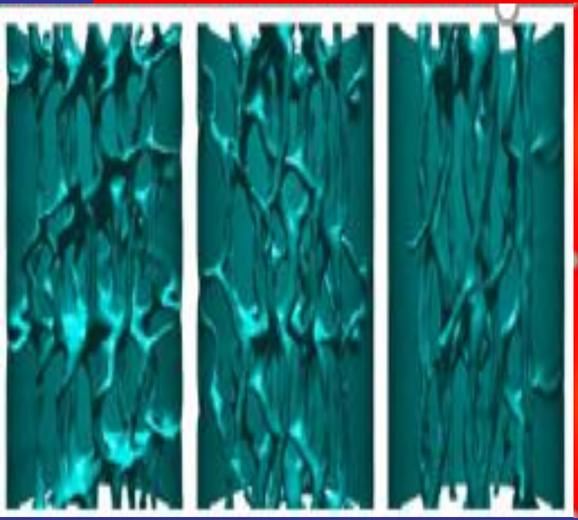
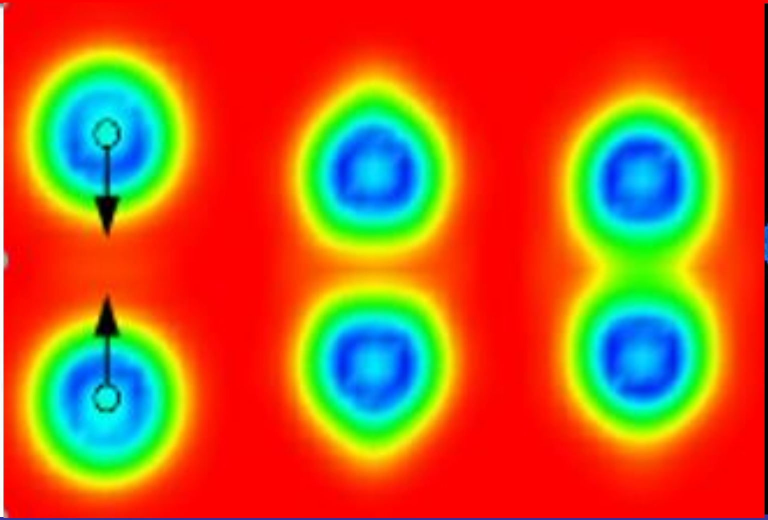
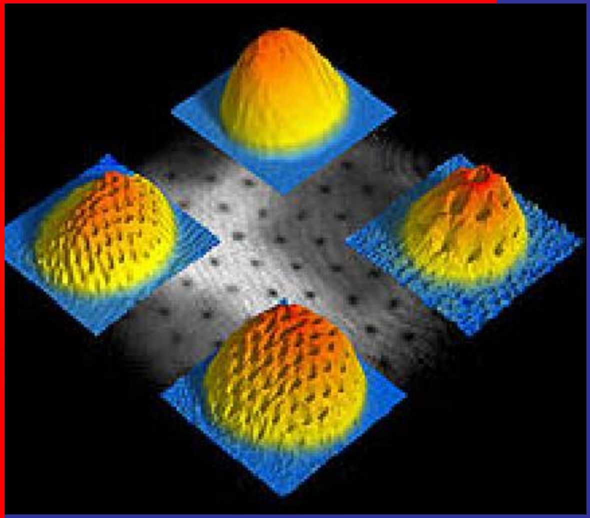
Exotic Features of Superfluidity in Ultracold Atomic Gases

Piotr Magierski

Warsaw University of Technology

Funding:: NATIONAL SCIENCE CENTRE POLAND

HPC:: PRACE

OAK RIDGE National Laboratory | LEADERSHIP COMPUTING FACILITY

東京工業大学 Tokyo Institute of Technology

GSIC Global Scientific Information and Computing Center

University of Tsukuba Center for Computational Sciences 筑波大学 計算科学研究センター

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Klejdja Xhani

What are ultracold atomic gases?


Photo from the Nobel Foundation archive.

Claude Cohen-Tannoudji


Photo from the Nobel Foundation archive.

Steven Chu

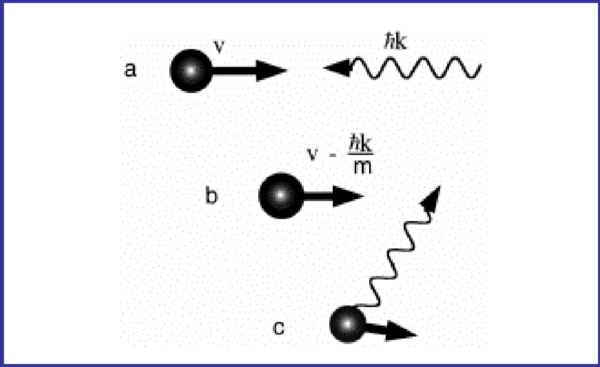

Photo from the Nobel Foundation archive.

William D. Phillips

Nobel Prize in Physics 1997

„for development of methods to cool and trap atoms with laser light”

Idea od laser cooling



Schematic realization of *optical molasses*

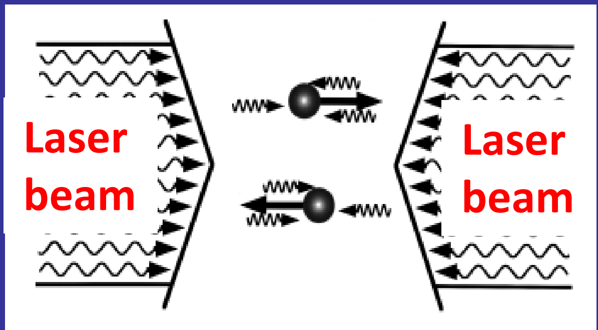
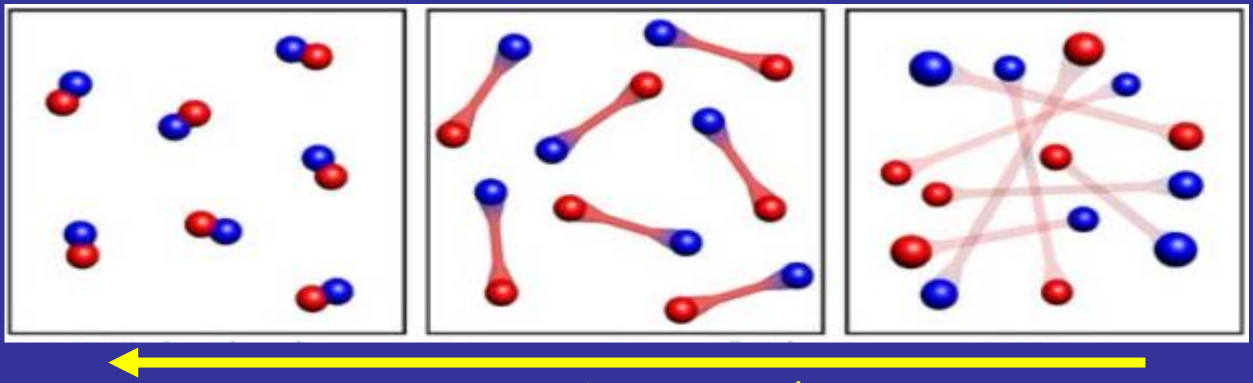


FIG. 12. Doppler cooling in one dimension.

