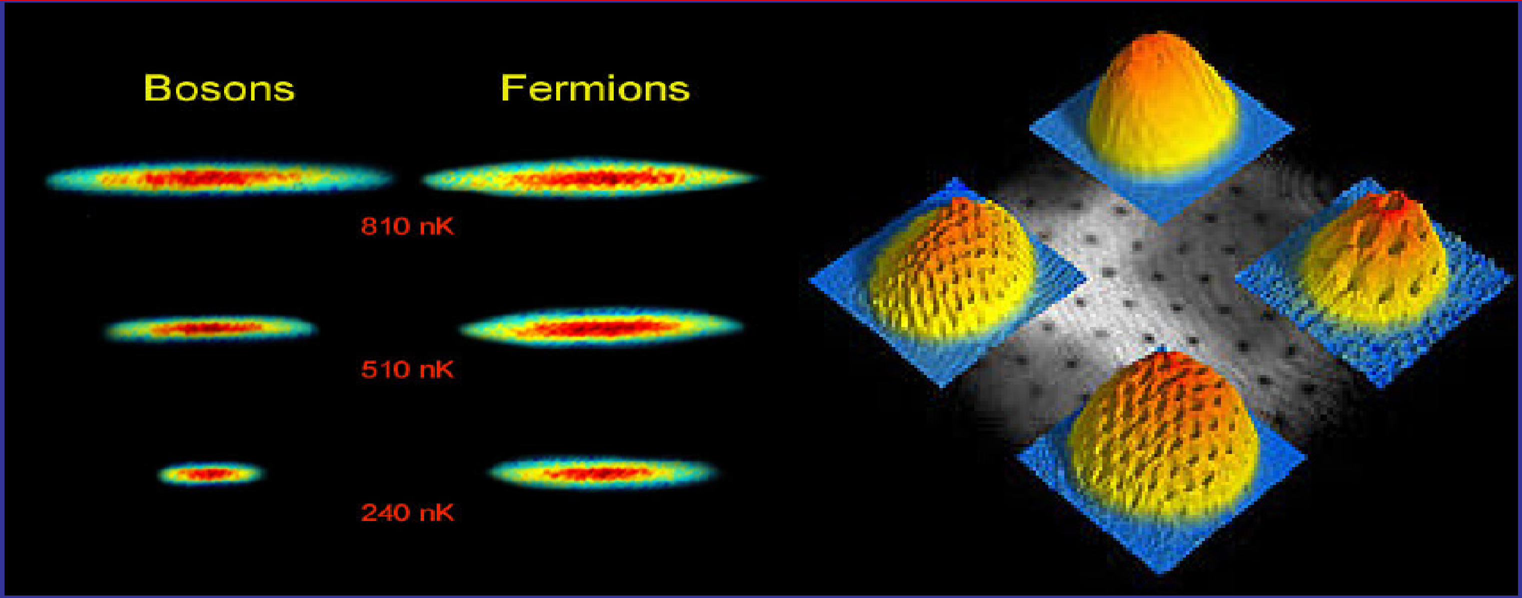
Thermodynamics of a trapped unitary Fermi gas



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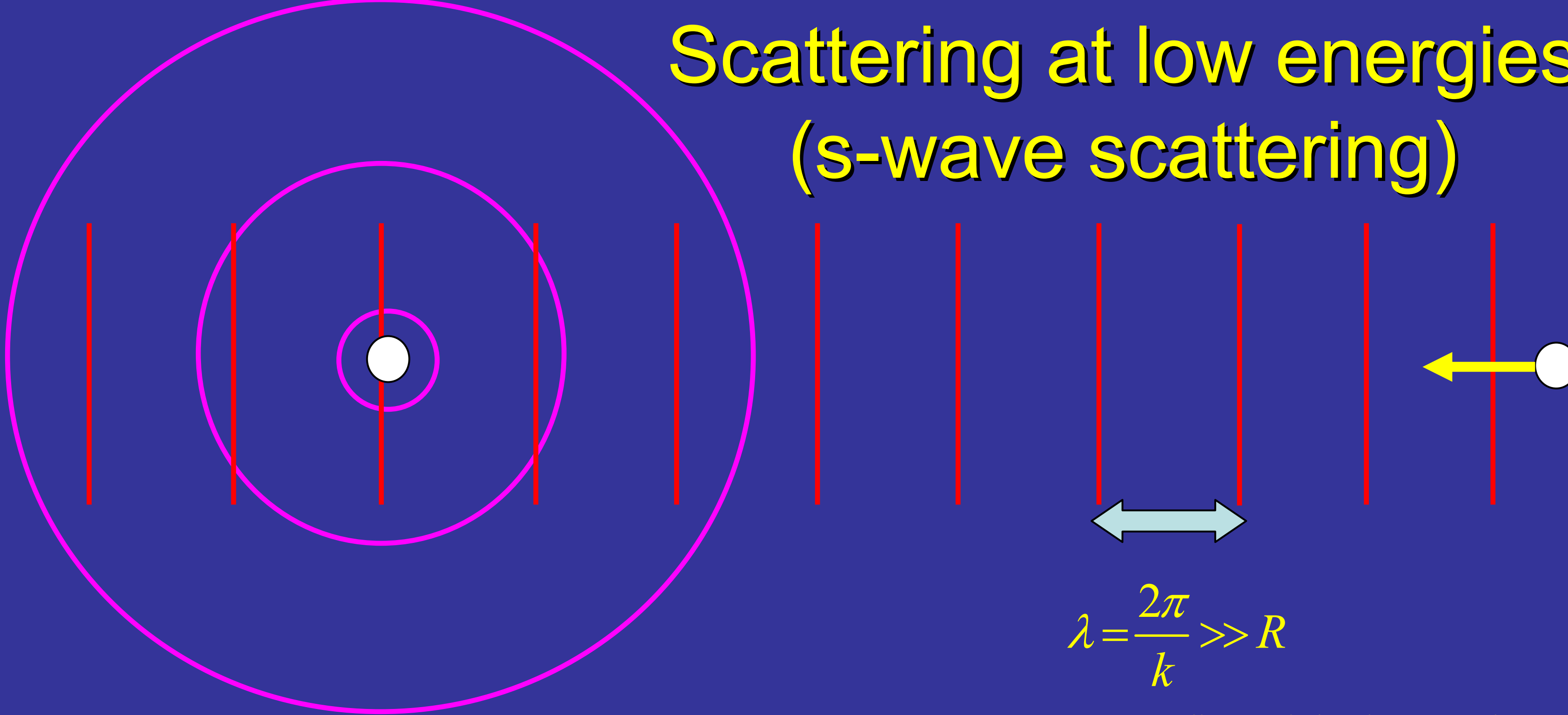
Scattering at low energies (s-wave scattering)



$$\lambda = \frac{2\pi}{k} >> R$$

R - radius of the interaction potential

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + f\frac{e^{ikr}}{r};$$ f - scattering amplitude

$$f = \frac{1}{-ik - 1/a + \frac{1}{2}r_0 k^2},$$ a - scattering length, r_0 - effective range

