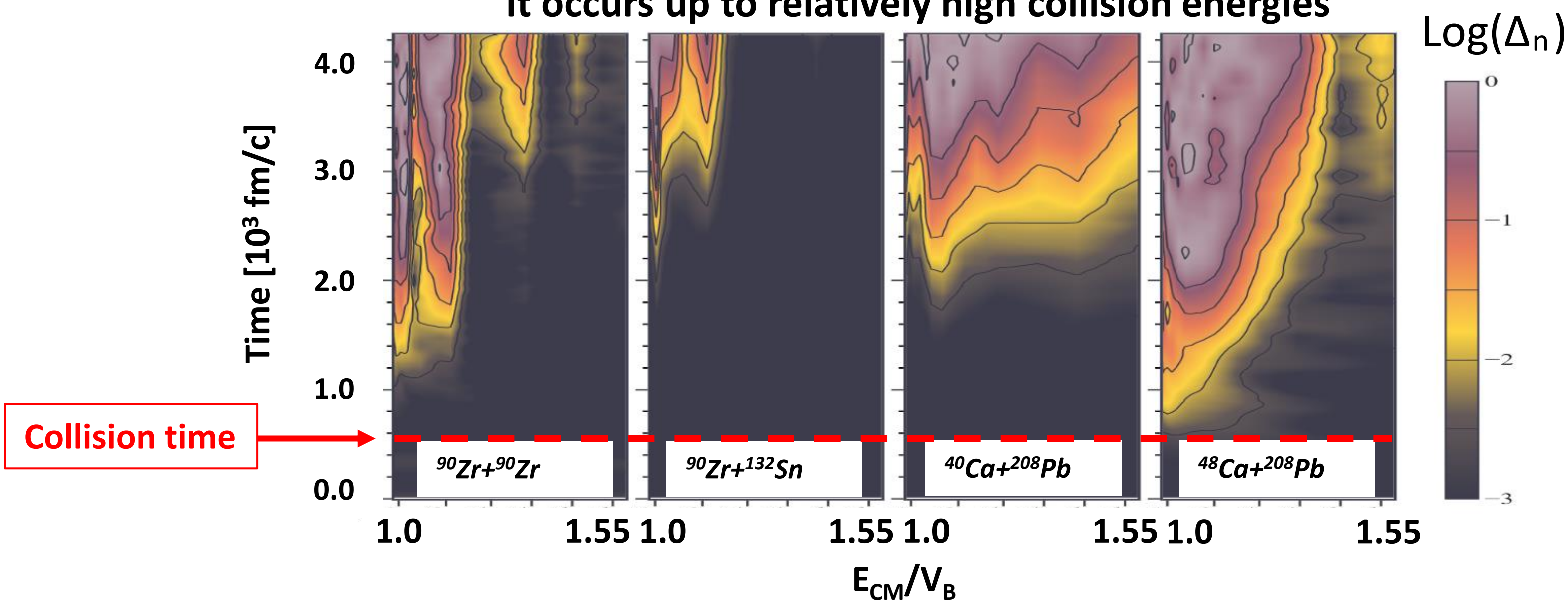
Interestingly, the effect is generic and occurs for various collisions of magic nuclei.



Exponential increase of pairing gap after collision indicating **pairing instability** in di-nuclear system.
Time scale of pairing enhancement:

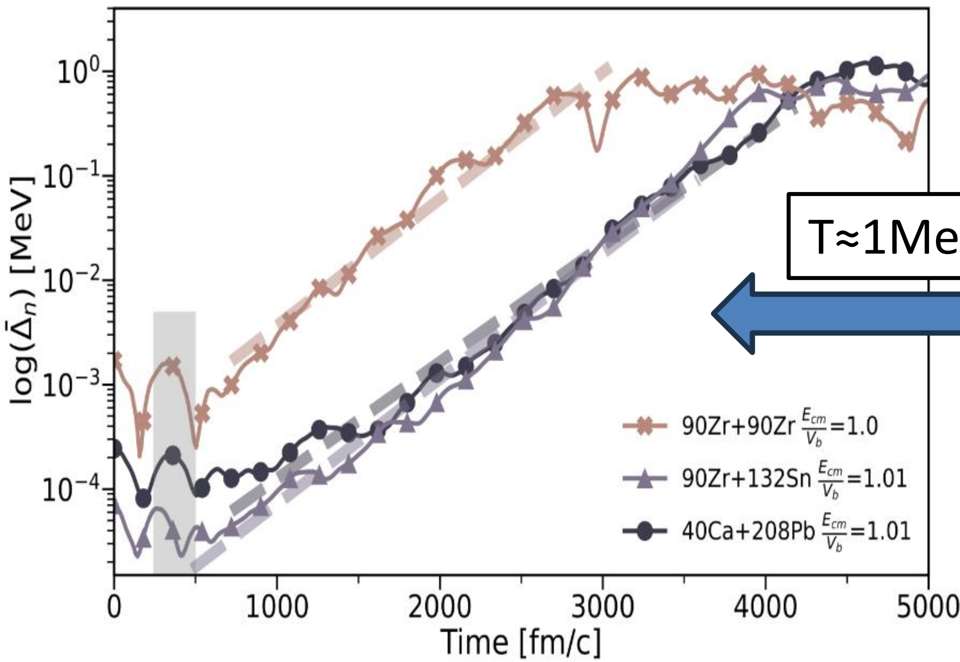
$$\tau \gg \frac{\hbar}{\Delta_0}$$

It occurs up to relatively high collision energies

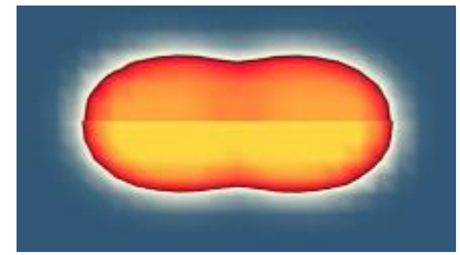


The excitation energy of a compound system after merging exceeds **20-30 MeV**.
It corresponds to temperatures **close to or even higher than the critical temperature for superfluid-to-normal transition.** Therefore it is unlikely that the system develops superfluid phase and it is rather nonequilibrium enhancement of pairing correlations.

Dynamic pairing enhancement



T≈1MeV



Temperatures, associated with excitation energies relative to the nuclear configuration after merging, are about **1 MeV .**
They **exceed** the critical temperature for **the superfluid-to-normal transition**.

$$\underbrace{i\hbar\frac{d\rho}{dt} = [h,\rho]}_{\text{TDHF (collisionless part)}} + \underbrace{\Delta\chi^\dagger - \chi\Delta^\dagger}_{\text{Pairing („collision” term)}}$$

Pairing correlations appearing at relatively high excitation energy provide a nonnegligible modification of the density evolution.

Superconducting systems of interest

$$\frac{\Delta}{\varepsilon_F} \leq 0.5$$

Ultracold atomic (fermionic) gases. Unitary regime.
Dynamics of quantum vortices, solitonic excitations, quantum turbulence

$$\frac{\Delta}{\varepsilon_F} \leq 0.1-0.2$$

Astrophysical applications.
Modelling of neutron star interior (glitches): vortex dynamics, dynamics of inhomogeneous nuclear matter.

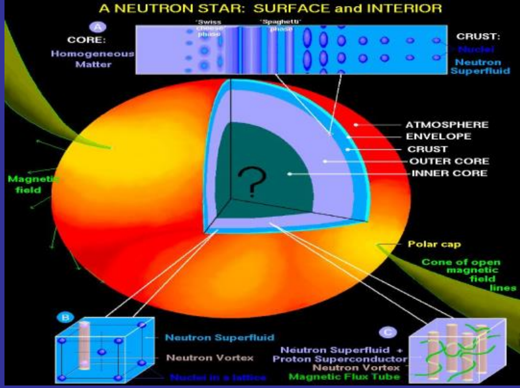
$$\frac{\Delta}{\varepsilon_F} \leq 0.03$$

Nuclear physics.
Induced nuclear fission, fusion, collisions.

Collisions of ultracold atomic clouds offer an insight into pairing-related effects relevant to nuclear collisions.