- Alkali atoms (fermions) used for trapping and cooling: ^{6}Li, ^{40}K
- Laser light (via Doppler effect) is used to create *optical molasses* which decrease the velocity width of atomic cloud and cool it down.
- Sub-doppler cooling is achieved through *Sisyphus cooling* and/or *evaporative cooling* and allows to reach temperatures below microkelvins
- Dilute cloud of 10^5-10^6 atoms is kept in magneto-optical trap (MOT)

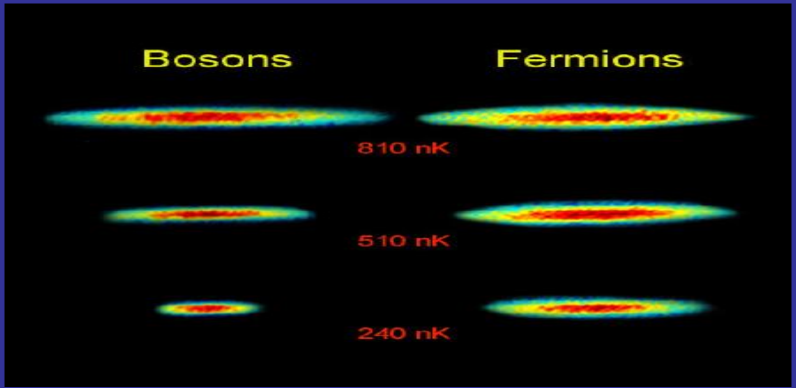
In dilute atomic systems experimenters can control nowadays almost anything:

- The number of atoms in the trap: typically about 10^5-10^6 atoms divided 50-50 among the lowest two hyperfine states.
- The density of atoms
- Mixtures of various atoms
- The temperature of the atomic cloud
- **The strength of this interaction is fully tunable!**

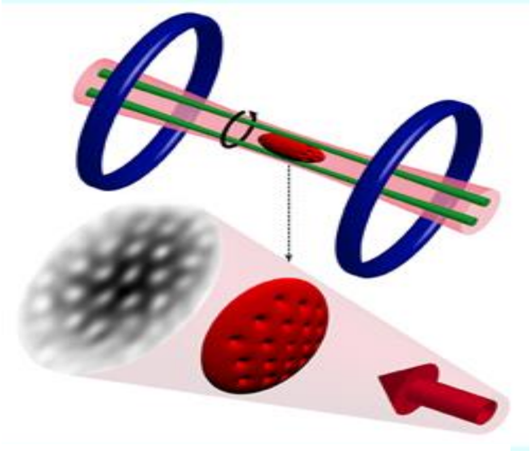


BEC regime **Unitary regime** **BCS regime**

Interaction strength



Evidence for fermionic superfluidity in ultracold atomic gases.



Rotation of a superfluid cloud leads to generation of **quantum vortices**

M.W. Zwierlein *et al.*, Nature, 435, 1047 (2005)

BEC regime:

BCS regime:

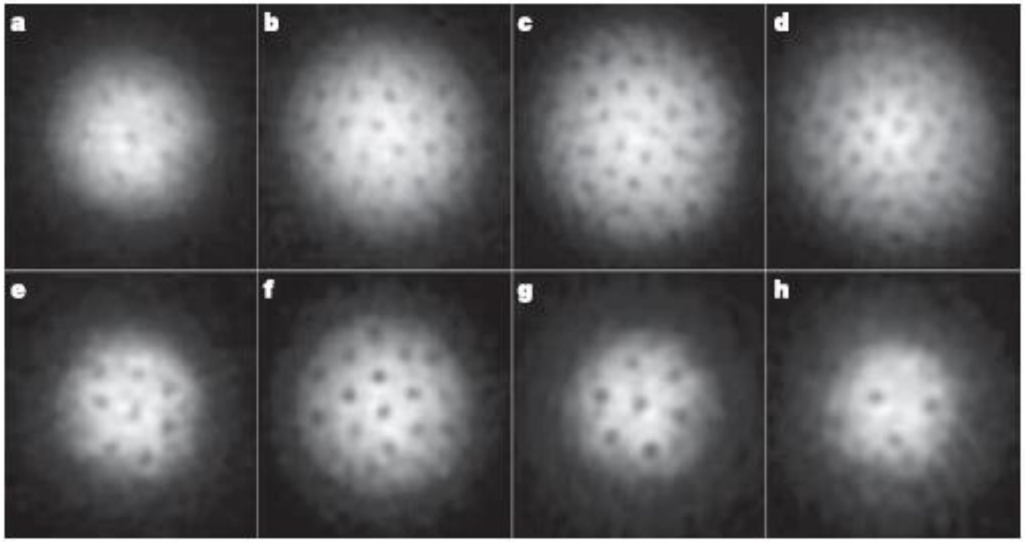
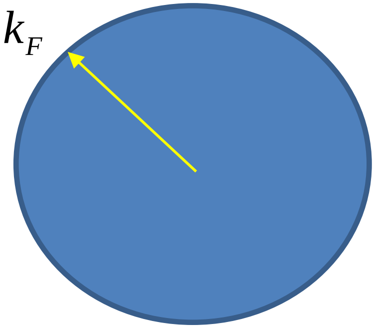
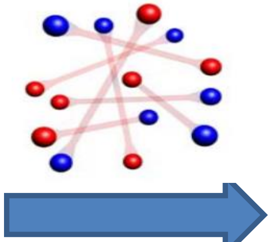


Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b**–**h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**), 812 G (**d**), 833 G (**e**), 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of view of each image is 880 μm × 880 μm.

$$\varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

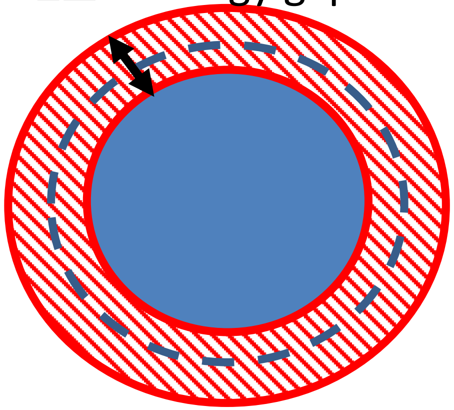


Fermi sphere



Attractive Interaction leads to formation of Cooper pairs

2Δ - energy gap



Why do we need supercomputers to simulate ultracold gases?

Evolution of Cooper pair field (pairing field) is at the heart of theoretical description of superfluids :

$$\Delta(\vec{r},t) = \left|\Delta\left(\vec{r},t\right)\right| e^{i\phi(\vec{r},t)}$$

Method: Time Dependent Density Functional Theory

Replacing hard-to-solve linear Schroedinger eq.

$$\boxed{i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi}$$

by huge set (10^5 -10^6) partial differential, nonlinear eqs.

(**Superfluid Local Density Approximation**):

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) & 0 & 0 & \Delta(\boldsymbol{r},t) \\ 0 & h_b(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h_a^*(\boldsymbol{r},t) & 0 \\ \Delta^*(\boldsymbol{r},t) & 0 & 0 & -h_b^*(\boldsymbol{r},t) \end{pmatrix}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t) \\ u_{n,b}(\boldsymbol{r},t) \\ v_{n,a}(\boldsymbol{r},t) \\ v_{n,b}(\boldsymbol{r},t) \end{pmatrix}$$

where *h* and Δ depends on “densities”:

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |v_{n,\sigma}(\boldsymbol{r},t)|^2, \qquad \tau_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r},t)|^2,$$

$$\nu(\boldsymbol{r},t) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r},t)v^*_{n,\downarrow}(\boldsymbol{r},t), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c} \mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r},t)\nabla v_{n,\sigma}(\boldsymbol{r},t)],$$

More details:
A. Bulgac, M.M. Forbes, P. Magierski,
The Unitary Fermi Gas: From Monte Carlo to Density Functionals,
Lecture Notes in Physics 836
ed. W. Zwerger, Springer (2011).

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https://wslda.fizyka.pw.edu.pl

Warsaw University of Technology | W-SLDA Toolkit

W-SLDA Toolkit

Self-consistent solver of mathematical problems which have structure formally equivalent to Bogoliubov-de Gennes equations.

static problems: st-wslda

$$\begin{pmatrix} h_a(\boldsymbol{r})-\mu_a & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_b^*(\boldsymbol{r})+\mu_b \end{pmatrix}\begin{pmatrix}u_n(\boldsymbol{r})\\ v_n(\boldsymbol{r})\end{pmatrix} = E_n \begin{pmatrix}u_n(\boldsymbol{r})\\ v_n(\boldsymbol{r})\end{pmatrix}$$

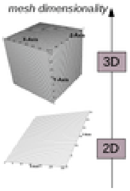
time-dependent problems: td-wslda

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}u_n(\boldsymbol{r},t)\\ v_n(\boldsymbol{r},t)\end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t)-\mu_a & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_b^*(\boldsymbol{r},t)+\mu_b \end{pmatrix}\begin{pmatrix}u_n(\boldsymbol{r},t)\\ v_n(\boldsymbol{r},t)\end{pmatrix}$$

Speed-up calculations by exploiting High Performance Computing

Functionals for studies of BCS and unitary regimes

Dimensionalities of problems: 3D, 2D and 1D



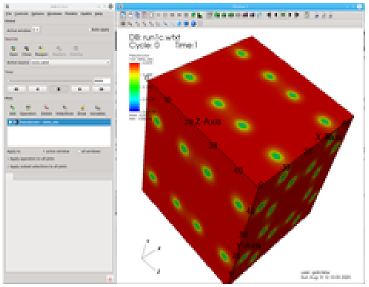
W-SLDA allows to solve problems

in 3D without any symmetry restrictions: Ψ=φ(x,y,z)

in 2D with translational invariance along z direction: Ψ=

in 1D with translational invariance along y and z direction

Integration with VisIt: visualization, animation and analysis tool



W-SLDA is integrated with open-source VisIt tool. It allows for:

visualizing 3D, 2D and 1D results,

data processing,

creating animations for time-dependent simulations.