If $k \to 0$ then the interaction is determined by the scattering length alone.

$$E/E_{FG} = 1 + \frac{10}{9\pi}(k_F a)\left[1 + \frac{6}{35\pi}(k_F a)(11 - 2\ln 2) + \dots\right] + \text{pairing}$$

Perturbation series

$$E_{FG} = \frac{3}{5}\varepsilon_F N$$ - Energy of the noninteracting Fermi gas

➤ What is the unitary regime?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$$n\, r_0^3 \ll 1 \qquad n\, |a|^3 \gg 1$$

n - particle density
a - scattering length
r_0 - effective range

$$i.e.\ \ r_0 \to 0,\ a \to \pm\infty$$

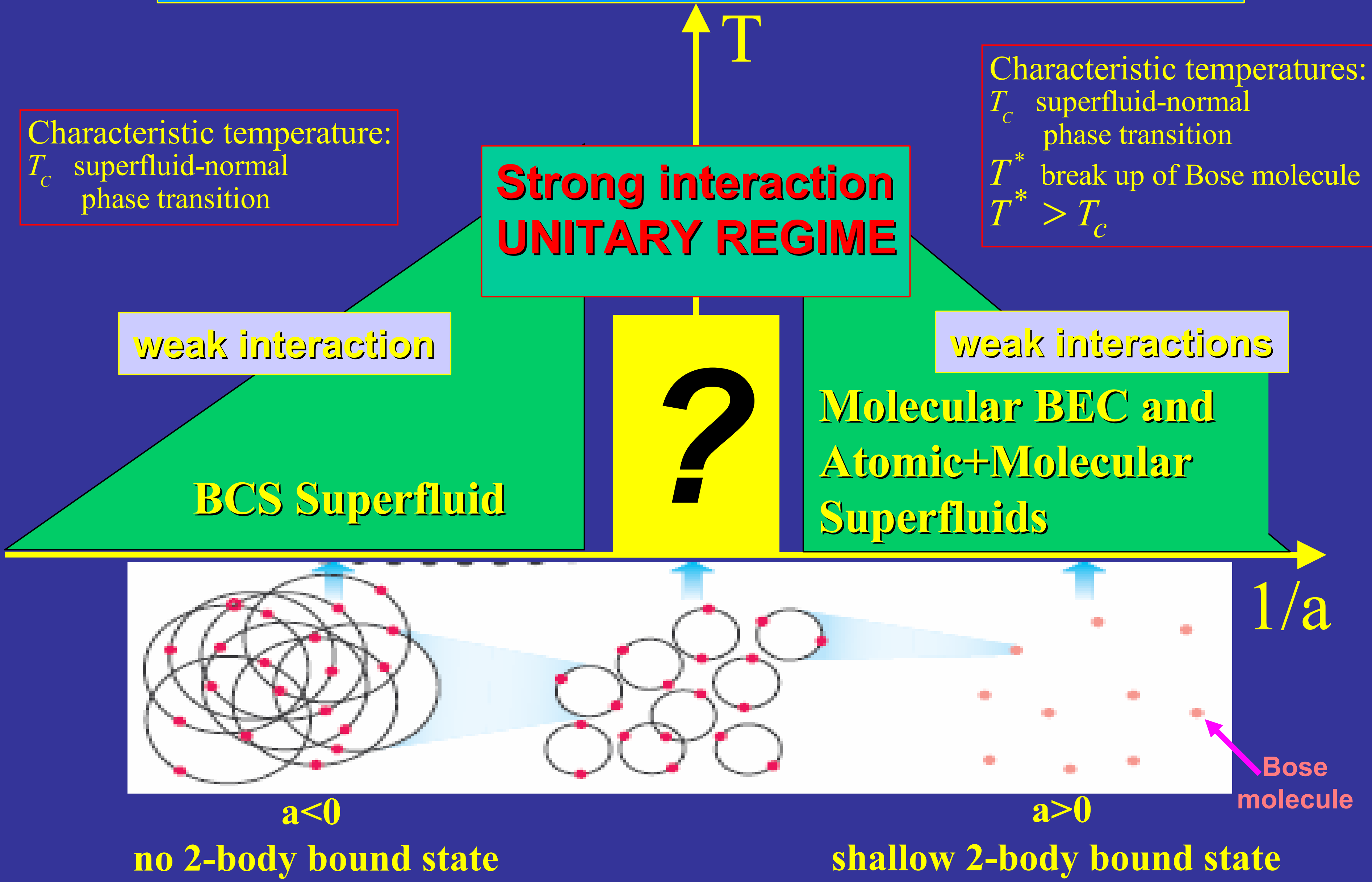
NONPERTURBATIVE REGIME

System is dilute but strongly interacting!

UNIVERSALITY: $$E = \xi_0 E_{FG}$$

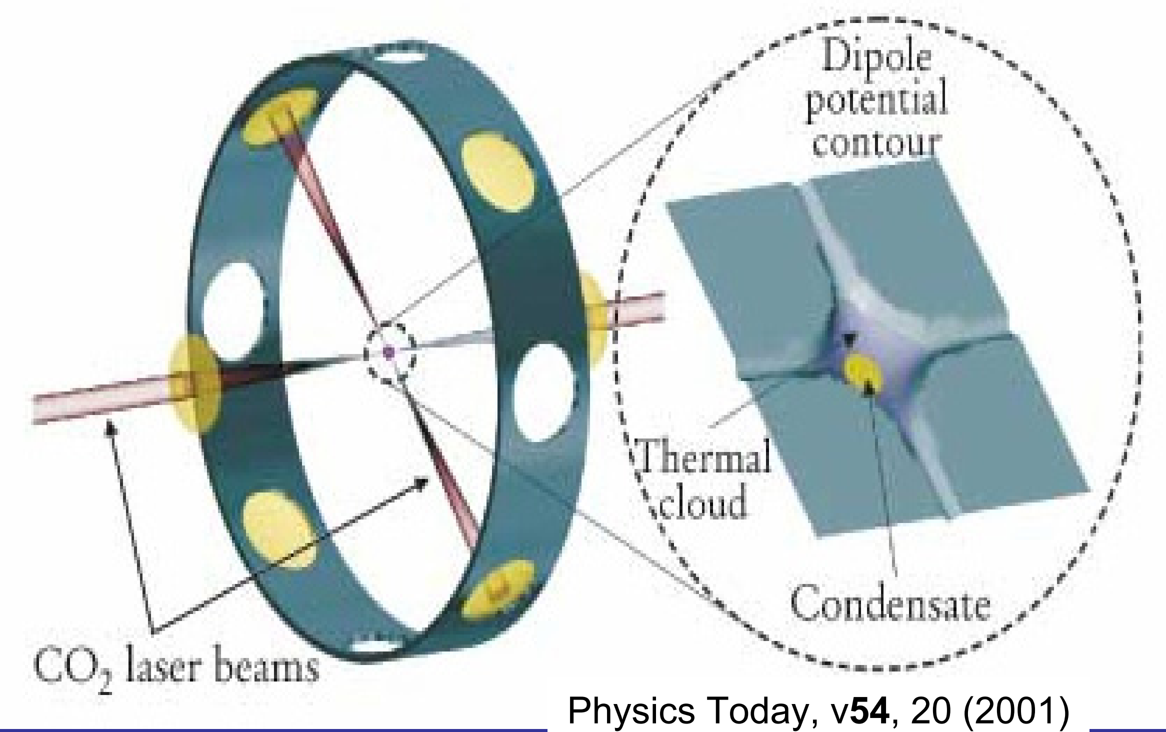
AT FINITE TEMPERATURE: $$E(T) = \xi\left(\frac{T}{\varepsilon_F}\right)E_{FG},\quad \xi(0) = \xi_0$$

