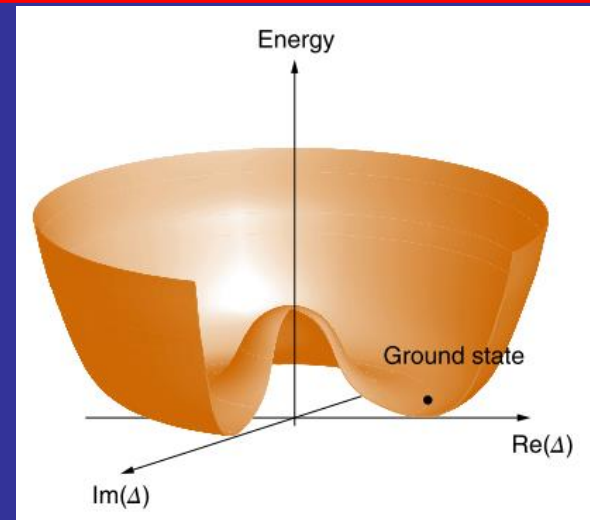
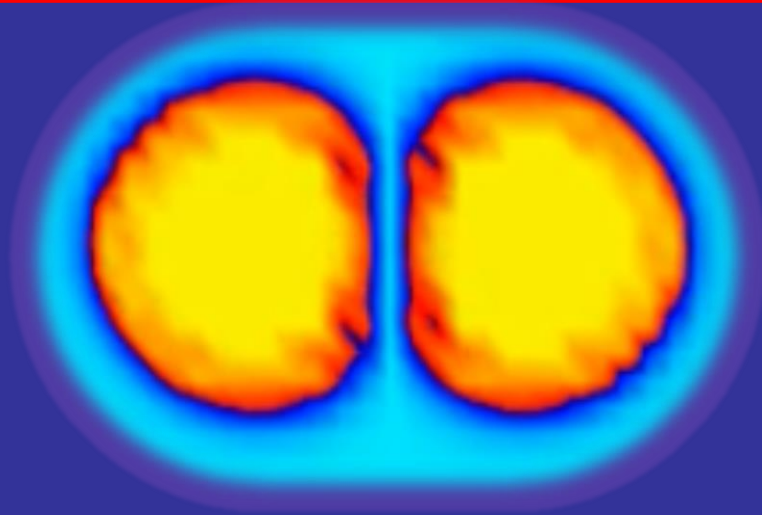


# *Mody Nambu-Goldstone'a i Higgosa w zderzeniach jąder atomowych*



*Piotr Magierski*  
*(Warsaw University of Technology)*

## Collaborators:

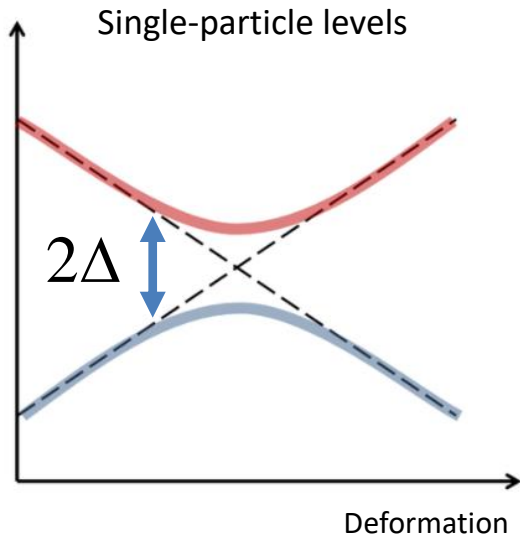
Matthew Barton

Andrzej Makowski (Ph.D. Student)

Kazuyuki Sekizawa (Tokyo Inst. of Tech.)

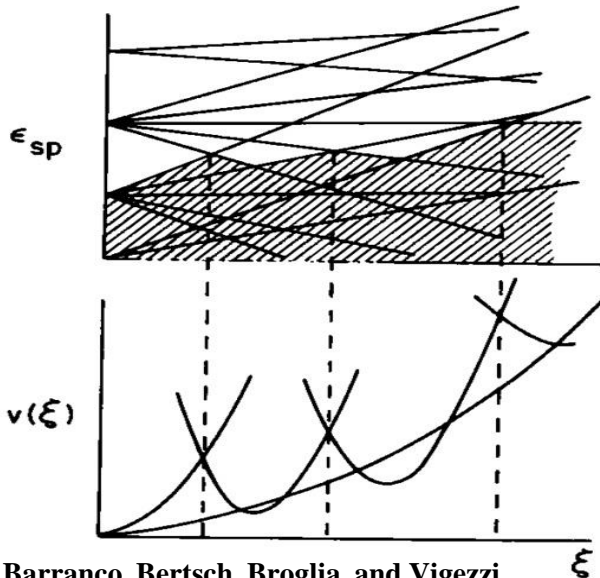
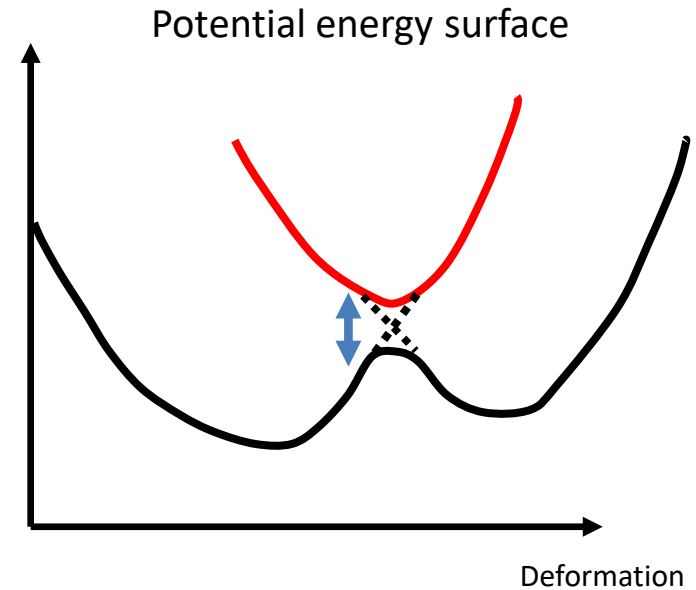
Gabriel Wlazłowski

# Pairing as an energy gap



Quasiparticle energy:

$$E_{qp} = \sqrt{(\varepsilon - \mu)^2 + |\Delta|^2}$$



As a consequence of pairing correlations large amplitude nuclear motion becomes more adiabatic.

While a nucleus elongates its Fermi surface becomes oblate and its sphericity must be restored  
 Hill and Wheeler, PRC, 89, 1102 (1953)  
 Bertsch, PLB, 95, 157 (1980)