Contributors

Project leader

Gabriel Wlazłowski

Warsaw University of Technology, Faculty of Physics
Main developer of the W-SLDA Toolkit

Theory expertise

Aurel Bulgac

Department of Physics, University of Washington
Aurel Bulgac derived Superfluid Local Density Approximation (SLDA) equations for cold atoms, presently implemented in W-SLDA Toolkit. He also supervised implementation of core algorithms of td-wslda codes.

Piotr Magierski

Warsaw University of Technology, Faculty of Physics
Piotr Magierski contributed to development of Superfluid Local Density Approximation (SLDA) method.

Michael McNeil Forbes

Department of Physics and Astronomy, Washington State University
Michael Forbes together with Aurel Bulgac developed Asymmetric Superfluid Local Density Approximation (ASLDA) for spin-imbalanced atomic gases.

HPC expertise

Kenneth J. Roche

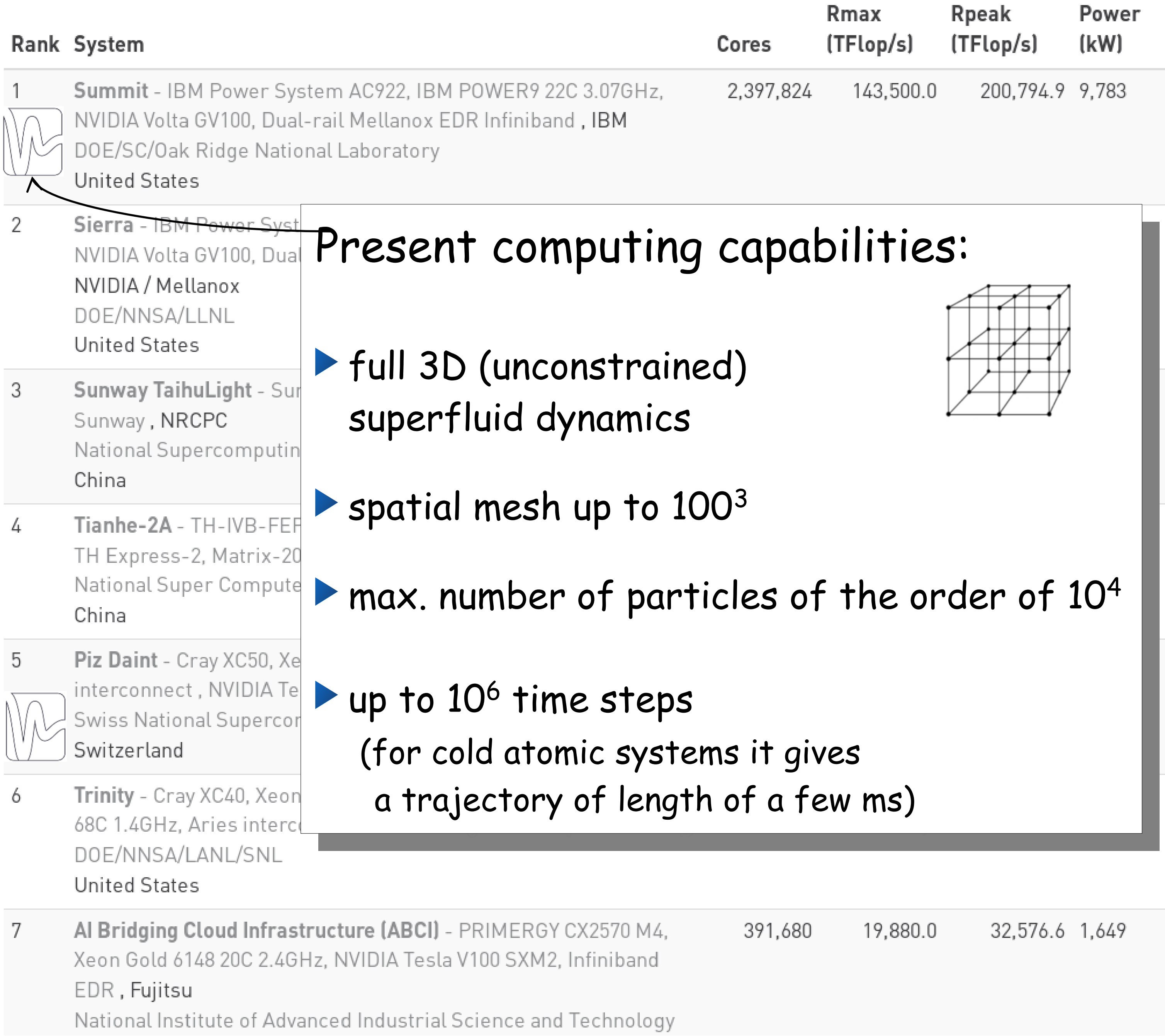
Pacific Northwest National Laboratory
Kenneth J. Roche supervised parallelization process (MPI + CUDA) of main engine of td-wslda codes.

Maciej Marchwiany

Interdisciplinary Centre for Mathematical and Computational Modelling (ICM)
Implementation of parallel I/O in td-wslda codes (2016-2018)

To execute superfluid TDDFT we need supercomputers...

https://www.topa500.org/



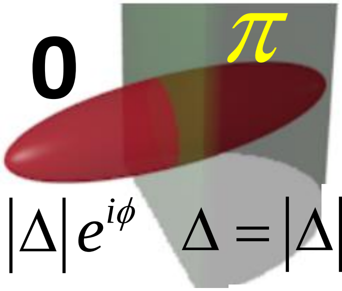
Rank	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	**Summit** - IBM Power System AC922, IBM POWER9 22C 3.07GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband , IBM DOE/SC/Oak Ridge National Laboratory United States	2,397,824	143,500.0	200,794.9	9,783
2	**Sierra** - IBM Power Syst NVIDIA Volta GV100, Dual NVIDIA / Mellanox DOE/NNSA/LLNL United States				
3	**Sunway TaihuLight** - Sun Sunway , NRCPC National Supercomputin China				
4	**Tianhe-2A** - TH-IVB-FEP TH Express-2, Matrix-20 National Super Compute China				
5	**Piz Daint** - Cray XC50, Xe interconnect , NVIDIA Te Swiss National Supercor Switzerland				
6	**Trinity** - Cray XC40, Xeon 68C 1.4GHz, Aries interc DOE/NNSA/LANL/SNL United States				
7	**AI Bridging Cloud Infrastructure (ABCI)** - PRIMERGY CX2570 M4, Xeon Gold 6148 20C 2.4GHz, NVIDIA Tesla V100 SXM2, Infiniband EDR , Fujitsu National Institute of Advanced Industrial Science and Technology	391,680	19,880.0	32,576.6	1,649

Present computing capabilities:

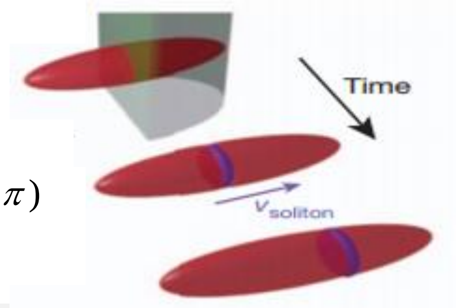
- full 3D (unconstrained) superfluid dynamics
- spatial mesh up to 100^3
- max. number of particles of the order of 10^4
- up to 10^6 time steps
  (for cold atomic systems it gives
  a trajectory of length of a few ms)

All further results shown here were generated on Piz Daint (CSCS)