Expected phases of a two species dilute Fermi system
BCS-BEC crossover



In dilute atomic systems experimenters can control nowadays almost anything:

- The number of atoms in the trap: typically about 10^5-10^6 atoms divided 50-50 among the lowest two hyperfine states.
- The density of atoms
- Mixtures of various atoms
- The temperature of the atomic cloud
- **The strength of this interaction is fully tunable!**

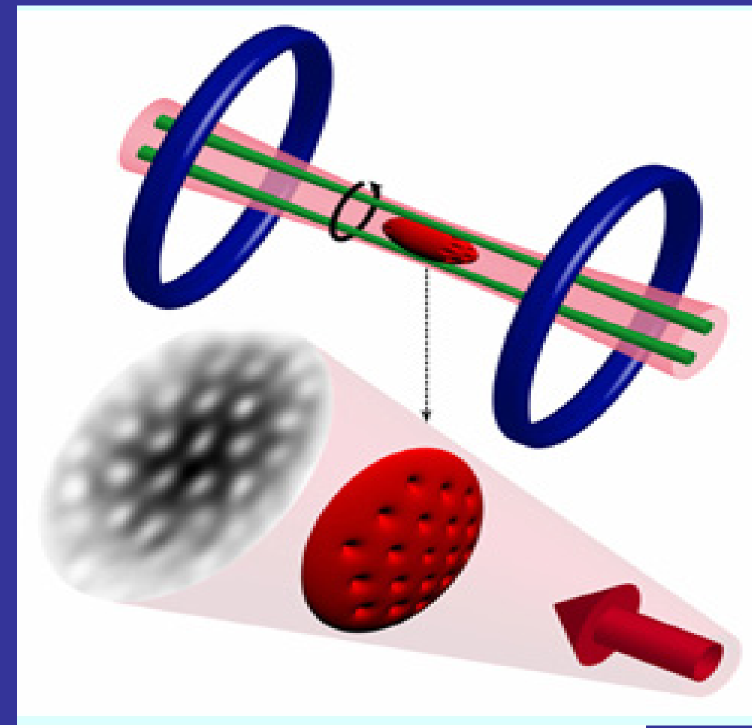


Physics Today, v**54**, 20 (2001)

Who does experiments?

- **Jin's group at Boulder**
- **Grimm's group in Innsbruck**
- **Thomas' group at Duke**
- **Ketterle's group at MIT**
- **Salomon's group in Paris**
- **Hulet's group at Rice**

Evidence for fermionic superfluidity: vortices!

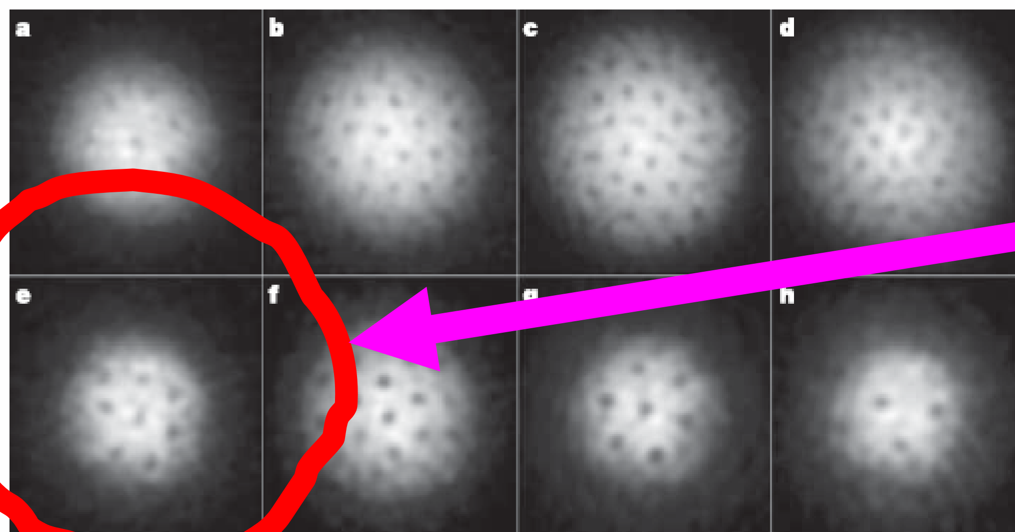


system of fermionic ^6Li atoms

Feshbach resonance: B=834G

BEC side: a>0

BCS side a<0

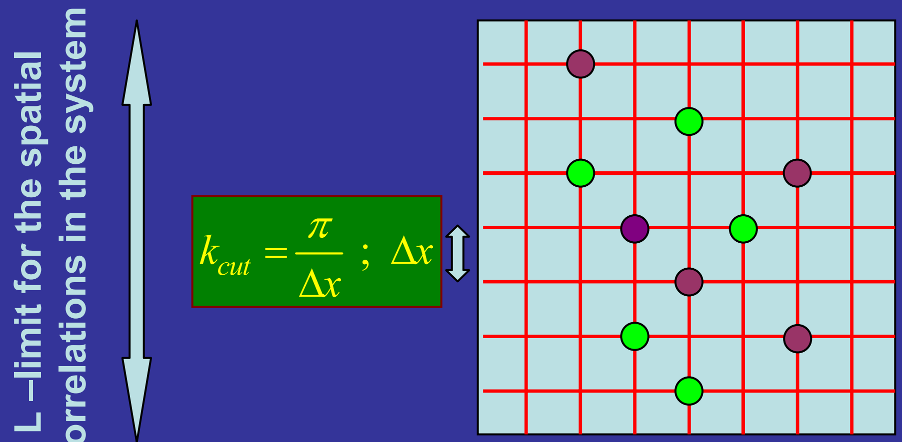


UNITARY REGIME

M.W. Zwierlein *et al.*, Nature, 435, 1047 (2005)

Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b–h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**), 812 G (**d**), 833 G (**e**), 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of view of each image is 880 μm × 880 μm.