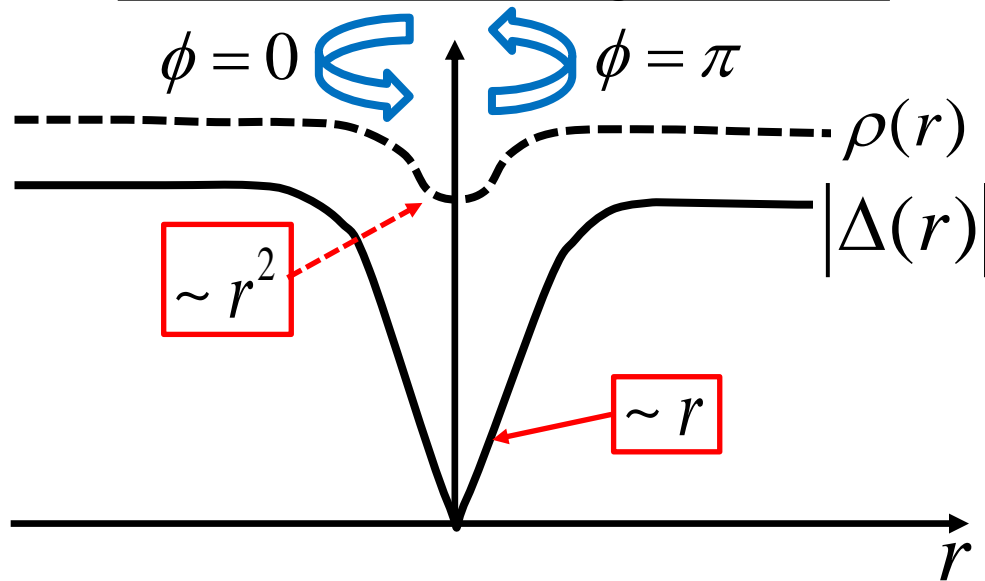
# Pairing as a field

$$\Delta(\vec{r}, t) = |\Delta(\vec{r}, t)| e^{i\phi(\vec{r}, t)}$$

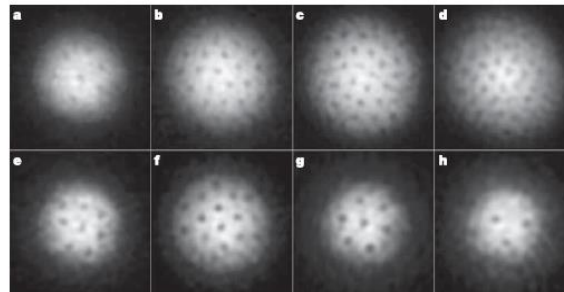
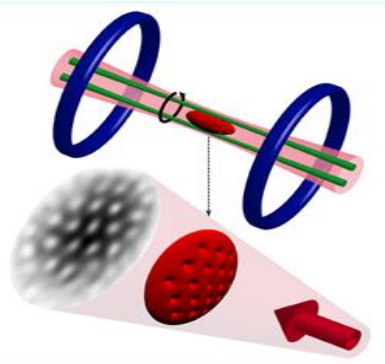
Both magnitude and phase may have a nontrivial spatial and time dependence.

Example of a nontrivial spatial dependence: *quantum vortex*

Vortex structure – section through the vortex core



Example of a topological excitation: magnitude of the pairing gap vanishes in the vortex core.



Experiments with ultracold Li-6 atoms: pictures of the vortex lattice.

Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) or 500 ms (b–h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the

magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (a), 766 G (b), 792 G (c), 812 G (d), 833 G (e), 843 G (f), 853 G (g) and 863 G (h). The field of view of each image is  $880 \mu\text{m} \times 880 \mu\text{m}$ .

M.W. Zwierlein *et al.*,  
Nature, 435, 1047 (2005)

# The well known effects in superconductors where the simplified BCS approach fails

1) Quantum vortices, solitonic excitations related to pairing field (e.g. domain walls)

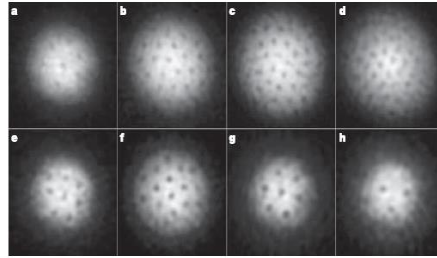
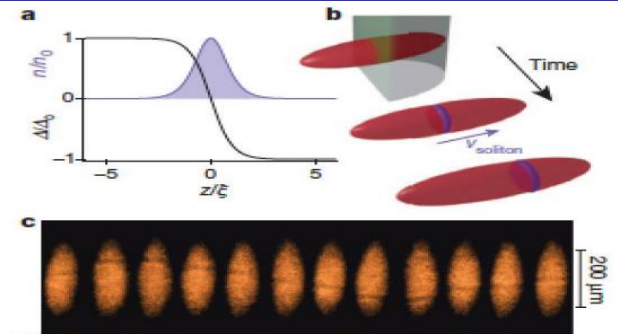
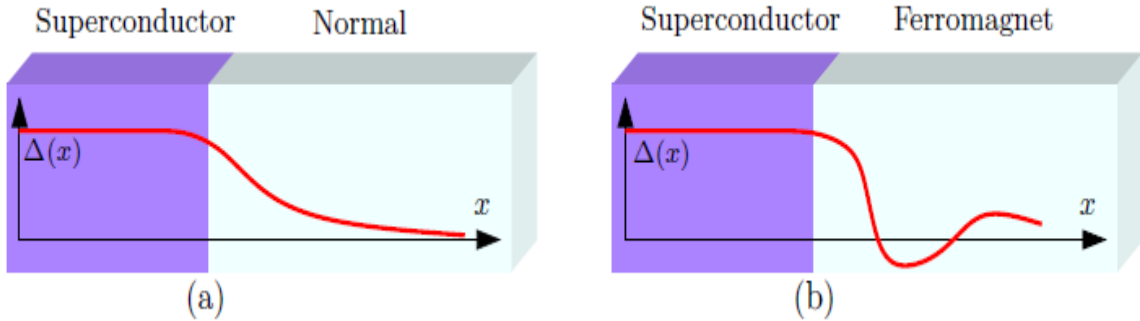


Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) or 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (a), 766 G (b), 792 G (c), 812 G (d), 833 G (e), 843 G (f), 853 G (g) and 863 G (h). The field of view of each image is  $880 \mu\text{m} \times 880 \mu\text{m}$ .

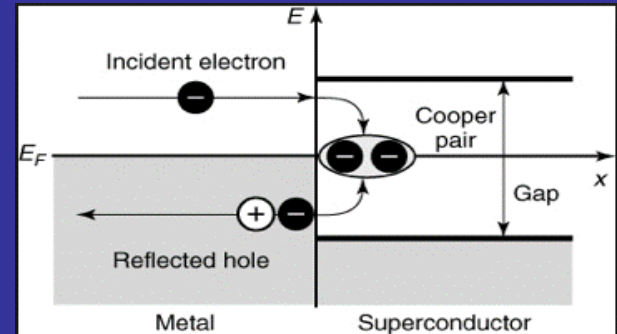
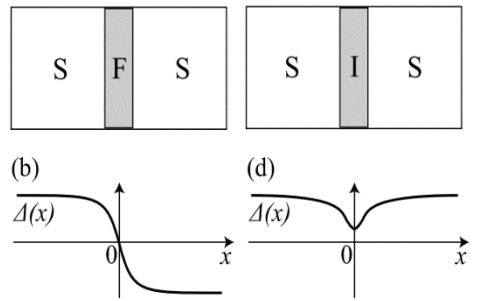


2) Bogoliubov – Anderson phonons

3) proximity effects: variations of the pairing field on the length scale of the coherence length.



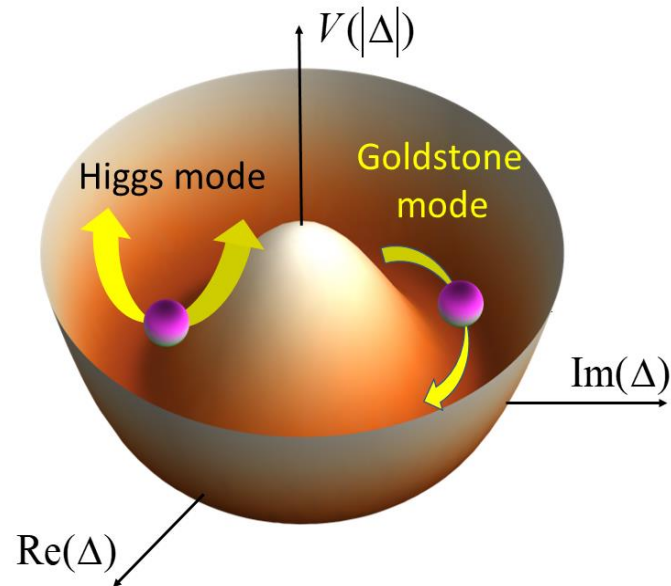
4) physics of Josephson junction (superfluid - normal metal), pi-Josephson junction (superfluid - ferromagnet)



5) Andreev reflection (particle-into-hole and hole-into-particle scattering) Andreev states cannot be obtained within BCS

$$\Delta(\vec{r}, t) = |\Delta(\vec{r}, t)| e^{i\phi(\vec{r}, t)}$$

Appearance of pairing field in Fermi systems is associated with U(1) symmetry breaking.