Example 1: Atomic cloud collisions - decay of solitonic excitations



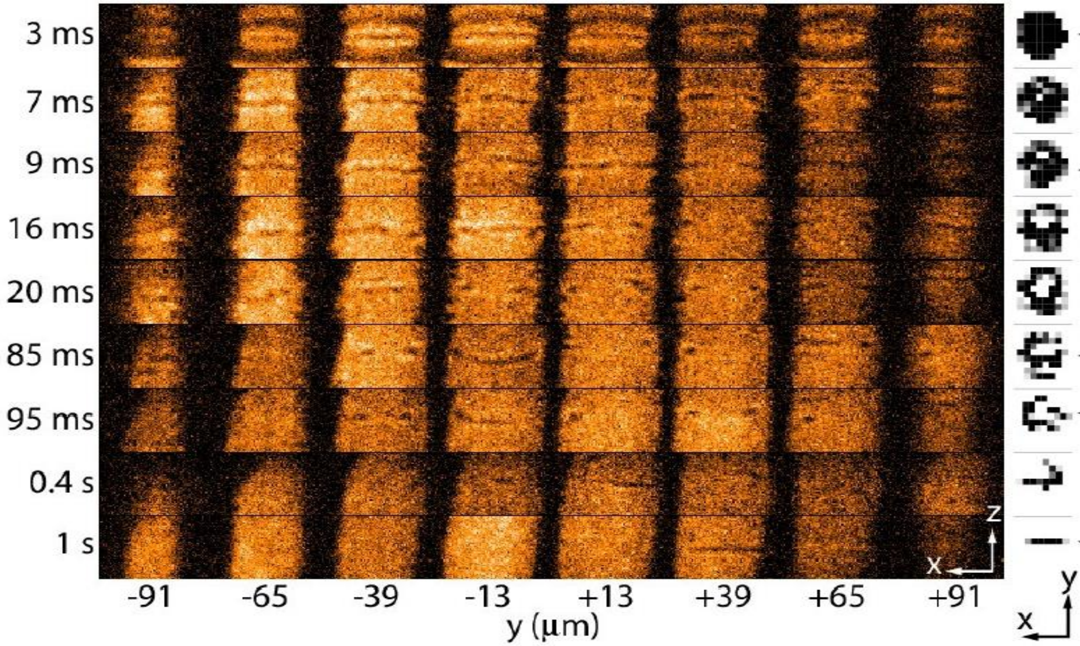
$$\Delta = |\Delta| e^{i\phi} \qquad \Delta = |\Delta| e^{i(\phi+\pi)}$$

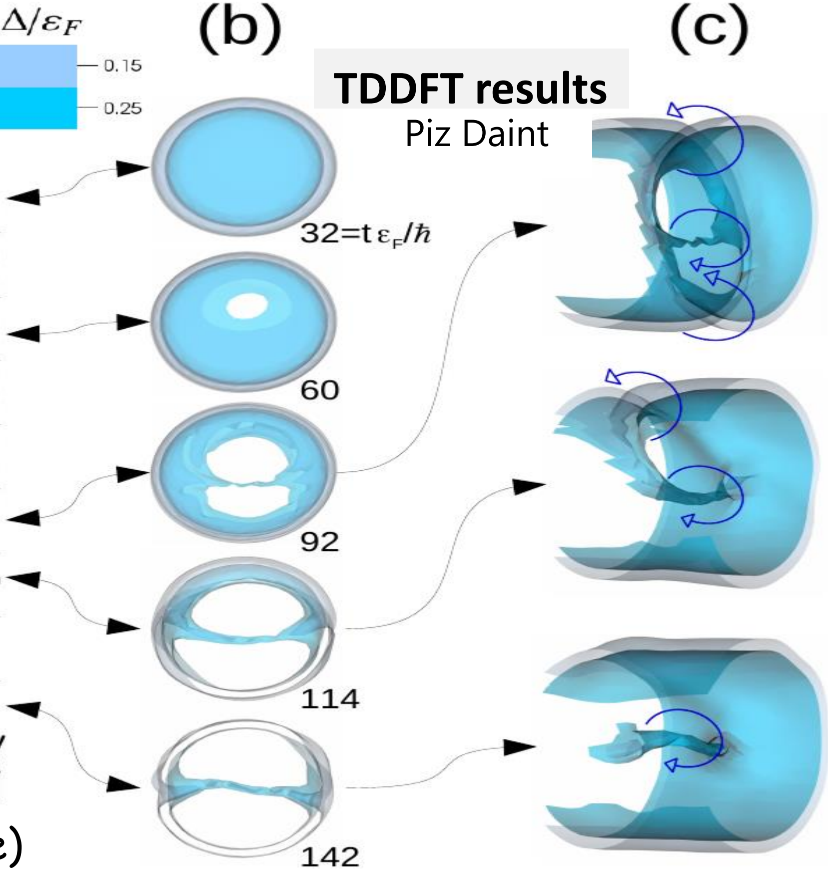


Series of MIT experiments:
Nature 499, 426 (2013);
PRL 113, 065301 (2014);
PRL 116, 045304 (2016);
→ observation of decay
of a dark soliton into a vortex line

MIT experiment

Phys. Rev. Lett. 116, 045304 (2016)





TDDFT results

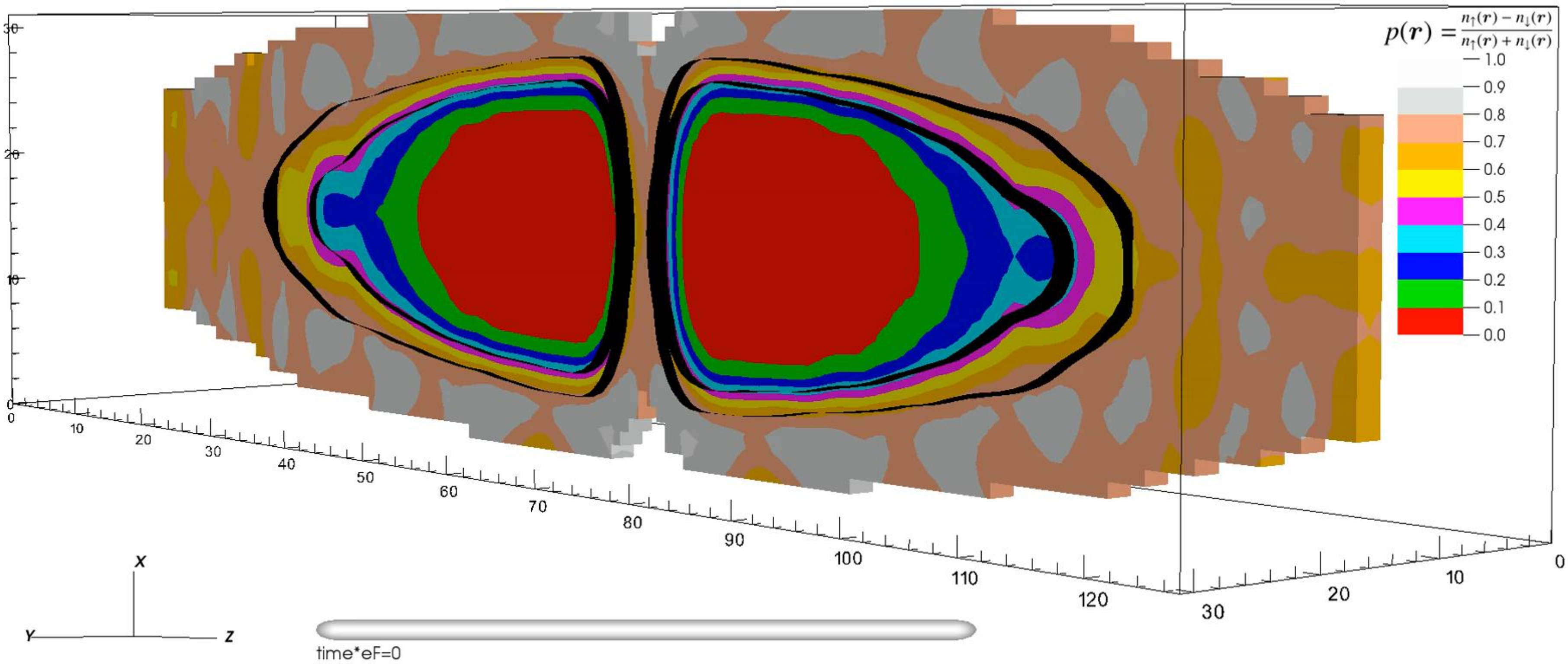
Piz Daint

Decay of solitonic excitation (pairing nodal structure) generates a sequence of topological excitations involving: “Phi”-soliton and vortex line.

G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. Magierski,
Phys. Rev. Lett. **120**, 253002 (2018)

Spin imbalanced ultracold atomic gas in the unitary regime

$N_\uparrow = 304, \; N_\downarrow = 202, \; P = 20\%$



The vortex core becomes polarized.

This may be understood noting that the most energetically favorable place to store excess of unpaired spins is at the core of the vortex where, Δ = 0 - no Cooper pairs need to be broken.