Coordinate space



$Volume = L^3$

$lattice\ spacing = \Delta x$

- Spin up fermion

- Spin down fermion

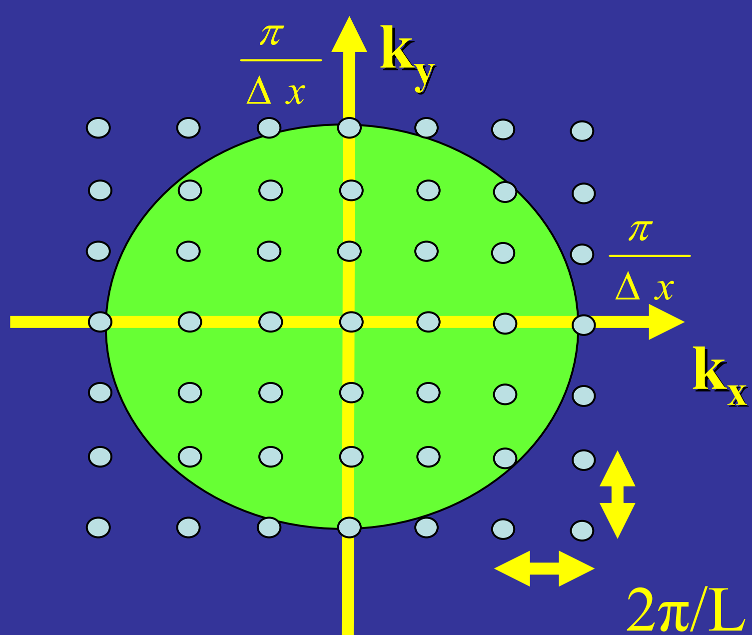
Periodic boundary conditions imposed

External conditions:

T - temperature

μ - chemical potential

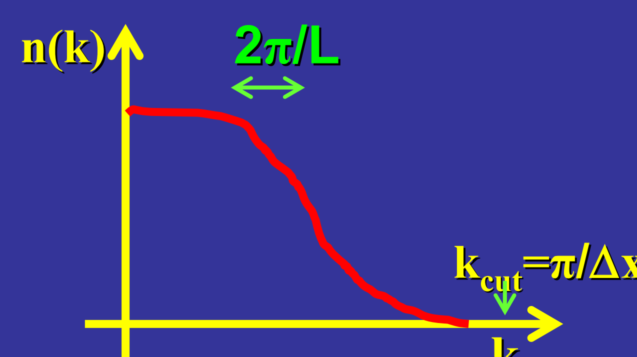
Momentum space



UV momentum cutoff $\Lambda_{UV} = \frac{\pi}{\Delta x}$

IR momentum cutoff $\Lambda_{IR} = \frac{2\pi}{L}$

$$\frac{\hbar^2 \Lambda_{IR}^2}{2m} << \varepsilon_F,\ \Delta << \frac{\hbar^2 \Lambda_{UV}^2}{2m}$$



$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \hat{\psi}_s(\vec{r}) \; - \; g \int d^3r \; \hat{n}_\uparrow(\vec{r}) \hat{n}_\downarrow(\vec{r})$$

$$\hat{N} = \int d^3r \; \left(\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r}) \right); \quad \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r}) \hat{\psi}_s(\vec{r})$$

$$\frac{1}{g} = -\frac{m}{4\pi\hbar^2 a} + \frac{m k_{cut}}{2\pi^2 \hbar^2}$$

Running coupling constant g defined by lattice

$$\frac{1}{g} = \frac{m}{2\pi\hbar^2 \Delta x}$$ - UNITARY LIMIT

Trotter expansion (*trotterization* of the propagator)

$$Z(\beta) = \mathrm{Tr}\; \exp\left[-\beta \left(\hat{H} - \mu \hat{N} \right) \right] = \mathrm{Tr}\; \left\{ \exp\left[-\tau \left(\hat{H} - \mu \hat{N} \right) \right] \right\}^{N_\tau}, \qquad \beta = \frac{1}{T} = N_\tau \tau$$

$$E(T) = \frac{1}{Z(T)} \mathrm{Tr}\; \hat{H} \exp\left[-\beta \left(\hat{H} - \mu \hat{N} \right) \right]$$

$$N(T) = \frac{1}{Z(T)} \mathrm{Tr}\; \hat{N} \exp\left[-\beta \left(\hat{H} - \mu \hat{N} \right) \right]$$

$$\exp\left[-\tau\left(\hat{H}-\mu\hat{N}\right)\right]\approx\exp\left[-\tau\left(\hat{T}-\mu\hat{N}\right)/2\right]\exp(-\tau\hat{V})\exp\left[-\tau\left(\hat{T}-\mu\hat{N}\right)/2\right]$$

$$+O(\tau^3)$$

Discrete Hubbard-Stratonovich transformation

$$\exp(-\tau\hat{V})=\prod_{\vec{r}}\sum_{\sigma(\vec{r})=\pm 1}\frac{1}{2}\left[1+\sigma(\vec{r})A\hat{n}_{\uparrow}(\vec{r})\right]\left[1+\sigma(\vec{r})A\hat{n}_{\downarrow}(\vec{r})\right],\quad A=\sqrt{\exp(\tau g)-1}$$

σ-fields fluctuate both in space and imaginary time

$$\hat{U}(\sigma)=\prod_{j=1}^{N_\tau}\hat{W}_j(\sigma);$$

$$\hat{W}_j(\sigma)=\exp\left[-\tau\left(\hat{T}-\mu\hat{N}\right)/2\right]\prod_{\vec{r}}\left[1+\sigma(\vec{r})A\hat{n}_{\uparrow}(\vec{r})\right]\left[1+\sigma(\vec{r})A\hat{n}_{\downarrow}(\vec{r})\right]\exp\left[-\tau\left(\hat{T}-\mu\hat{N}\right)/2\right]$$

$$Z(T) = \int D\sigma(\vec{r},\tau)\ \mathrm{Tr}\,\hat{U}(\{\sigma\});$$

$$\int D\sigma(\vec{r},\tau) \equiv \sum_{\{\sigma(\vec{r},1)=\pm 1\}} \sum_{\{\sigma(\vec{r},2)=\pm 1\}} \cdots \sum_{\{\sigma(\vec{r},N_\tau)=\pm 1\}} \quad ; \quad N_\tau \tau = \frac{1}{T}$$

$$\hat{U}(\{\sigma\}) = T_\tau \exp\{-\int_0^\beta d\tau[\hat{h}(\{\sigma\}) - \mu]\}$$