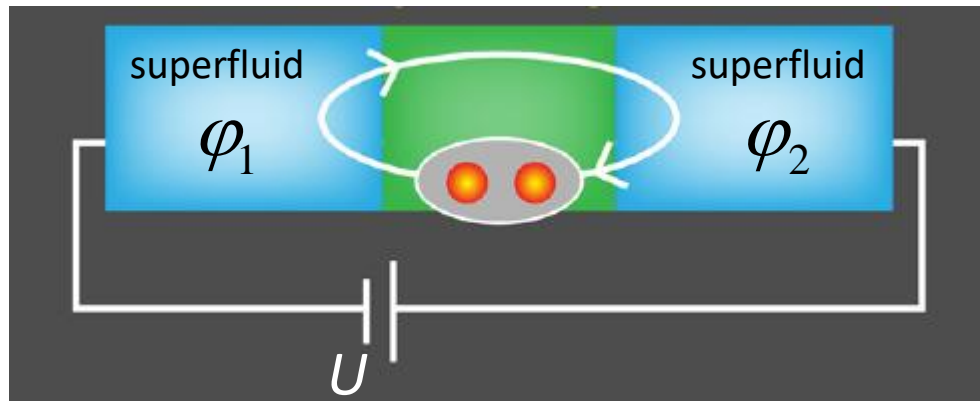
There are two characteristic modes associated with  $\Delta(\vec{r}, t)$

- 1) **Nambu-Goldstone mode** explores the degree of freedom associated with the phase:  $\phi(\vec{r}, t)$
- 2) **Higgs mode** explores the degree of freedom associated with the magnitude:  $|\Delta(\vec{r}, t)|$

# Probing phase degree of freedom of pairing field

The well known example is Josephson junction:

- DC Josephson junction:  $U = 0$
- AC Josephson junction:  $U \neq 0$



$$\Delta\varphi = \varphi_1 - \varphi_2$$

$$J(t) = J_c \sin(\Delta\varphi(t))$$
$$\frac{d(\Delta\varphi)}{dt} = \frac{2eU}{\hbar}$$

Relation between Josephson current and phase differences Between pairing fields.

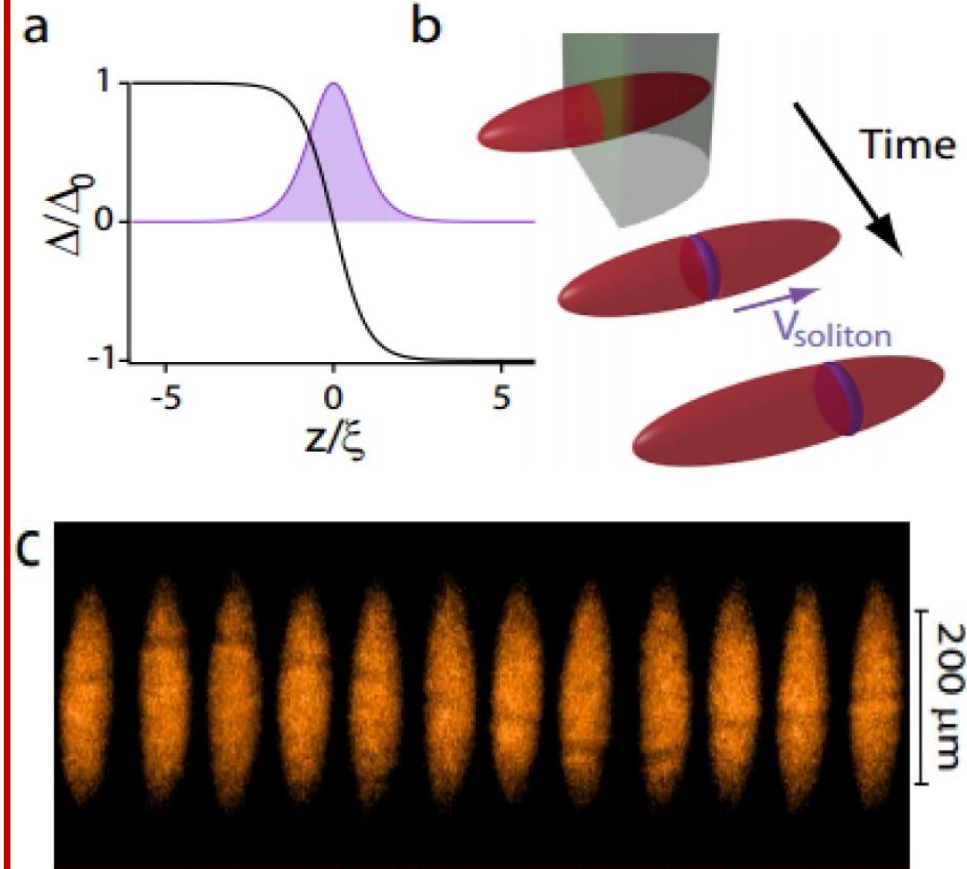
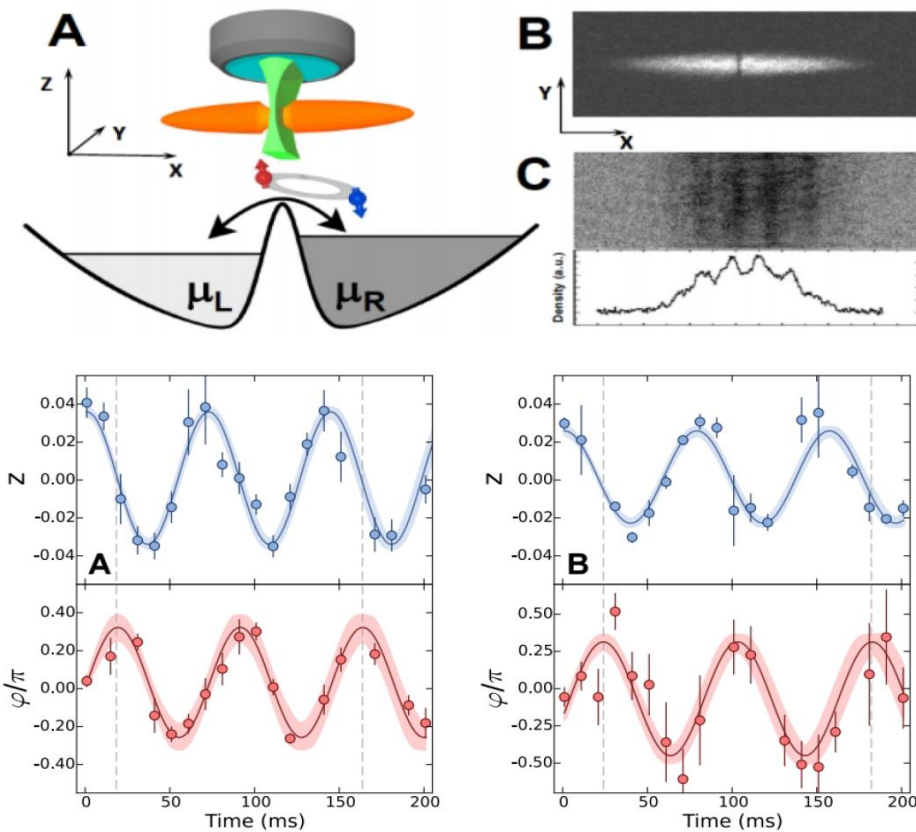
**Important:** Josephson junction means usually so-called *weak link*.

**Pairing condensates on both sides are assumed to remain unperturbed by the Josephson current.**

# Ultracold atomic gases: two regimes for realization of the Josephson junction

Weak coupling (weak link)

Strong coupling



Observation of **AC Josephson effect** between two  $6\text{Li}$  atomic clouds.

It need not to be accompanied by creation of a topological excitation.

G. Valtolina et al., Science 350, 1505 (2015).

Creation of a „heavy soliton“ after merging two superfluid atomic clouds.