New effects predicted for spin-polarized systems:

Impact on the solitonic cascade:

final product of the cascade depends on the spin imbalance in the system

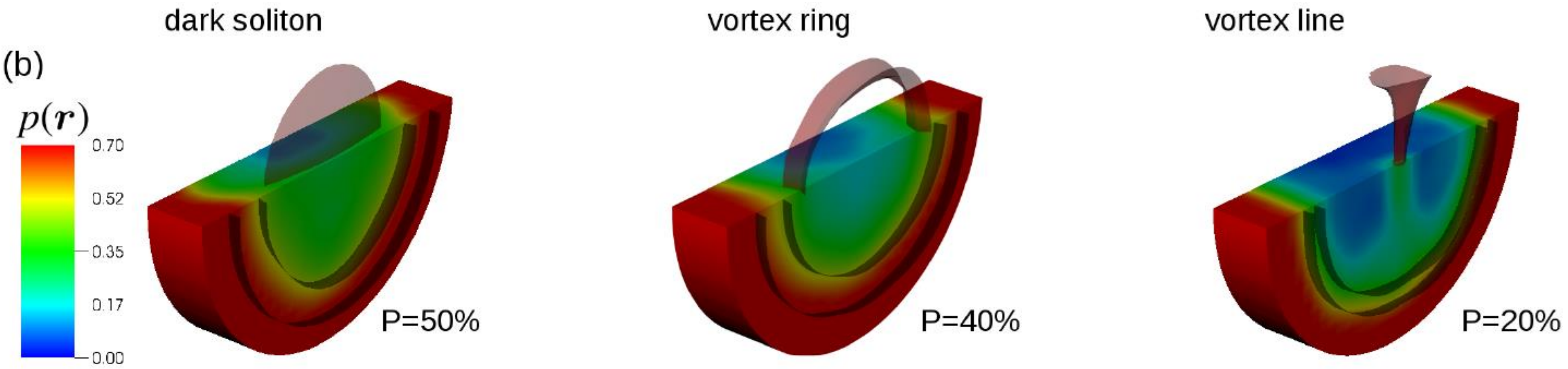
(can be verified experimentally with present setups)

P=20%: Dark soliton ⇨ Vortex ring ⇨ Vortex line

P=40%: Dark soliton ⇨ Vortex ring

P=50%: Dark soliton

Cascade is suppressed by the polarization effects



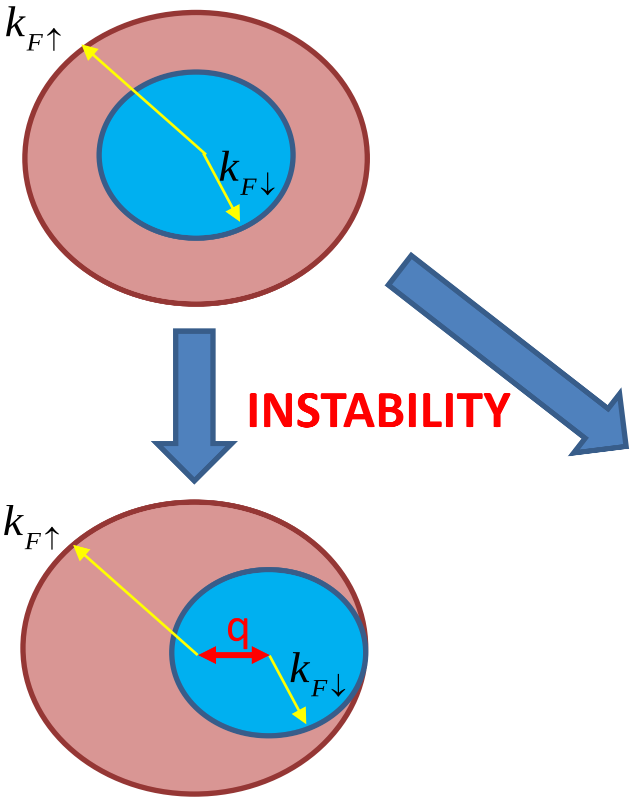
Stability of topological defect depends on its internal structure…

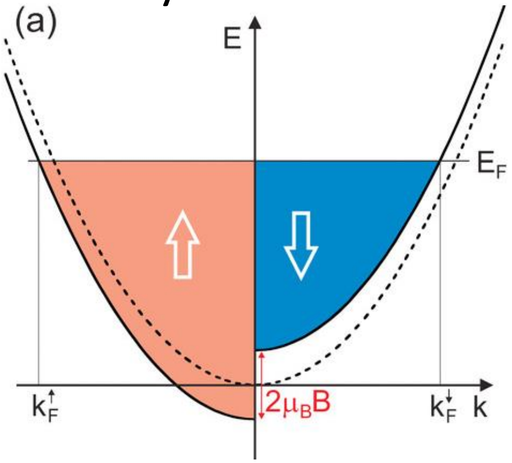
→ For sufficiently large spin-imbalance dark solitons become stable (no snake instability) (see also: Reichl & Mueller, PRA 95, 053637; Lombardi, et. al., PRA 96, 033609)

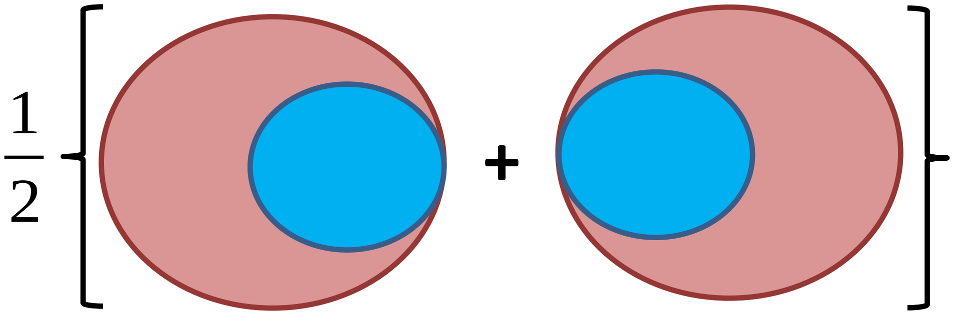
G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. Magierski, Phys. Rev. Lett. **120**, 253002 (2018)

Example 2: Spin polarized impurites in superfluids

Pairing in spin imbalanced systems







Fulde-Ferrell phase:

Larkin-Ovchinnikov phase:

Translational symmetry is broken:

Larkin-Ovchinnikov (LO): $\Delta(r) \sim cos(\vec{q}\cdot\vec{r})$

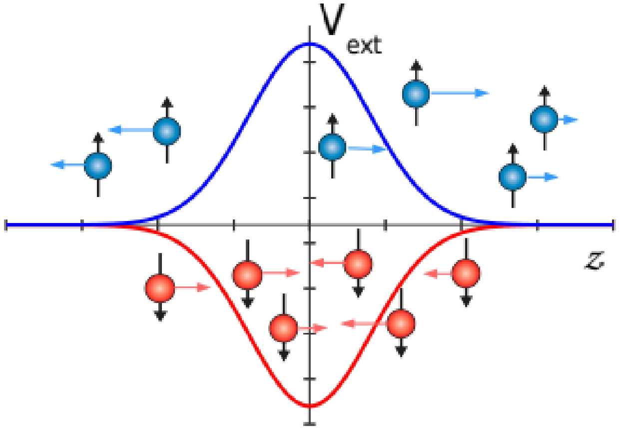
Fulde-Ferrell (FF): $\Delta(r) \sim \exp(i\vec{q}\cdot\vec{r})$

A.I. Larkin and Y. N. Ovchinnikov,
Sov. Phys. JETP 20, 762 (1965)
P. Fulde and R. A. Ferrell,
Phys. Rev. 135, A550 (1964)

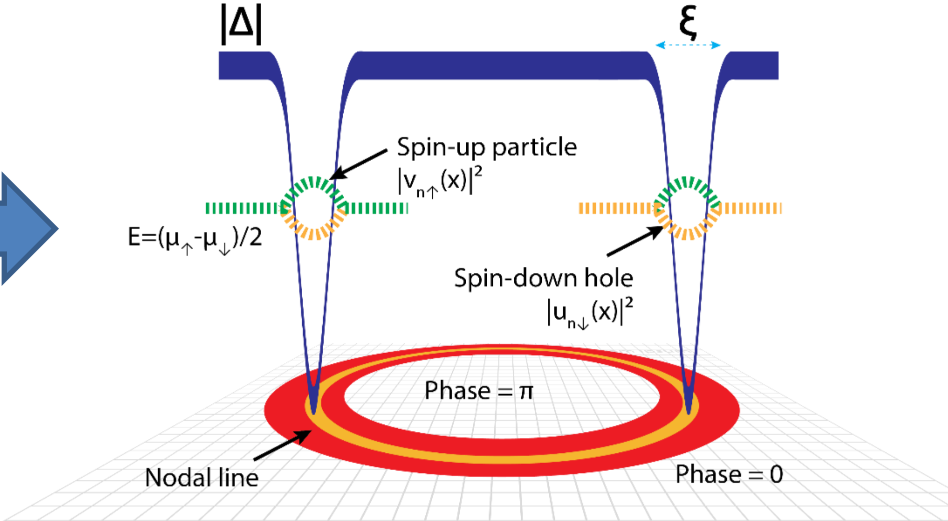
Predictions for atomic gas in the unitary regime: A. Bulgac, M.M.Forbes, Phys. Rev. Lett. 101,215301 (2008)