$\frac{\Delta}{\varepsilon_F}$ **- Pairing gap to Fermi energy ratio**

Determination of the neutron star crust properties: dynamics of nuclear Coulomb crystal



Effective mass of a nucleus in superfluid neutron environment

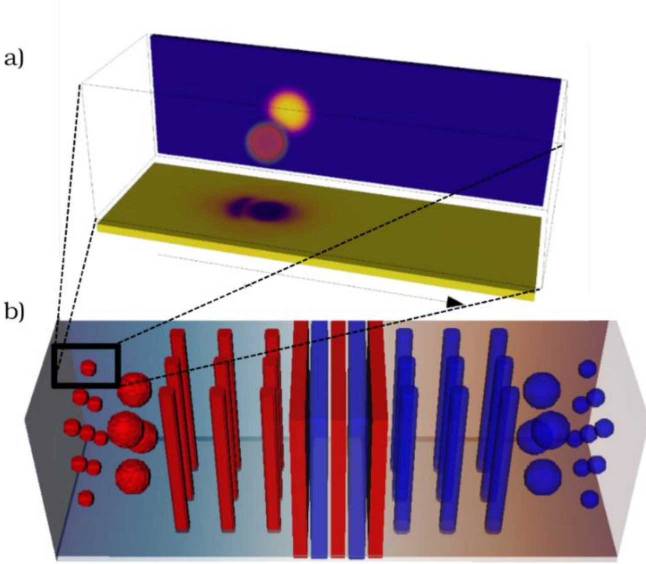
Suppose we would like to evaluate an effective mass of a heavy particle immersed in a Fermi bath.
Can one come up with the effective (classical) equation of motion of the type:

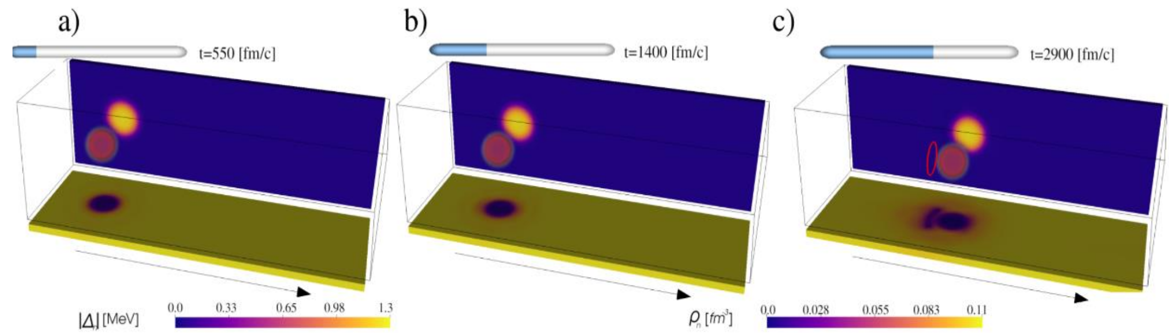
$$M\frac{d^2q}{dt^2} - F_D\left(\frac{dq}{dt},\ldots\right) + \frac{dE}{dq} = 0 \quad ?$$

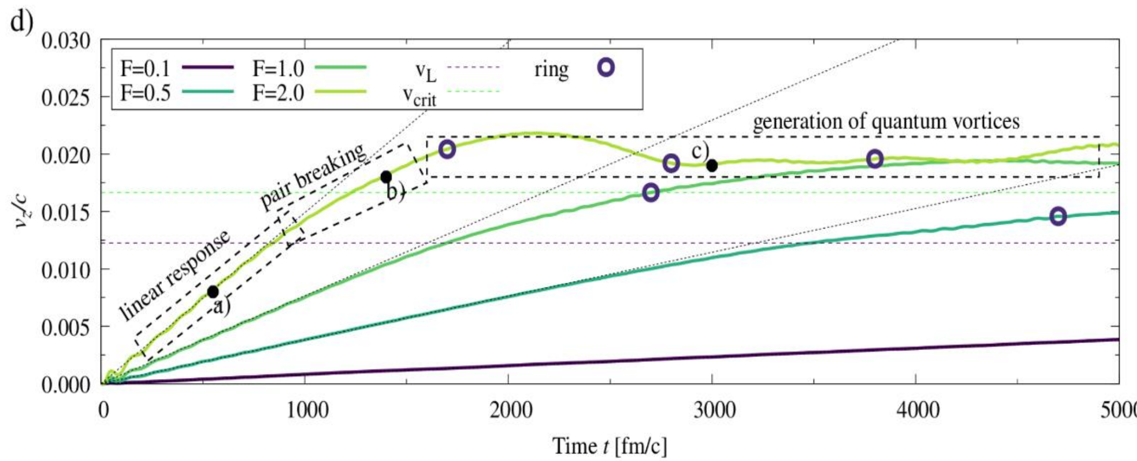
In general it is a complicated task as the first and the second term may not be unambiguously separated.