T. Yefsah et al., Nature 499, 426 (2013).

# Collisions of two superfluid nuclei

## Physics of two nuclear, coupled superconductors

Little bit of history:

Volume 1, number 7

PHYSICS LETTERS

1 July 1962

### POSSIBLE NEW EFFECTS IN SUPERCONDUCTIVE TUNNELLING \*

B. D. JOSEPHSON

Cavendish Laboratory, Cambridge, England

Received 8 June 1962

We here present an approach to the calculation of tunnelling currents between two metals that is sufficiently general to deal with the case when both metals are superconducting. In that case new effects are predicted, due to the possibility that electron pairs may tunnel through the barrier leaving the quasi-particle distribution unchanged.

$$J(t) = J_c \sin(\Delta\varphi(t))$$

$$\frac{d(\Delta\varphi)}{dt} = \frac{2eU}{\hbar}$$

Dynamics of the Josephson effect:



*AN ANALOG OF THE JOSEPHSON EFFECT IN NUCLEAR TRANSFORMATIONS*

V. I. GOL'DANSKIĬ and A. I. LARKIN

Institute of Chemical Physics, Academy of Science, U.S.S.R.

Submitted March 30, 1967

Zh. Eksp. Teor. Fiz. 53, 1032-1037 (September, 1967)

When nuclei are bombarded by heavy ions, various processes of nucleon tunneling through the potential barrier that separates the interacting nuclei at the smallest possible classical distance are observed. It is shown that nucleon pairing may give rise to a significant increase of the cross section for the transition of neutron or proton pairs, a phenomenon which in some respects is analogous to the Josephson effect in superconductors. Pairing is taken into account in the calculation of the probability for the excitation of various levels by one-nucleon exchange, which has been calculated earlier by Breit and Ebel<sup>[1]</sup> without such corrections. The probability for two-nucleon exchange is determined. An expression is obtained for the two-proton radioactivity with account of any number of arbitrary levels, which goes over into the Galitskii-Chel'tsov formula<sup>[2]</sup> in the limiting case of a single S level.

Volume 32B, number 6

PHYSICS LETTERS

17 August 1970

## ON A NUCLEAR JOSEPHSON EFFECT IN HEAVY ION SCATTERING

K. DIETRICH

*Niels Bohr Institute, Copenhagen\*, Denmark*

Received 3 June 1970

The transfer of a pair of nucleons in sub-Coulomb scattering of two heavy ions is treated in a semi-classical theory. If both reaction partners are superconducting, a large enhancement factor is found.

## Brief Reports

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*Brief Reports are short papers which report on completed research or are addenda to papers previously published in the Physical Review. A Brief Report may be no longer than  $3\frac{1}{2}$  printed pages and must be accompanied by an abstract.*

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### Weak evidence for a nuclear Josephson effect in the $^{34}\text{S}(^{32}\text{S}, ^{32}\text{S})$ elastic scattering reaction

Michel C. Mermaz

*Service de Physique Nucléaire—Métrologie Fondamentale, Centre d'Etudes Nucléaires de Saclay,  
91191 Gif-sur-Yvette Cedex, France*

(Received 30 March 1987)

Optical model and exact finite range distorted-wave Born approximation analyses were performed on neutron pair exchange and alpha particle exchange reactions between two identical colliding cores. The possibility of a nuclear Josephson effect is discussed.

### Neutron pair and proton pair transfer reactions between identical cores in the sulfur region

Michel C. Mermaz

*Commissariat à l'Energie Atomique, Service de Physique Nucléaire, Centre d'études de Saclay, 91191 Gif sur Yvette, Cedex, France*

Michel Girod

*Commissariat à l'Energie Atomique, Service de Physique et Techniques Nucléaires, Boîte Postale 12, 91680 Bruyères-le-Châtel, France*

(Received 1 December 1995)

Optical model and exact finite range distorted-wave Born approximation analyses were performed on neutron pair exchange between identical cores for  $^{32}\text{S}$  and  $^{34}\text{S}$  nuclei and on proton pair exchange between identical cores for  $^{30}\text{Si}$  and  $^{32}\text{S}$ . The extracted spectroscopic factors were compared with theoretical ones deduced from Hartree-Fock calculations on these pairs of nuclei. The enhancement of the experimental cross sections with respect to the theoretical ones strongly suggests evidence for a nuclear Josephson effect.

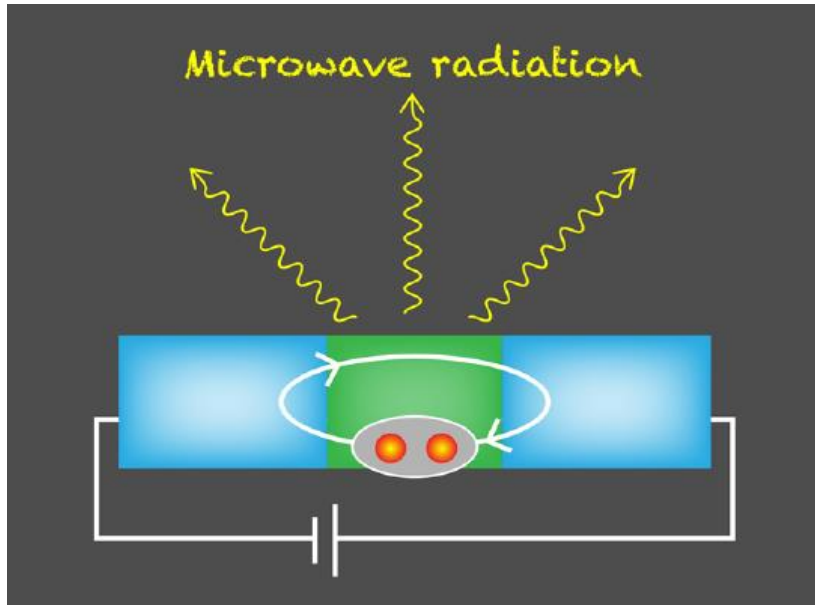
# Recent evidence for nuclear AC Josephson junction through gamma emission

## IDEA:

AC Josephson current produces microwave radiation

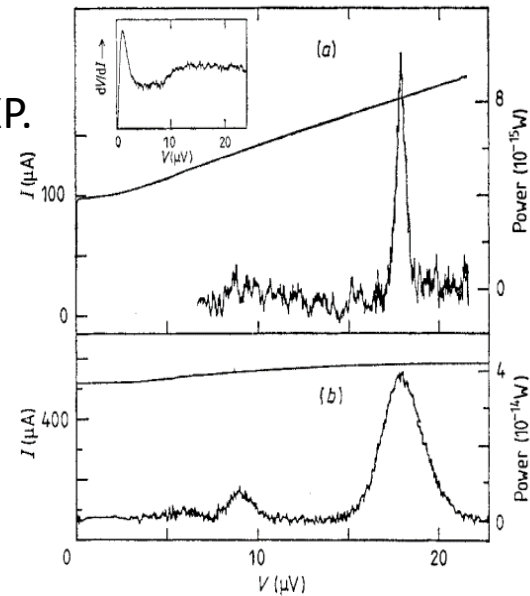
S. Shapiro, "Josephson currents in superconducting tunneling: The effect of microwaves and other observations," *Phys. Rev. Lett.* **11**, 80 (1963).

$$J(t) = J_c \sin\left(\frac{2eV}{\hbar}t\right) \xrightarrow{\text{Josephson radiation}} \omega = \frac{2eV}{\hbar}$$



From P. Magierski, *Physics* 14 (2021) 27.

EXP.



**Figure 17.** The Josephson radiation detected at 9.2 GHz from an indium microbridge as a function of voltage. The two traces (a) and (b) are taken at temperatures  $T=3.42$  K and 3.35 K, respectively. At submultiples of the voltage, at which a harmonic of the Josephson frequency equals the detector frequency, microwave power is observed. The linewidths of the signals are an increasing function of the dynamic resistance at the bias point. To indicate this, the current-voltage characteristics are also shown. The radiation detection is performed using a broadband microwave transformer coupling which avoids problems with non-Josephson radiation and self-resonance steps (see §5.5). The inserted  $d\langle V \rangle/dI$  against  $\langle V \rangle$  curve shows that there are no self-induced cavity steps (Soerensen *et al* 1977).