Engineering the structure of nodal surfaces in ultracold atomic gas

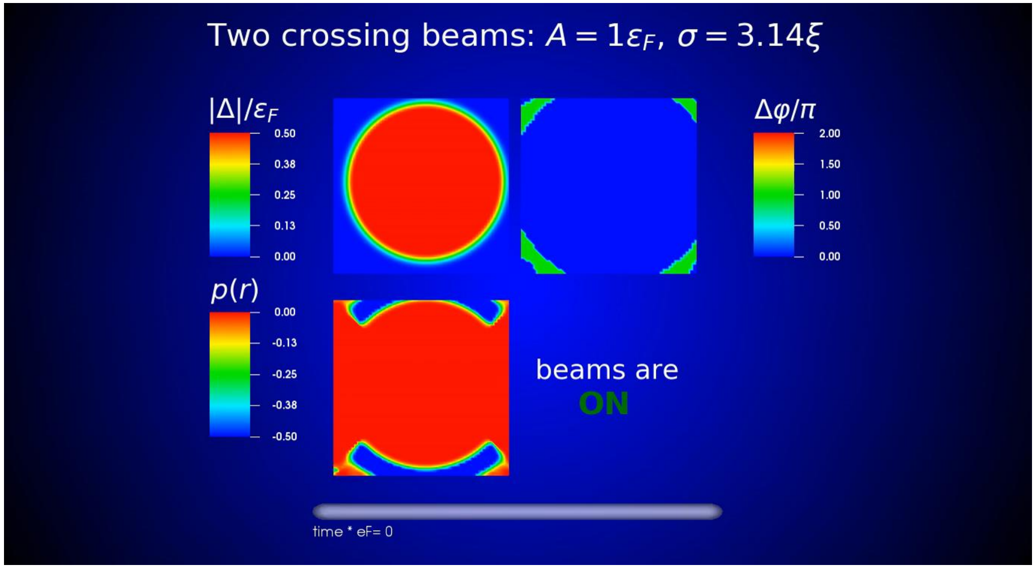
Spin-selective potential applied locally leads to Cooper pair breaking



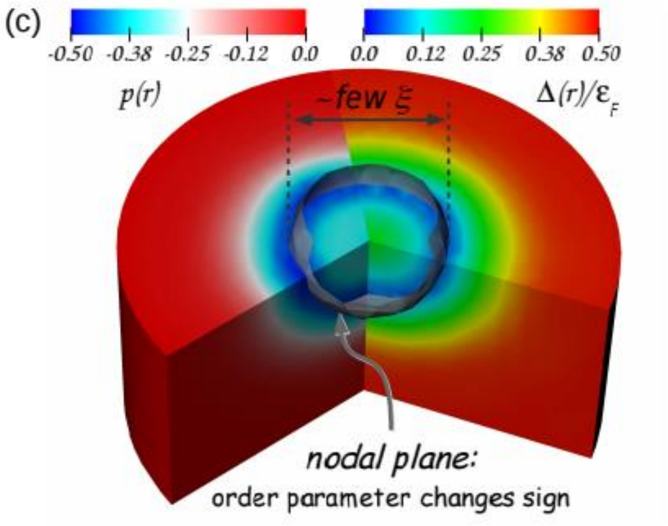
Pairing field nodal structure



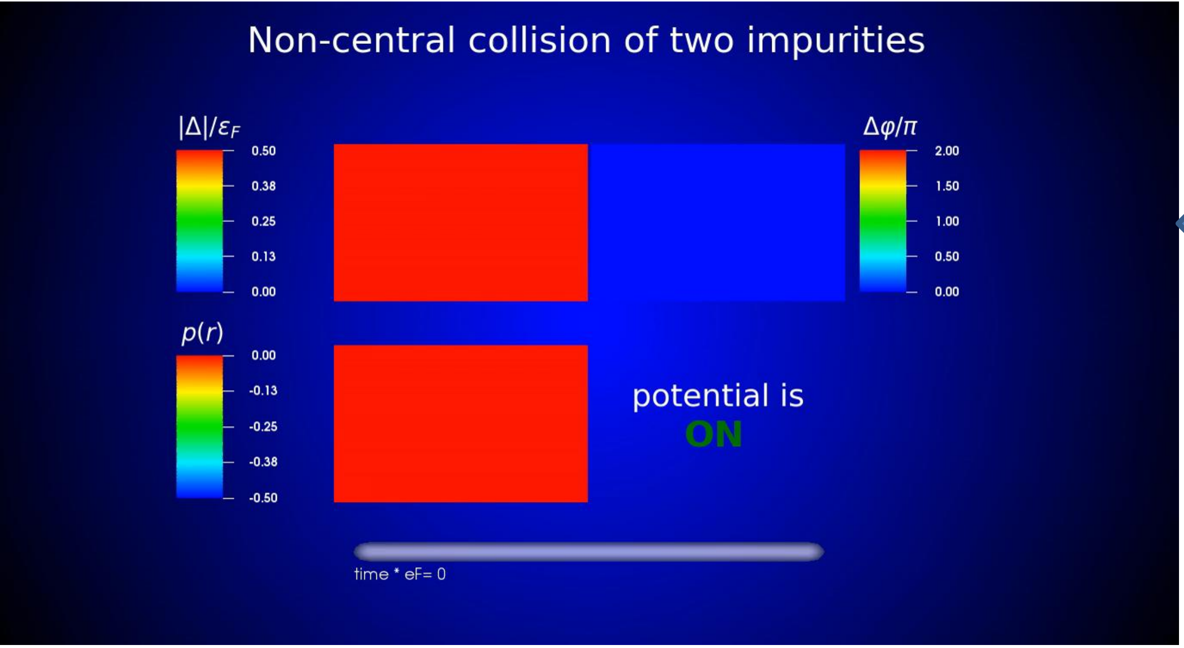
Generation of *ferron* in the unitary regime



Ferron structure



nodal plane:
order parameter changes sign



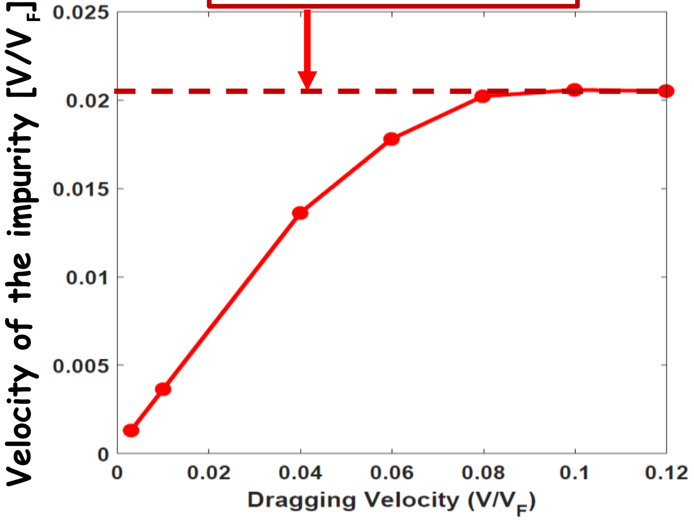
Surprisingly, the nodal structure remains stable even during collisions

P. Magierski, B. Tüzemen, G. Wlazłowski,
Phys. Rev. A100, 033613 (2019)

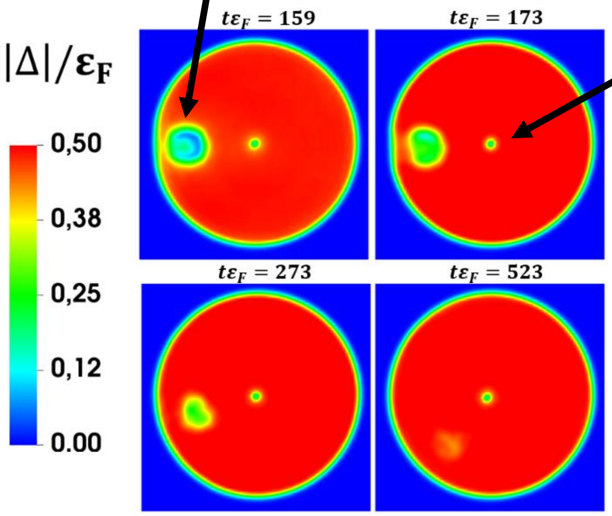
The velocities of impurites are about 30% of the velocity of sound.