One-body evolution operator in imaginary time

$$E(T) = \int \frac{D\sigma(\vec{r},\tau)\mathrm{Tr}\,\hat{U}(\{\sigma\})}{Z(T)} \frac{\mathrm{Tr}\left[\hat{H}\hat{U}(\{\sigma\})\right]}{\mathrm{Tr}\,\hat{U}(\{\sigma\})}$$

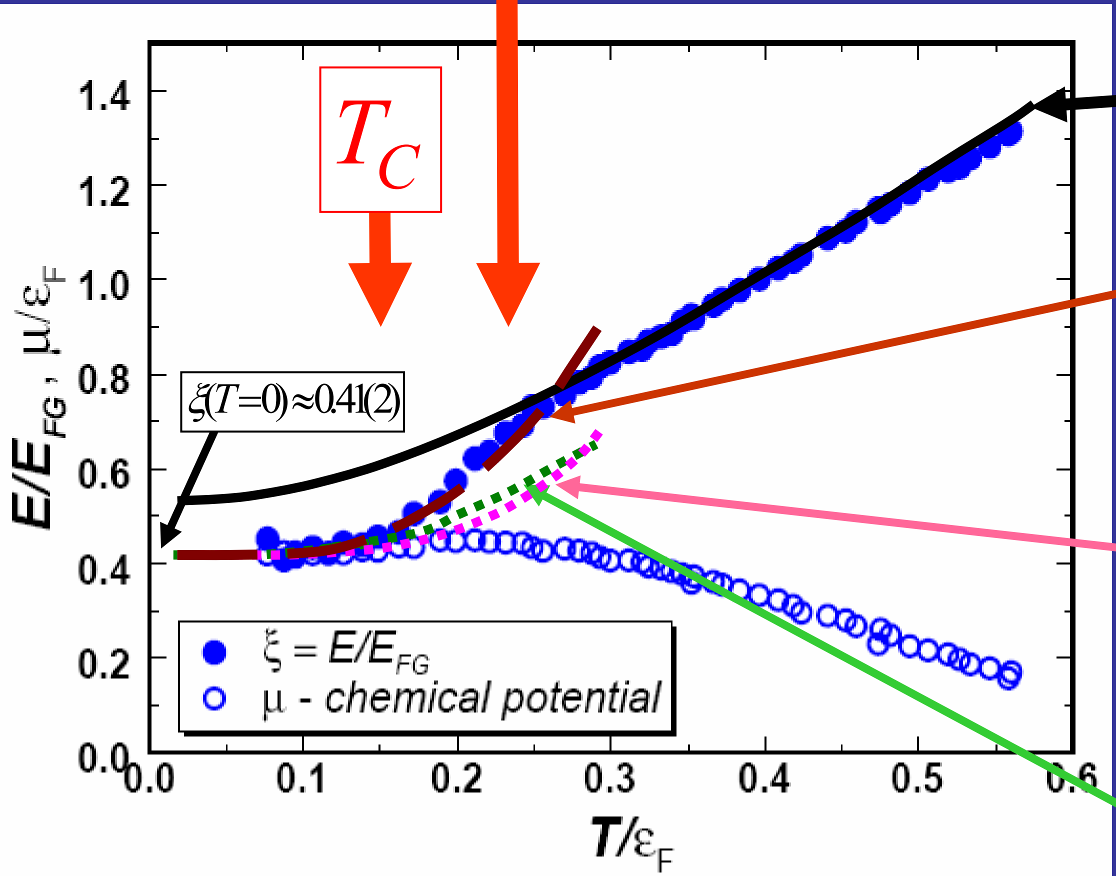
$$\mathrm{Tr}\,\hat{U}(\{\sigma\}) = \{\det[1 + \hat{U}_\uparrow(\sigma)]\}^2 = \exp[-S(\{\sigma\})] > 0$$

No sign problem!

$$n_\uparrow(\vec{x},\vec{y}) = n_\downarrow(\vec{x},\vec{y}) = \sum_{k,l<k_c} \psi_{\vec{k}}(\vec{x}) \left[\frac{U(\{\sigma\})}{1 + U(\{\sigma\})}\right]_{\vec{k}\vec{l}} \psi^*_{\vec{l}}(\vec{y}), \qquad \psi_{\vec{k}}(\vec{x}) = \frac{\exp(i\vec{k}\cdot\vec{x})}{\sqrt{L^3}}$$

<u>All traces can be expressed through these single-particle density matrices</u>

$a = \pm\infty$ Deviation from Normal Fermi Gas



Normal Fermi Gas (with vertical offset, solid line)

Bogoliubov-Anderson phonons and quasiparticle contribution (dashed line)

Bogoliubov-Anderson phonons contribution only (dotted line)

Quasi-particle contribution only (dotted line)

$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}} \exp\left(-\frac{\Delta}{T}\right)$$

$$\Delta = \left(\frac{2}{e}\right)^{7/3} \varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.41$$

A. Bulgac, J.E. Drut, P. Magierski, PRL96,090404(2006)

Thermodynamics of the unitary Fermi gas

$$\text{ENERGY:}\quad E(x)=\frac{3}{5}\xi(x)\varepsilon_F N;\quad x=\frac{T}{\varepsilon_F}$$

$$C_V=T\frac{\partial S}{\partial T}=\frac{\partial E}{\partial T}=\frac{3}{5}N\xi'(x)\Rightarrow S(x)=\frac{3}{5}N\int_0^x\frac{\xi'(y)}{y}dy$$

$$\text{ENTROPY/PARTICLE:}\quad \sigma(x)=\frac{S(x)}{N}=\frac{3}{5}\int_0^x\frac{\xi'(y)}{y}dy$$

$$\text{FREE ENERGY:}\ F=E-TS=\frac{3}{5}\varphi(x)\varepsilon_F N$$

$$\varphi(x)=\xi(x)-x\sigma(x)$$

$$\text{PRESSURE:}\quad P=-\frac{\partial E}{\partial V}=\frac{2}{5}\xi(x)\varepsilon_F\frac{N}{V}$$

$$PV=\frac{2}{3}E$$

Note the similarity to the ideal Fermi gas

Low temperature behaviour of a Fermi gas in the unitary regime

$$F(T) = \frac{3}{5}\varepsilon_F N\varphi\left(\frac{T}{\varepsilon_F}\right) = E - TS \quad \text{and} \quad \frac{\mu(T)}{\varepsilon_F} \approx \xi_s \approx 0.41(2) \text{ for } T < T_C$$

$$\mu(T) = \frac{dF(T)}{dN} = \varepsilon_F\left[\varphi\left(\frac{T}{\varepsilon_F}\right) - \frac{2}{5}\frac{T}{\varepsilon_F}\varphi'\left(\frac{T}{\varepsilon_F}\right)\right] \approx \varepsilon_F \xi_s$$

$$\varphi\left(\frac{T}{\varepsilon_F}\right) = \varphi_0 + \varphi_1\left(\frac{T}{\varepsilon_F}\right)^{5/2}$$

$$E(T) = \frac{3}{5}\varepsilon_F N\left[\xi_s + \varsigma_s\left(\frac{T}{\varepsilon_F}\right)^n\right]$$

Lattice results disfavor either n≥3 or n≤2 and suggest n=2.5(0.25)

This is the same behavior as for a gas of noninteracting (!) bosons below the condensation temperature.