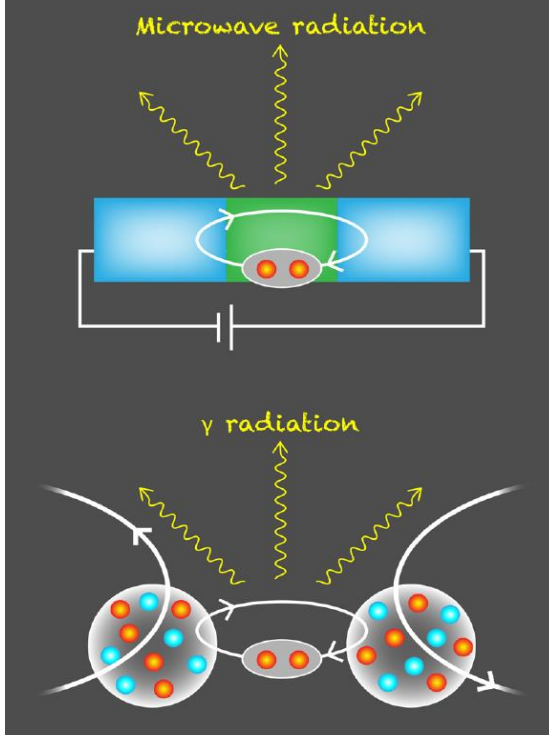
$^{116}\text{Sn} + ^{60}\text{Ni}$  ( $140.6 < E_{\text{cm}} < 167.95$  MeV) has been analyzed by:

C.Potel, F.Barranco, E.Vigezzi, R.A. Broglia, "Quantum entanglement in nuclear Cooper-pair tunneling with gamma rays," Phys.Rev. C103, L021601 (2021)  
 R. Broglia, F. Barranco, G. Potel, E. Vigezzi  
 „Transient Weak Links between Superconducting Nuclei: Coherence Length”  
 Nuclear Physics News 31, 25 (2021)

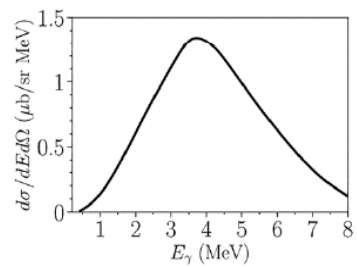
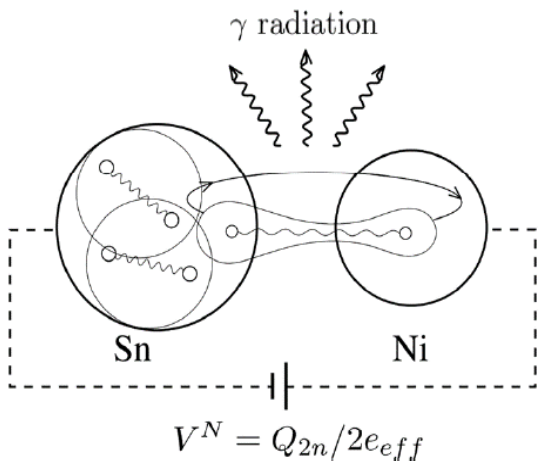
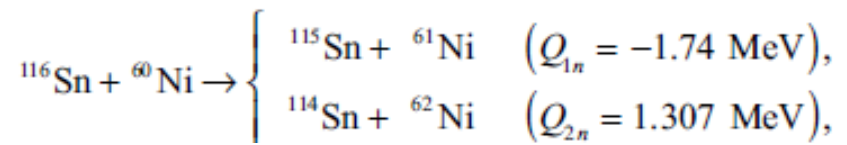
They realized that due to the fact that energy of a neutron pair is different in each nucleus it should create an effective „voltage” between nuclei and consequently to AC Josephson junction.

As a result one should witness oscillatory motion of neutron Cooper pairs between nuclei (only about 3 oscillations can occur).

This in turn would induce proton charge oscillations and give rise gamma emission.



From P. Magierski, *Physics 14* (2021) 27.



The authors state:  
 „...theory predicts the reduced gamma-strength [...] corresponding to an observable gamma-strength function [...] peaked at  $\approx 4\text{MeV}$ .  
 It can be concluded that a nuclear analogue to the (ac) Josephson junction has been identified.”  
 Phys.Rev. C103, L021601(2021)

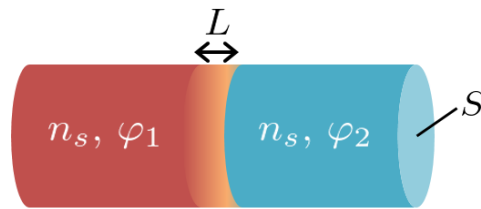
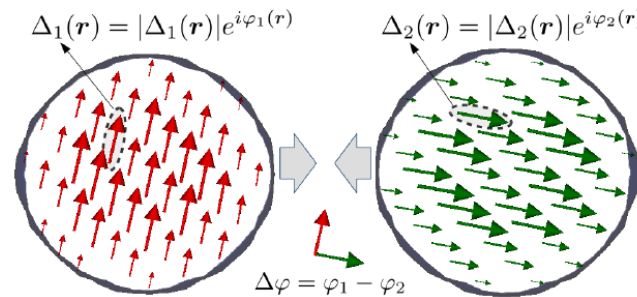
# „Josephson junction” above the barrier for capture

Collisions of superfluid nuclei having different phases of the pairing fields

The main questions are:

- how a possible solitonic structure can be manifested in nuclear system?
- what observable effect it may have on heavy ion reaction:  
kinetic energy distribution of fragments, capture cross section, etc.?

Clearly, we cannot control phases of the pairing field in nuclear experiments and the possible signal need to be extracted after averaging over the phase difference.