Peculiarity of ferron dynamics: there is a limiting velocity propotional to its polarization.

$$\boxed{v_{crit} \propto \mu_{\uparrow} - \mu_{\downarrow}}$$



Instability of ***ferron*** in the vicinity of ***quantum vortex***



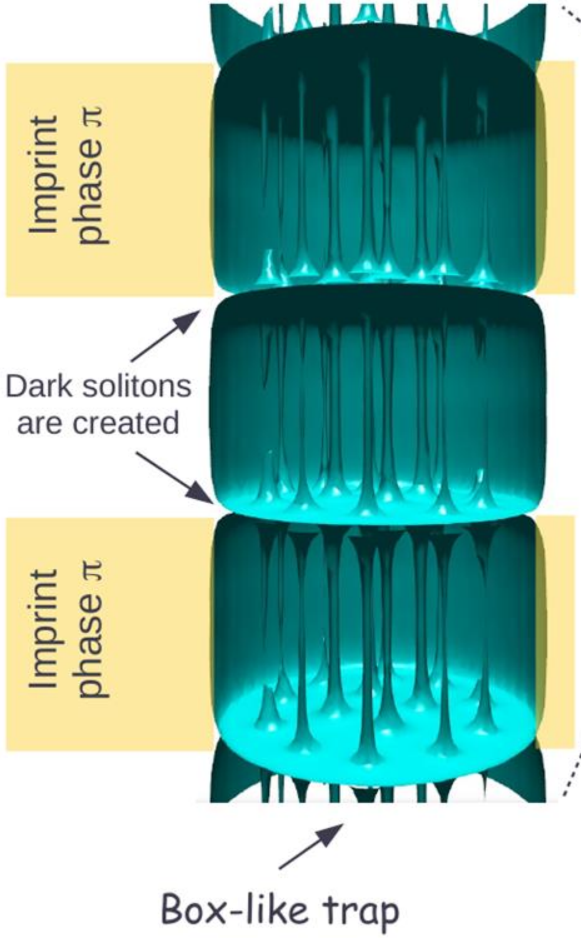
P. Magierski,
B. Tüzemen,
G. Wlazłowski,
arXiv:2102.04833

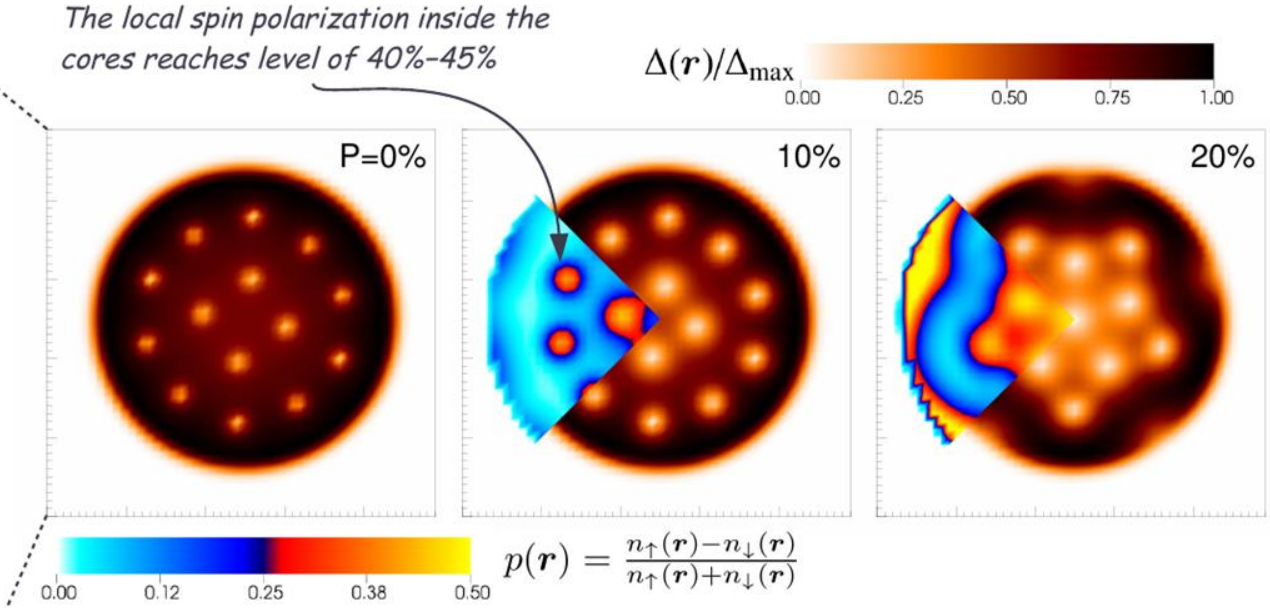
Example 3: Generation and decay of quantum turbulent state

Problem: how to generate the turbulence?

→ Our suggestion: *imprint few dark solitons on existing vortex lattice*

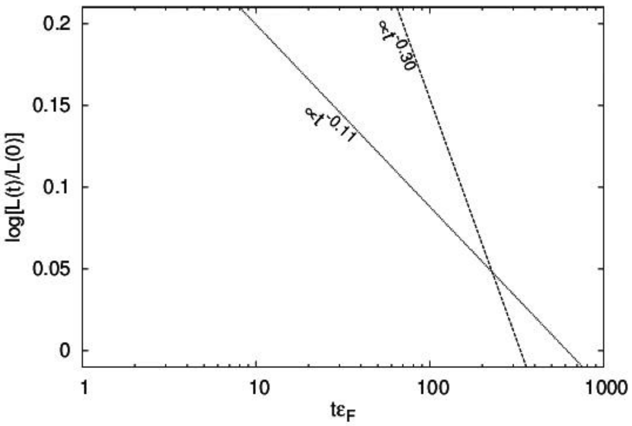
→ *rotating turbulence* *(nonzero total angular momentum)*

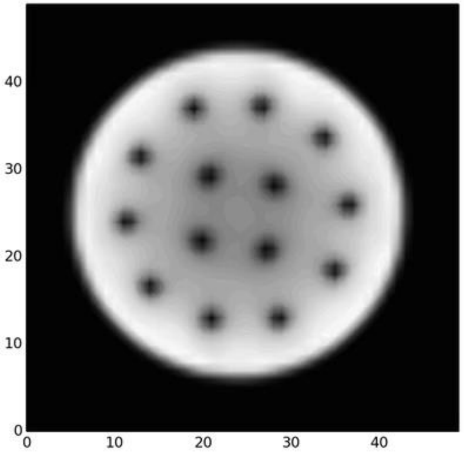


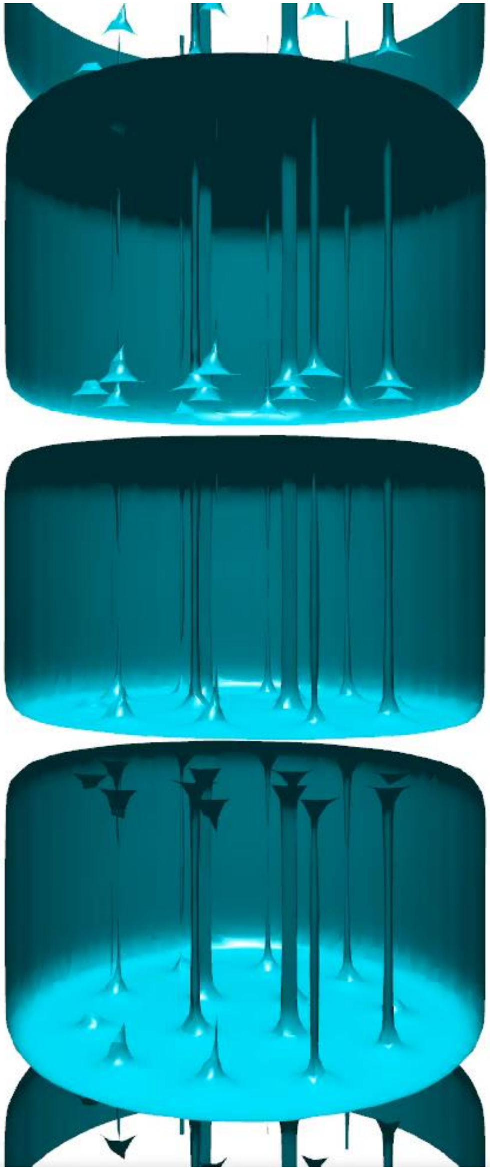

The local spin polarization inside the cores reaches level of 40%-45%

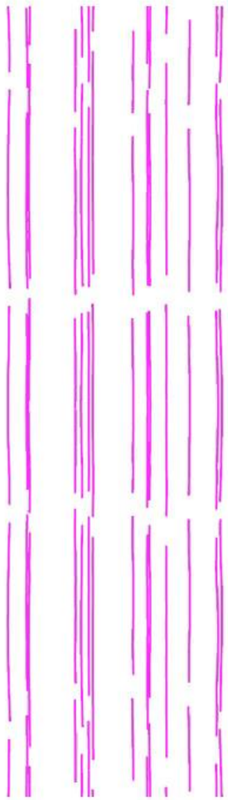
Interesting questions:

- *What are differences and similarities of turbulence in Fermi and Bose superfluids?*
- *What type of quantum turbulence is present in the fermionic systems?*
- *Turbulence in spin-imbalanced systems?*



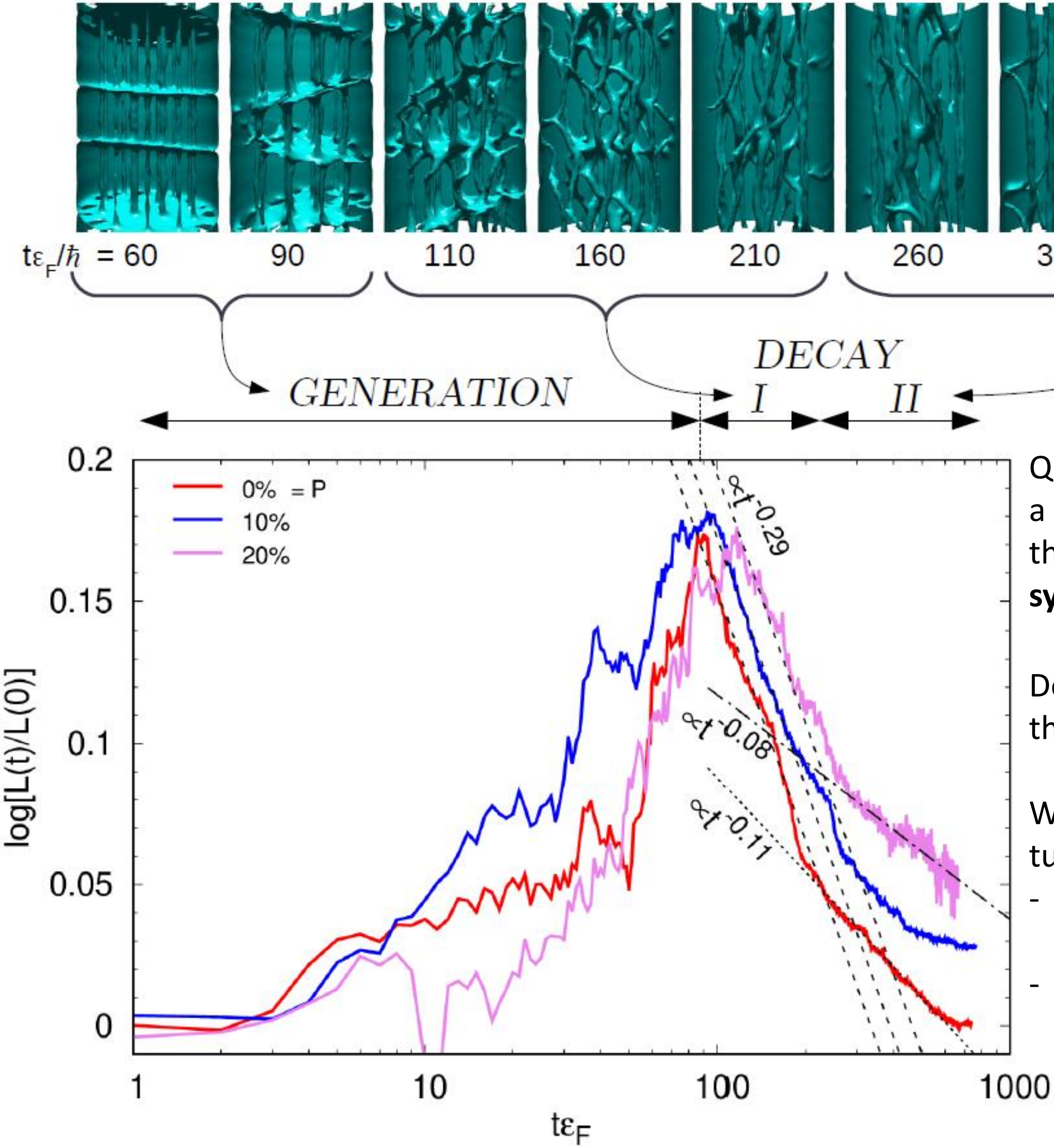






time = $0\ [\varepsilon_F^{-1}]$

K. Hossain, K. Kobuszewski, M.M. Forbes, P. Magierski, K. Sekizawa, G. Wlazłowski, arXiv:2010.07464



Qualitatively, the turbulence decay in a **spin-imbalanced system** exhibits the same properties as in **spin-symmetric** counterpart.

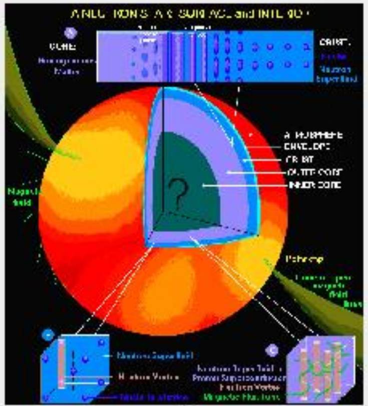
Decay rate is almost **insensitive** to the **spin-polarization** of the system.

We have identified **two regimes** of turbulence decay corresponding to:

- **Reconnections near the boundary** of the system
- **Decay of Kelvin waves**.

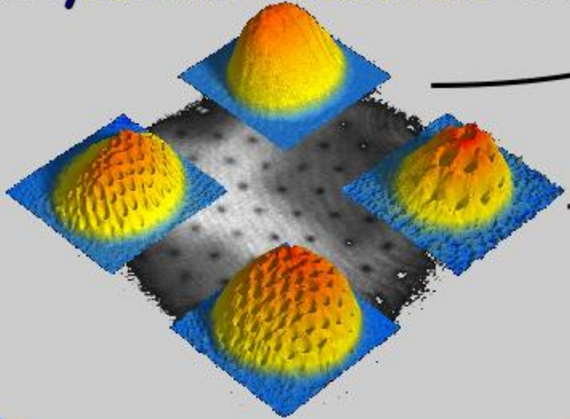
K. Hossain, K. Kobuszewski, M.M. Forbes, P. Magierski, K. Sekizawa, G. Wlazłowski, arXiv:2010.07464

Superfluid effects in neutron stars (glitches, turbulence)

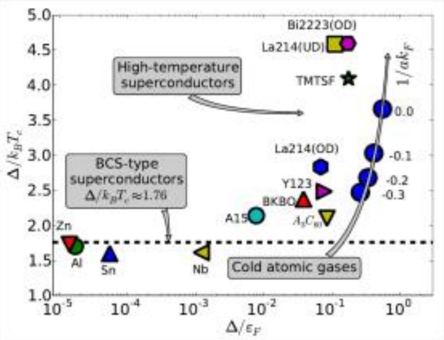


Quantum turbulence

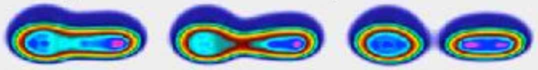
Ultracold fermionic gases as ***quantum simulators*** of...



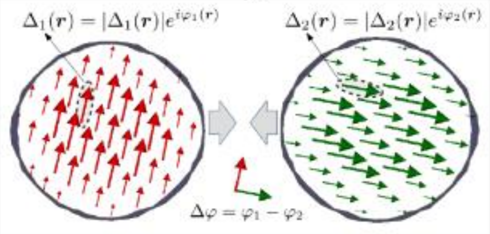
High-temperature superconductors



Impact of superfluidity on fission dynamics of heavy nucleus



Collisions of two superfluid nuclei



Time-dependent density functional theory for unified description of dynamics in these systems.

Thank you