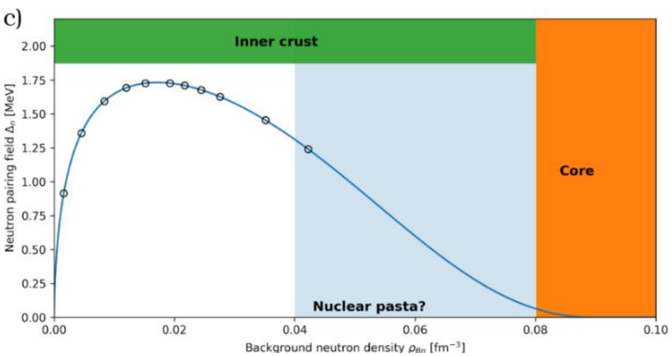
However for the superfluid system it can be done as for sufficiently slow motion (below the critical velocity) the second term may be neglected due to the presence of the pairing gap.

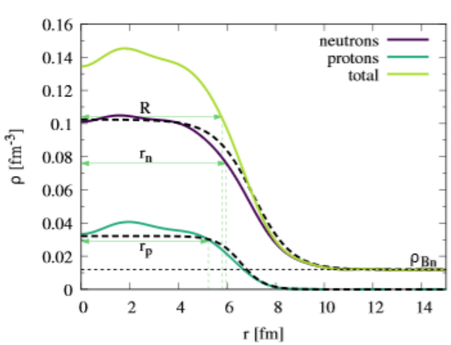
Dynamics of nuclear impurity in the neutron star crust: effective mass and energy dissipation







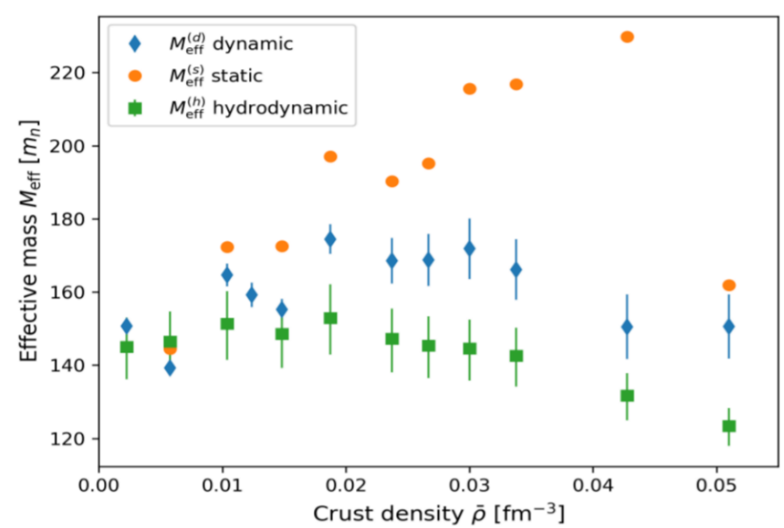




$$\boxed{M_{\mathrm{eff}}^{(d)} = a_z/F}$$

$$M_{\mathrm{eff}}^{(s)} = Zm_p + N_{\mathrm{bound}} m_n,$$

$$M_{\mathrm{eff}}^{(h)} = \frac{4}{3}\pi R_h^3 m_n \frac{(\rho_{\mathrm{in}} - \rho_{\mathrm{out}})^2}{\rho_{\mathrm{in}} + 2\rho_{\mathrm{out}}}.$$



D. Pęcak, A.Zdanowicz, N. Chamel, P. Magierski, G. Wlazłowski, arXiv:2403.17499

Summary and open questions

- Induced fission: the nuclear motion from sadle to scission **is not adiabatic, although it is slow.**
- **Excitation energy sharing**: depending on dynamics and density of states at scission - very severe test for TDDFT.
- TDHFB provides evidence for nontrivial behavior of pairing correlations in highly nonequilibrium conditions which includes **solitonic excitations** (dynamic barrier modification for capture) and **pairing enhancement** as a result of collision.
- There is certain experimental evidence for solitonic excitations, although not easy to extract (G. Scamps, Phys. Rev. C C 97, 044611 (2018)).
- **Pairing enhancement** in collision of magic nuclei is a generic feature of TDHFB appearing in collisions of magic nuclei at energies close to the Coulomb barrier.
- Impact of pairing enhancement on dynamics is unknown and requires more theoretical effort: **impact on quasifission process, interplay between pairing and shell effects in nuclear collisions**, …