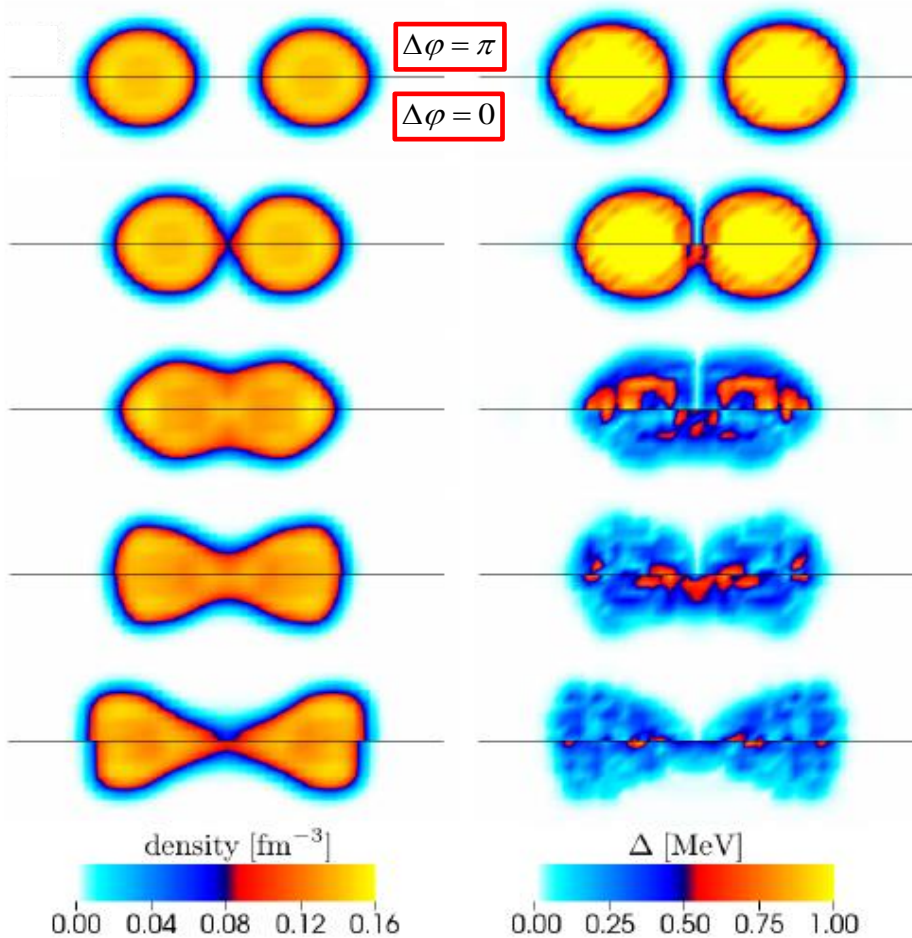
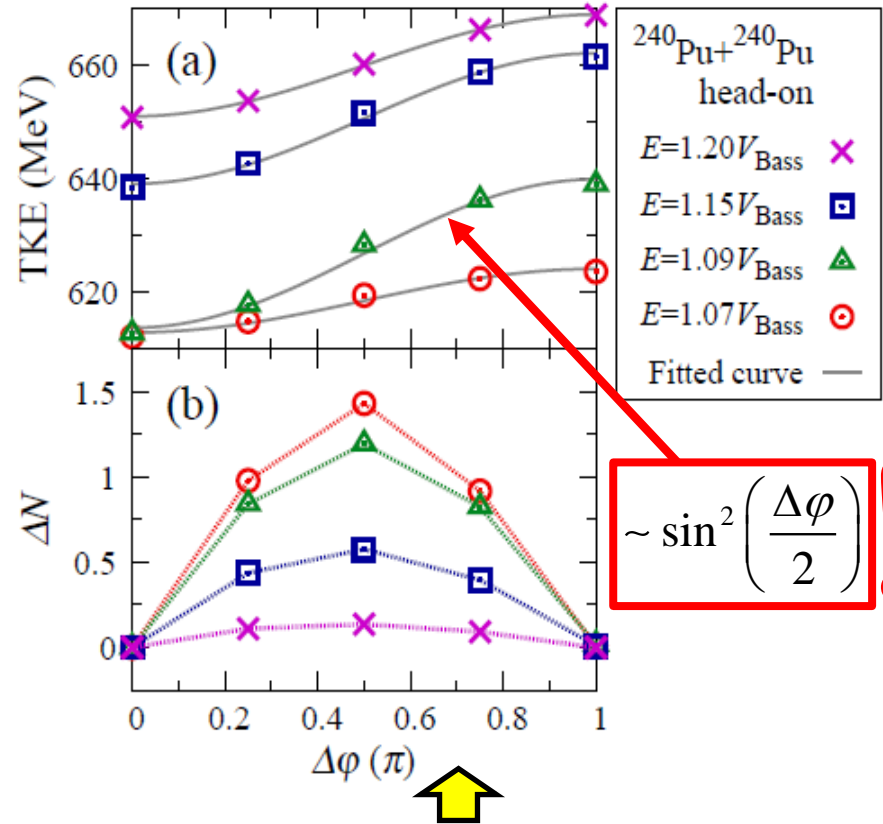


$$\Delta\varphi (\equiv \varphi_1 - \varphi_2)$$

From Ginzburg-Landau (G-L) approach:

$$E_j = \frac{S}{L} \frac{\hbar^2}{2m} n_s \sin^2 \frac{\Delta\varphi}{2}$$

For typical values characteristic for two medium nuclei:  $E_j \approx 30\text{MeV}$

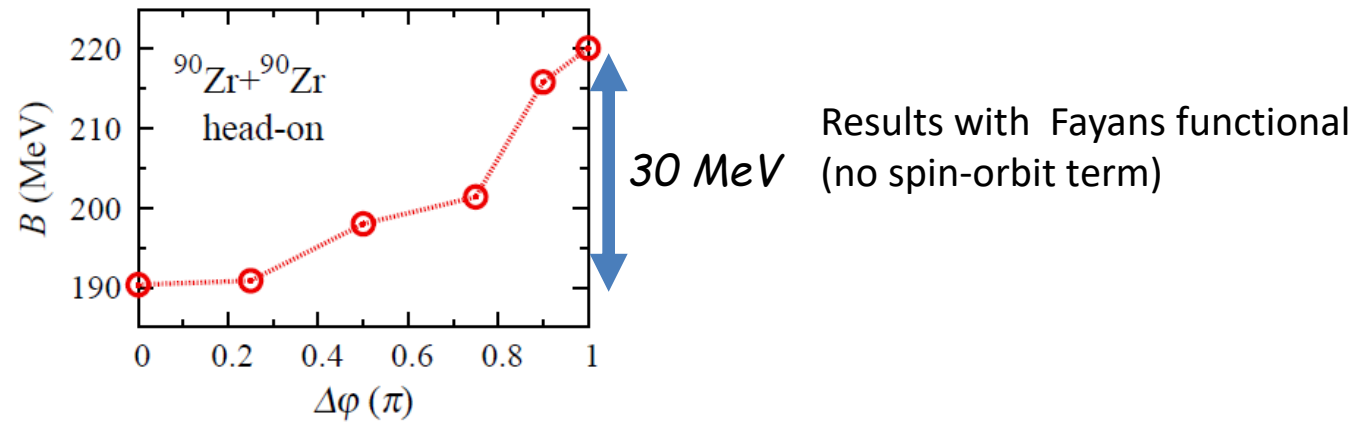
$^{240}\text{Pu}+^{240}\text{Pu}$ Total kinetic energy of the fragments (TKE)

Average particle transfer between fragments.

Creation of the solitonic structure between colliding nuclei prevents energy transfer to internal degrees of freedom and consequently enhances the kinetic energy of outgoing fragments.

Surprisingly, the gauge angle dependence from the G-L approach is perfectly well reproduced in the kinetic energies of outgoing fragments!

# Effective barrier height for fusion as a function of the phase difference



What is an average extra energy needed for the capture?

$$E_{extra} = \frac{1}{\pi} \int_0^{\pi} (B(\Delta\phi) - V_{Bass}) d(\Delta\phi) \approx 10 \text{ MeV}$$

The effect is found (within TDDFT) to be of the order of 30 MeV for medium nuclei and occur for energies up to 20-30% of the barrier height.

P. M., K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119 042501 (2017)

G. Scamps, Phys. Rev. C 97, 044611 (2018): **barrier fluctuations extracted from experimental data indicate that the effect exists although is weaker than predicted by TDDFT**

## Additional properties related to the solitonic excitation

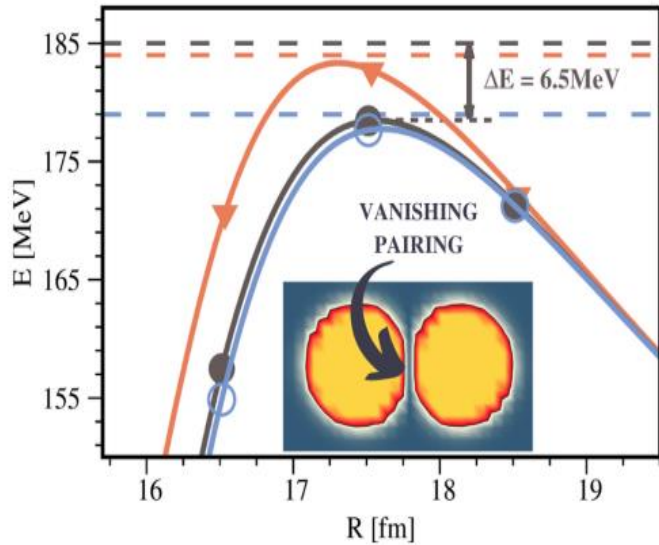


TABLE I: The minimum energies needed for capture in  $^{90}\text{Zr}+^{90}\text{Zr}$  and  $^{96}\text{Zr}+^{96}\text{Zr}$  for the case of  $\Delta\phi = 0$  [ $E_{\text{thresh}}(0)$ ] and  $\Delta\phi = \pi$  [ $E_{\text{thresh}}(\pi)$ ]. The energy difference between the two cases is shown in the last column. The average pairing gap  $\bar{\Delta}_i$  is defined by Eq. (4).