Experiment

John Thomas' group at Duke University,

L.Luo, et al. Phys. Rev. Lett. 98, 080402, (2007)

Dilute system of fermionic ^{6}Li atoms in a harmonic trap

- The number of atoms in the trap: N=1.3(0.2) x 10^5 atoms divided 50-50 among the lowest two hyperfine states.
- Fermi energy: $\varepsilon_F^{ho} = \hbar\Omega(3N)^{1/3}; \;\; \Omega = \left(\omega_x\omega_y\omega_z\right)^{1/3}$

$$\varepsilon_F^{ho} / k_B \approx 1\mu K$$

- Depth of the potential: $U_0 \approx 10\varepsilon_F^{ho}$
- How they measure: energy, entropy and temperature?

$$\left.\begin{array}{l} PV = \dfrac{2}{3}E \\ \vec{\nabla}P = -n(\vec{r})\vec{\nabla}U \end{array}\right\} \Rightarrow N\langle U\rangle = \frac{E}{2} \text{ - virial theorem}$$

$n(\vec{r})$ - local density

Holds at unitarity and for noninteracting Fermi gas

Theory: local density approximation (LDA)

Uniform system

$$\Omega = F - \lambda N = \frac{3}{5}\varphi(x)\varepsilon_F N - \lambda N$$

Nonuniform system *(gradient corrections neglected)*

$$\Omega = \int d^3 r \left[\frac{3}{5}\varepsilon_F(\vec{r})\varphi(x(\vec{r})) + U(\vec{r}) - \lambda \right] n(\vec{r})$$

$$x(\vec{r}) = \frac{T}{\varepsilon_F(\vec{r})}; \quad \varepsilon_F(\vec{r}) = \frac{\hbar^2}{2m}\left[3\pi^2 n(\vec{r})\right]^{2/3}$$

The overall chemical potential λ and the temperature T are constant throughout the system. The density profile will depend on the shape of the trap as dictated by:

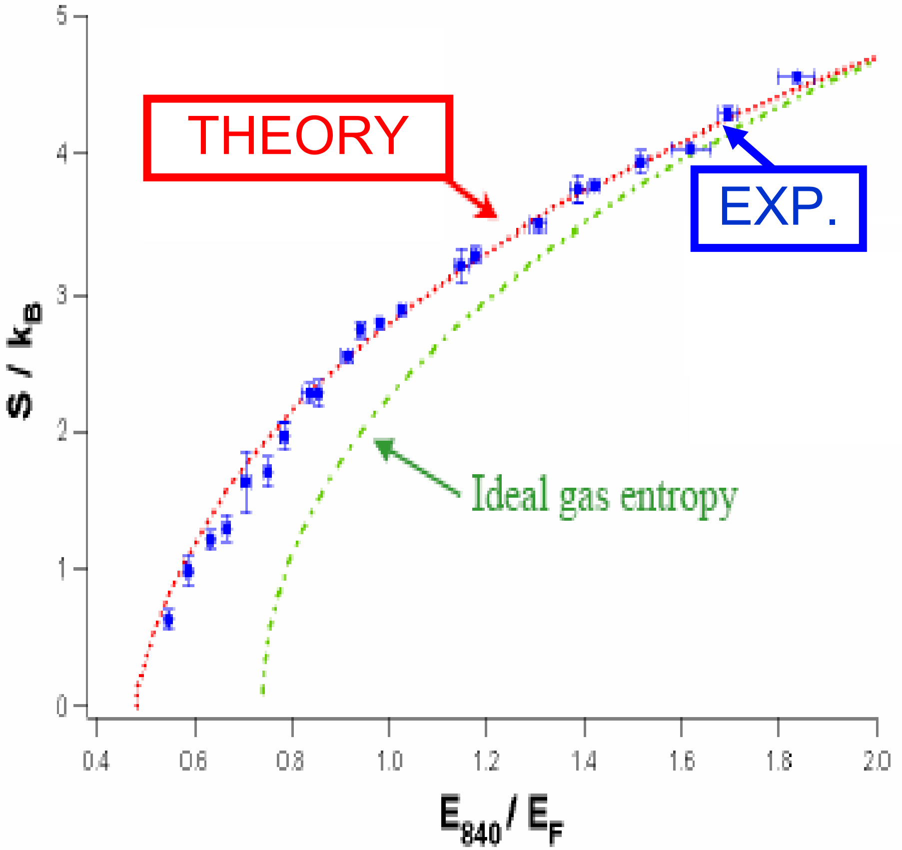
$$\frac{\delta\Omega}{\delta n(\vec{r})} = \frac{\delta(F - \lambda N)}{\delta n(\vec{r})} = \mu(x(\vec{r})) + U(r) - \lambda = 0$$

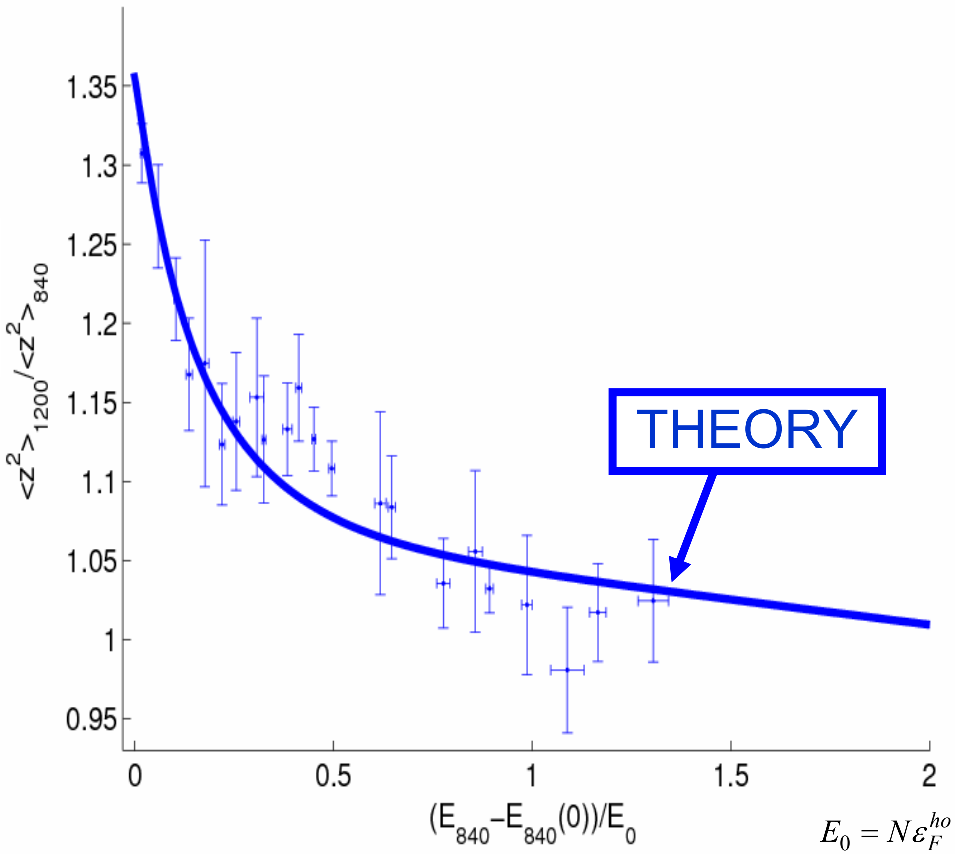
Using as an input the Monte Carlo results for the uniform system and experimental data (trapping potential, number of particles), we determine the density profiles.