	$\bar{\Delta}_q$ (MeV)	$E_{\text{thresh}}(0)$ (MeV)	$E_{\text{thresh}}(\pi)$ (MeV)	$\Delta E_s$
$^{90}\text{Zr}$	$\bar{\Delta}_n = 0.00$	184	184	0
	$\bar{\Delta}_p = 0.09$			
$^{96}\text{Zr}$	$\bar{\Delta}_n = 1.98$	179	185	6
	$\bar{\Delta}_p = 0.32$			
	$\bar{\Delta}_n = 2.44$	178	187	9
	$\bar{\Delta}_p = 0.33$			
$\bar{\Delta}_n = 2.94$	178	187	9	
$\bar{\Delta}_p = 0.34$				

### Dynamic nature of the effect:

Solid lines: static barrier between two nuclei (with pairing included):

$^{90}\text{Zr}+^{90}\text{Zr}$  - brown

$^{96}\text{Zr}+^{96}\text{Zr}$  - black (0-phase diff.) and blue (Pi-phase diff.)

**Static barriers are practically insensitive to the phase difference of pairing fields.**

Dashed lines: Actual threshold for capture obtained in dynamic calculations.

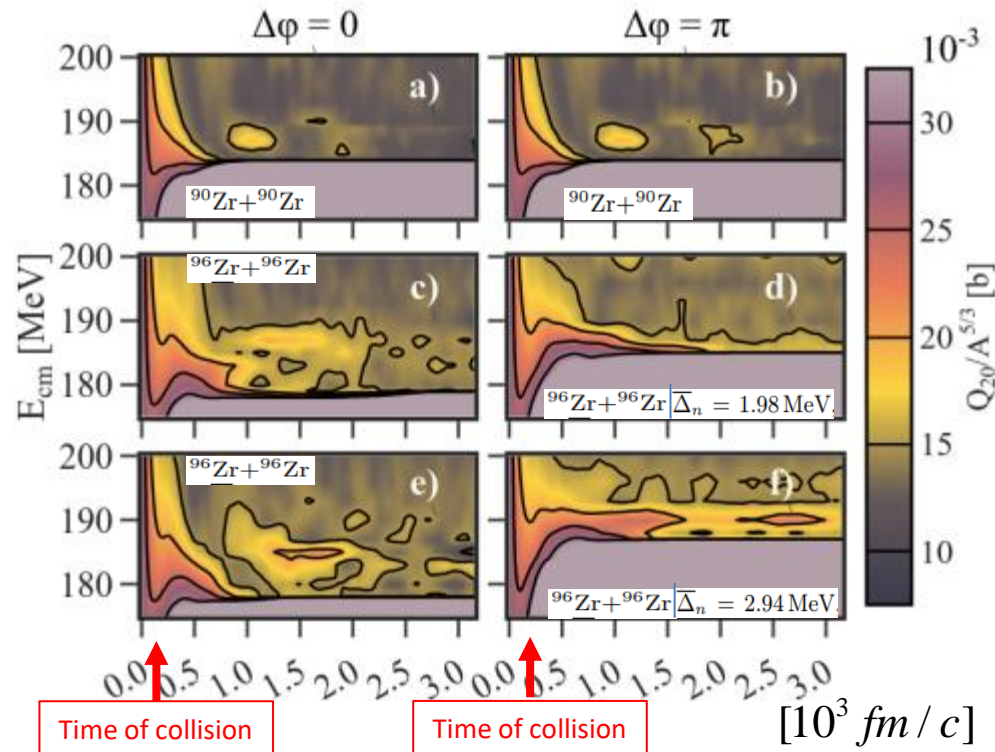
Hence  $\Delta E$  measures the additional energy which has to be added to the system to merge nuclei.

Dependence of the additional energy on pairing gap in colliding nuclei



TABLE II: Total excitation energies (TXEs) in  $^{96}\text{Zr}+^{96}\text{Zr}$  at c.m. energies  $E_{c.m.}$  just below the threshold for capture (see Table I) with  $\Delta\phi = 0$  [TXE(0)] and  $\Delta\phi = \pi$  [TXE( $\pi$ )] are shown. The average pairing gap  $\bar{\Delta}_i$  is defined by Eq. (4).

	$\bar{\Delta}_q$ (MeV)	$E_{c.m.}$ (MeV)	TXE (MeV)	
			$\Delta\phi = 0$	$\Delta\phi = \pi$
$^{96}\text{Zr}$	$\bar{\Delta}_n = 1.98$	178	37	25
	$\bar{\Delta}_p = 0.32$			
	$\bar{\Delta}_n = 2.44$	177	34	10
	$\bar{\Delta}_p = 0.33$			
	$\bar{\Delta}_n = 2.94$	177	34	8
	$\bar{\Delta}_p = 0.34$			



Collisions at energies just below the threshold for capture:

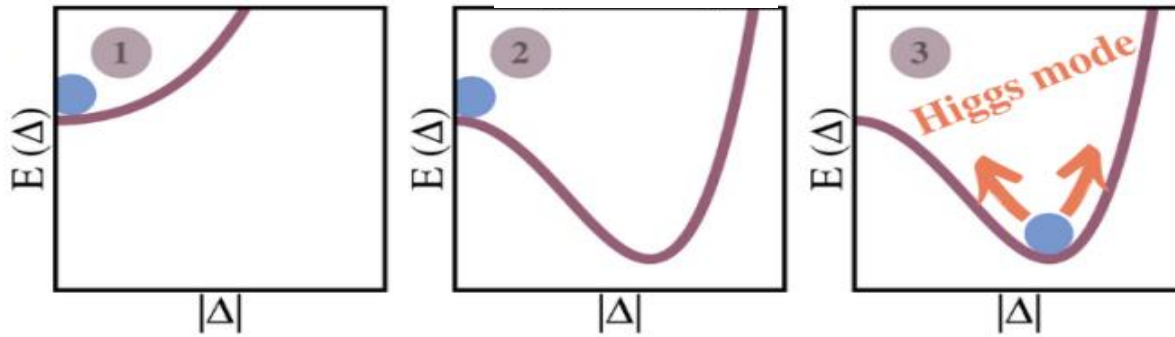
Note the **strong suppression of excitation energies** of re-separated fragments for collisions with Pi-phase difference.

**The magnitude of pairing is the same in both cases.**

Qualitative differences in shape evolution of compound system:

**Nonzero pairing field leads to slower evolution towards compact shape.**

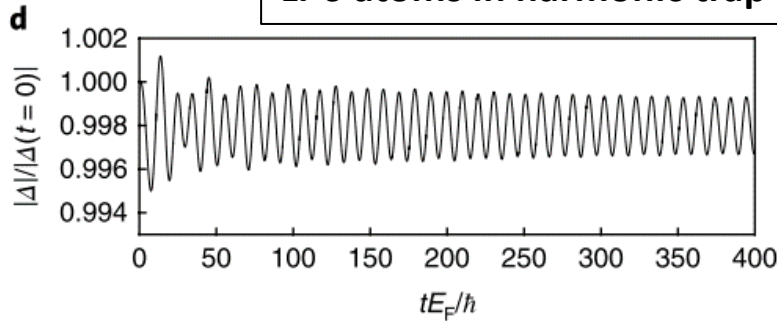
# Pairing Higgs mode



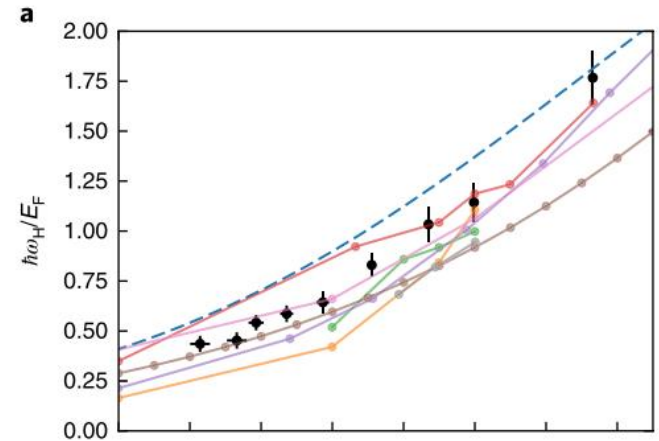
**How to move from the regime 1 to regime 3 in nuclear systems?**

In the ultracold atomic gas one can induce Higgs mode by varying coupling constant.

A. Behrle et al.  
*Higgs mode in a strongly interacting fermionic Superfluid*, Nature Physics **14**, 781 (2018).  
**Li-6 atoms in harmonic trap**



Uniform oscillation of pairing field  
 with frequency:  $2\Delta / \hbar$  (numerical simulations)



Measured peak position of the energy absorption spectra (black dots) and theory predictions for Higgs mode.

Contrary to low-energy Goldstone modes Higgs modes are in principle unstable and decay.  
 Precursors of Higgs modes exists even in few-body systems (J. Bjerlin et al. Phys. Rev. Lett. 116, 155302 (2016) )

# Nuclear pairing Higgs mode

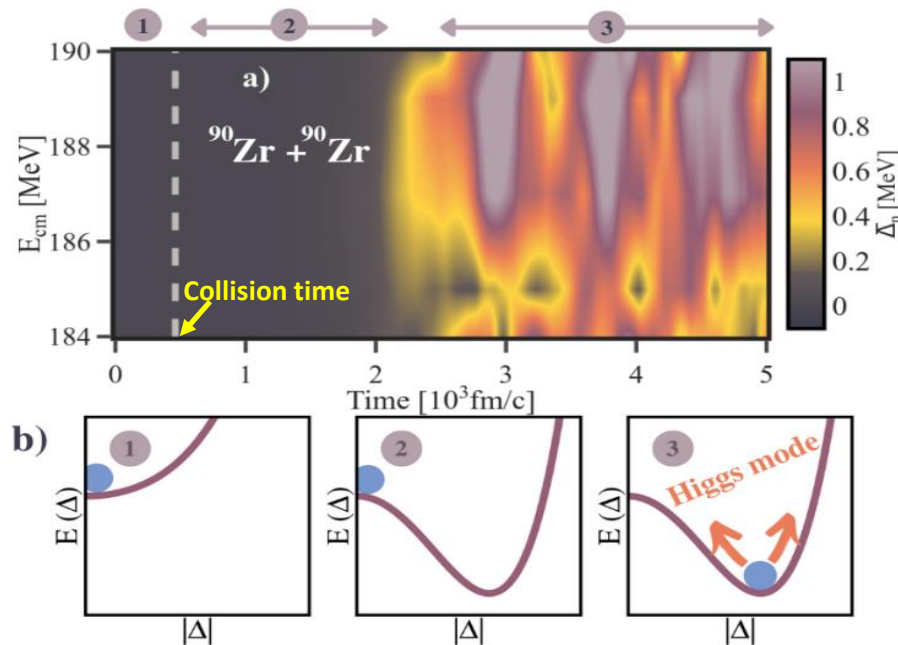
$$\Delta = \frac{8}{e^2} \varepsilon_F \exp\left(\frac{-2}{gN(\varepsilon_F)}\right) \quad - \quad \text{BCS formula – weak coupling limit}$$

$\varepsilon_F$  - Fermi energy

$g$  - Pairing coupling constant

$N(\varepsilon_F)$  - Density of states at the Fermi level

Although one cannot change coupling constant in atomic nuclei one may affect ***density of states at the Fermi surface and consequently trigger Higgs mode.***



Collision of two neutron magic systems creates an elongated di-nuclear system.

Within 1500 fm/c pairing is enhanced in the system and reveals oscillations with frequency:

$$\Delta < \hbar\omega < 2\Delta$$

## Dynamics of pairing instability

After collision the pairing configuration corresponding to initial magic system becomes unstable.

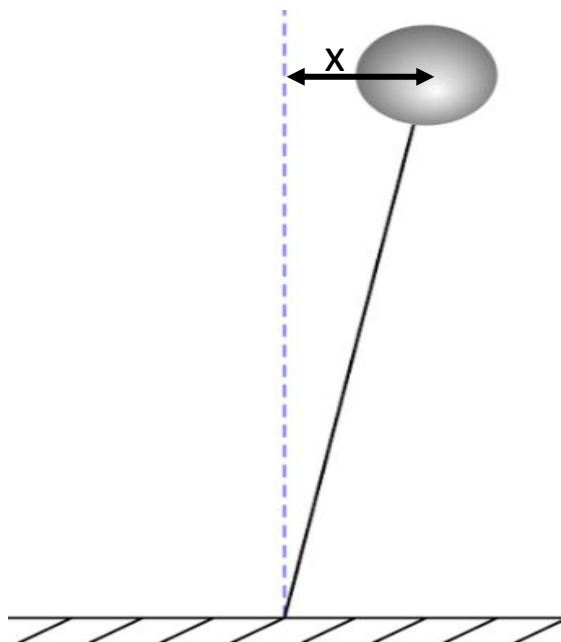
It is an analogue to pendulum which suddenly become inverted.

inverted pendulum eq. for small displacements i.e. close to unstable point of equilibrium:

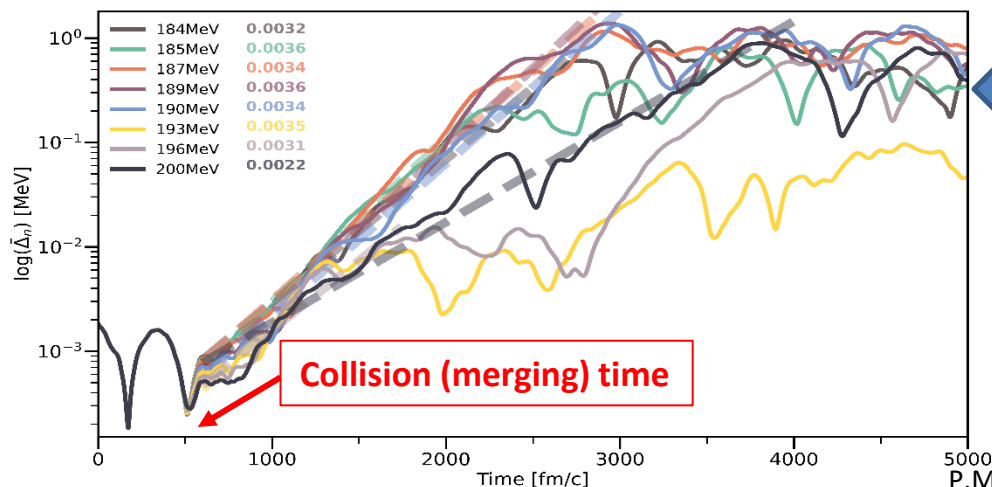
$$\frac{d^2 x}{dt^2} \approx \alpha^2 x \Rightarrow x(t) \approx \frac{x_0}{2} \exp(\alpha t)$$

Similarly: pairing gap behavior around the point of instability:

$$\frac{d^2 \Delta}{dt^2} \approx \alpha^2 \Delta \Rightarrow \Delta(t) \approx \frac{\Delta_0}{2} \exp(\alpha t)$$



$^{90}\text{Zr} + ^{90}\text{Zr}$  head-on collision above the threshold for capture



**Exponential increase of pairing gap** after collision indicating **pairing instability** in di-nuclear system.

Excited **Higgs mode** (uniform pairing) becomes **fragmented (decays)** already during the first period of oscillation.

# Summary and open questions

- It seems we have some experimental evidence for degree of freedom related to **the pairing phase**:  
AC Josephson junction (pair transfer) and solitonic excitations (barrier modification)
- It is likely that the **solitonic excitation** will contribute to Świątecki's **extra-push energy** (*W.J.Świątecki, Phys. Scr. 24 (1981) 113; Nucl.Phys. A376 (1982) 275, ...*)
- The **enhancement of pairing correlations** after collision and merging as a signature for **Higgs mode** is a qualitatively new startling effect.  
It is surprising as to date it was expected that TDHF approach is sufficient, in particular for collisions involving magic nuclei.
- **Pairing enhancement** in collision of magic nuclei is **a generic feature**:  
according to the theory (TDHFB) it appears in other collisions of magic nuclei at energies close to the Coulomb barrier.
- Impact of pairing enhancement on dynamics is unknown and requires more theoretical effort: investigation of noncentral collisions, considerations of pairing correlations during subsequent stages of compound nucleus formation.

Systematic investigations of medium and heavy nuclei collisions close to Coulomb barrier within TDDFT theory with inclusion of pairing correlations are needed!