Comparison with experiment

John Thomas' group at Duke University,

L.Luo, et al. Phys. Rev. Lett. 98, 080402, (2007)





Entropy as a function of energy (relative to the ground state) for the unitary Fermi gas in the harmonic trap.

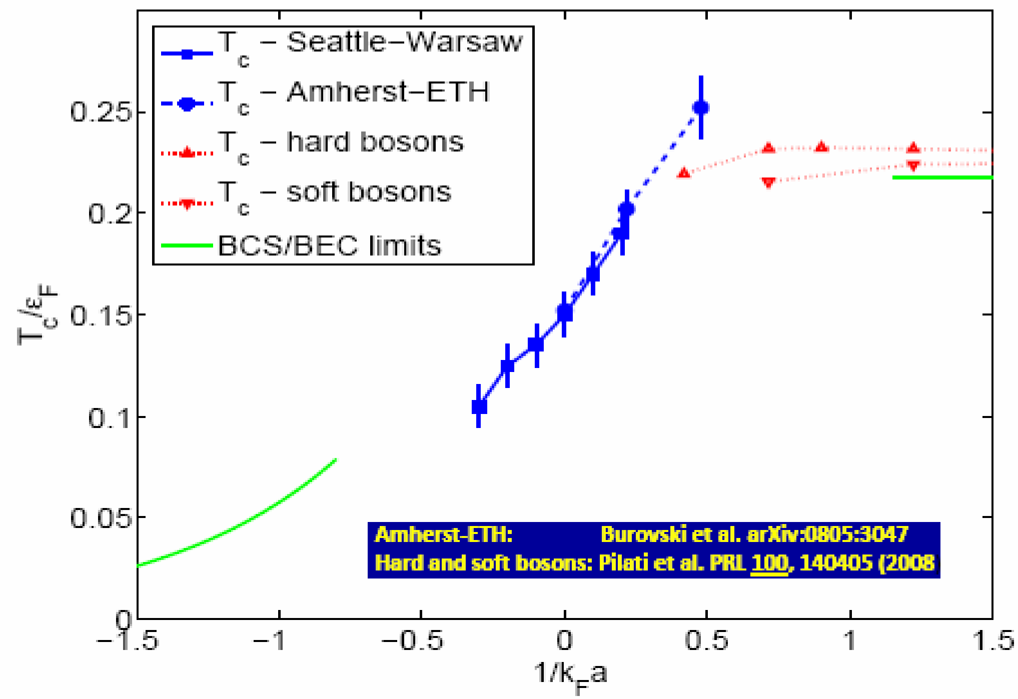
Ratio of the mean square cloud size at B=1200G to its value at unitarity (B=840G) as a function of the energy. Experimental data are denoted by point with error bars.

$$B = 1200G \Rightarrow 1/k_F a \approx -0.75$$

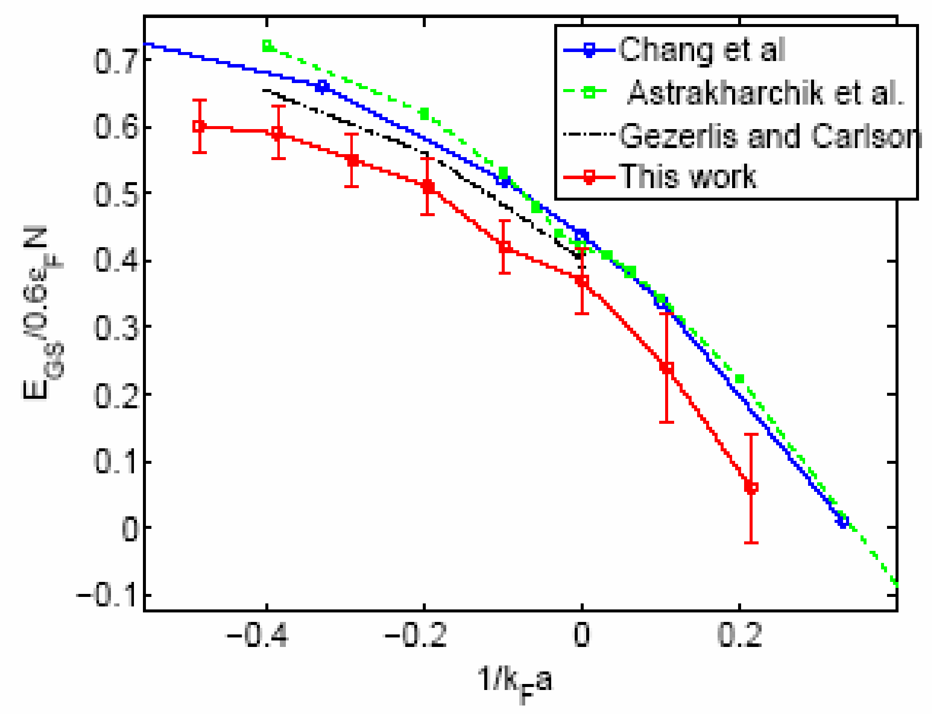
Theory: **Bulgac, Drut, and Magierski PRL 99, 120401 (2007)**

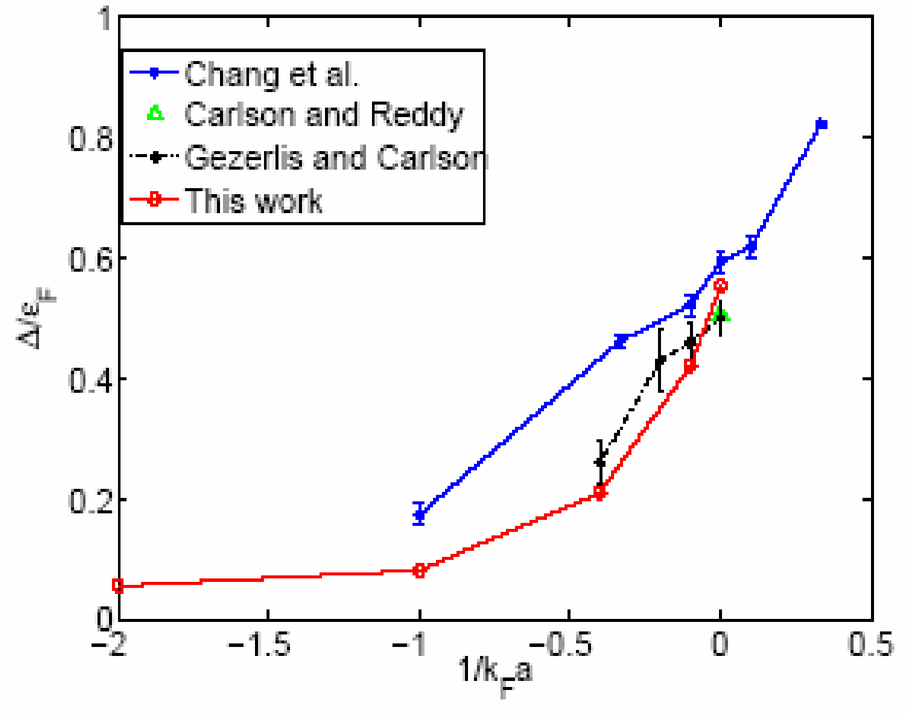
Results in the vicinity of the unitary limit:

- Critical temperature
- Ground state energy
- Pairing gap



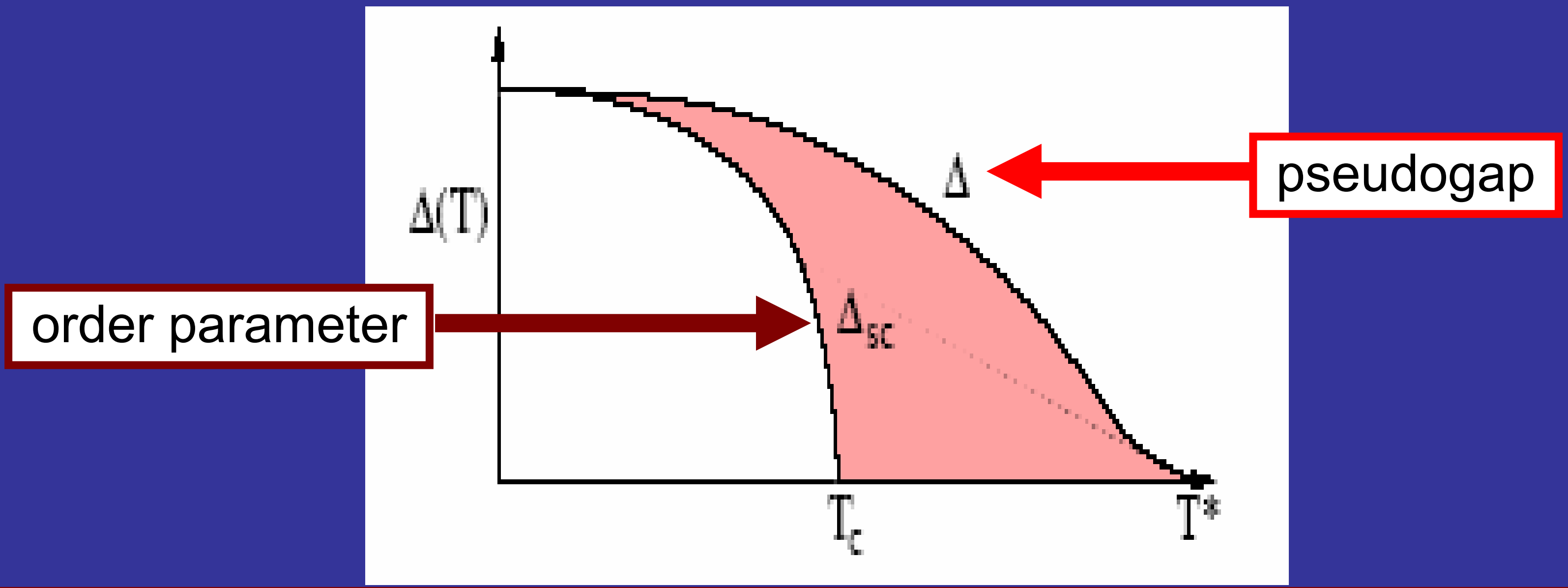
Bulgac, Drut, Magierski, PRA78, 023625(2008)





Pairing gap and pseudogap

Outside the BCS regime close to the unitary limit, but still before BEC, superconductivity/superfluidity emerge out of a very exotic, non-Fermi liquid normal state



The onset of superconductivity occurs in the presence of fermionic pairs!

Quantum Monte Carlo

Preliminary results indicate nonvanishing gap in the single-particle spectrum at the critical temperature.

$$\Delta(T_C) \approx 0.4\varepsilon_F$$

Bulgac, Drut, Magierski, and Wlazowski, arXiv:0801:1504

Conclusions

- ✓ Fully non-perturbative calculations for a spin ½ many fermion system in the unitary regime at finite temperatures are feasible and apparently the system undergoes a phase transition in the bulk at $T_c = 0.15\ (1)\ \varepsilon_F$.

- ✓ Between T_c and T_0 =0.23(2) ε_F the system is neither superfluid nor follows the normal Fermi gas behavior. Possibly due to pairing effects.

- ✓ Chemical potential is constant up to the T_0 – note similarity with Bose systems!

- ✓ Below the transition temperature, both phonons and fermionic quasiparticles contribute almost equaly to the specific heat. In more than one way the system is at crossover between a Bose and Fermi systems.

- ✓ Results (energy, entropy vs temperature) agree with recent measurments: L. Luo et al., PRL 98, 080402 (2007)

- ✓ There is an evidence for the existence of *pseudogap* at unitarity.

Superconductivity and superfluidity in Fermi systems

20 orders of magnitude over a century of (low temperature) physics

- ✓ **Dilute atomic Fermi gases** $T_c \approx 10^{-12} – 10^{-9}$ eV
- ✓ **Liquid ^{3}He** $T_c \approx 10^{-7}$ eV
- ✓ **Metals, composite materials** $T_c \approx 10^{-3} – 10^{-2}$ eV
- ✓ **Nuclei, neutron stars** $T_c \approx 10^{5} – 10^{6}$ eV
- **QCD color superconductivity** $T_c \approx 10^{7} – 10^{8}$ eV

units (1 eV $\approx 10^4$ K)