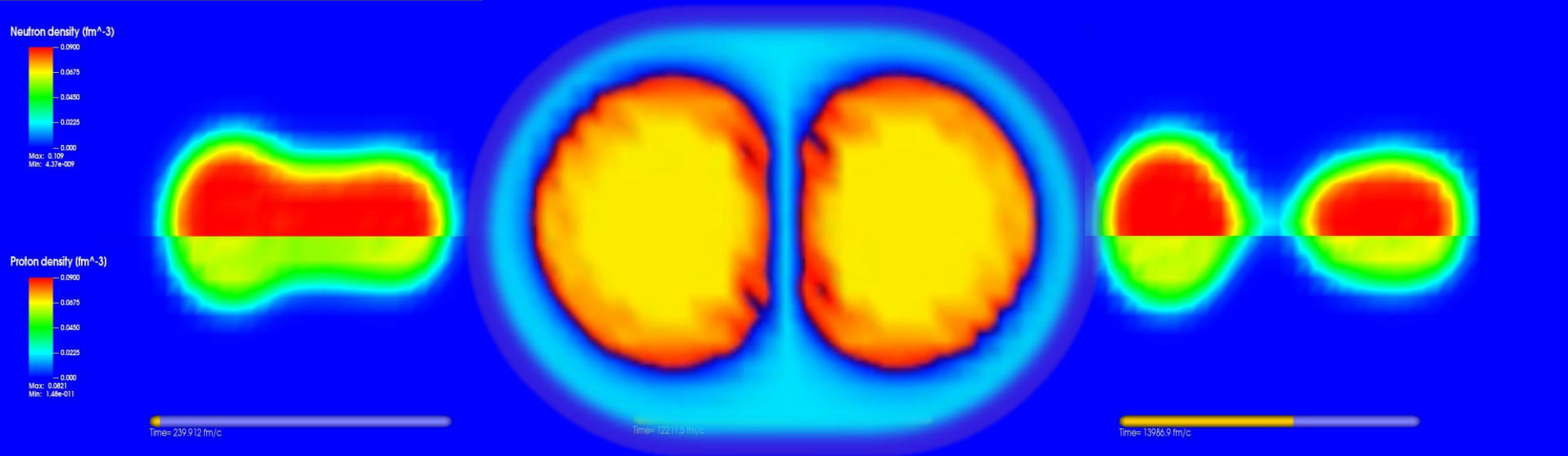


Pairing dynamics far from equilibrium



Piotr Magierski
(Warsaw University of Technology)

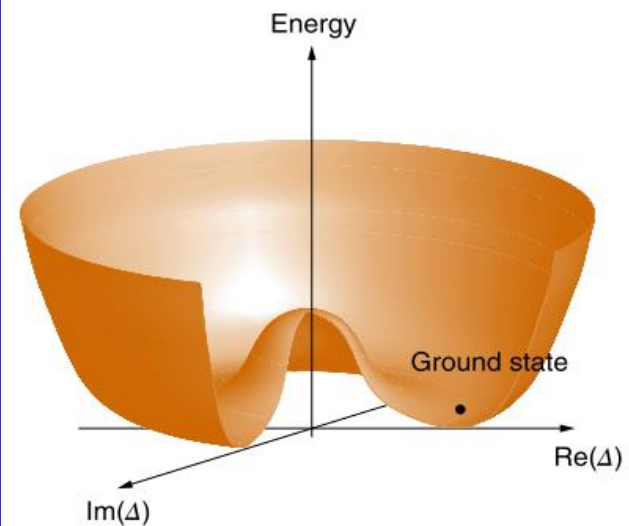
Collaborators:

Matthew Barton

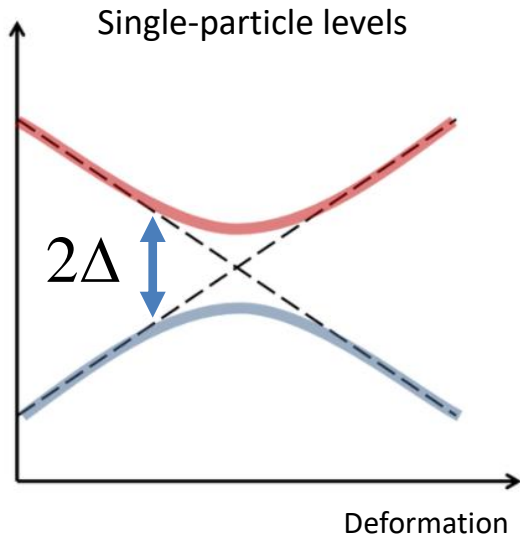
Andrzej Makowski (Ph.D. Student)

Kazuyuki Sekizawa (Tokyo Inst. of Tech.)

Gabriel Wlazłowski

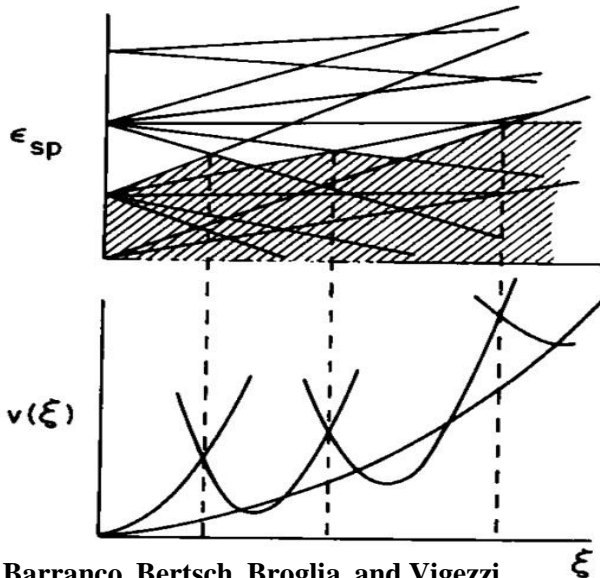
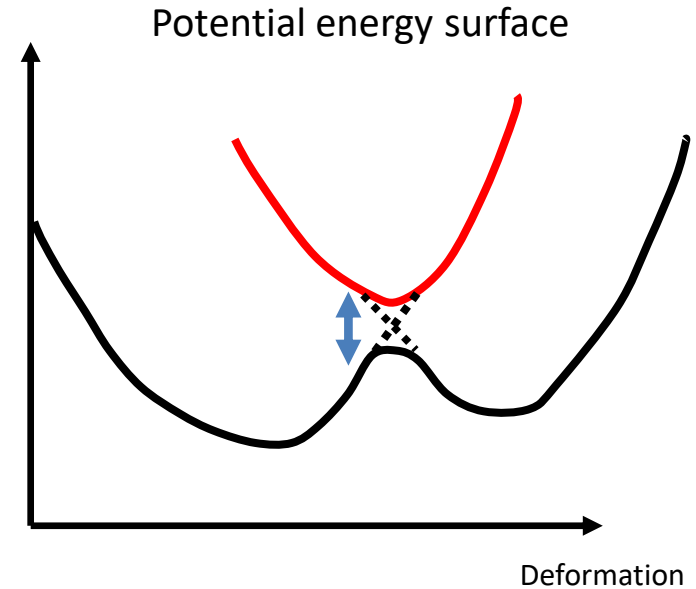


Pairing as an energy gap



Quasiparticle energy:

$$E_{qp} = \sqrt{(\varepsilon - \mu)^2 + |\Delta|^2}$$



As a consequence of pairing correlations large amplitude nuclear motion becomes more adiabatic.

While a nucleus elongates its Fermi surface becomes oblate and its sphericity must be restored
 Hill and Wheeler, PRC, 89, 1102 (1953)
 Bertsch, PLB, 95, 157 (1980)

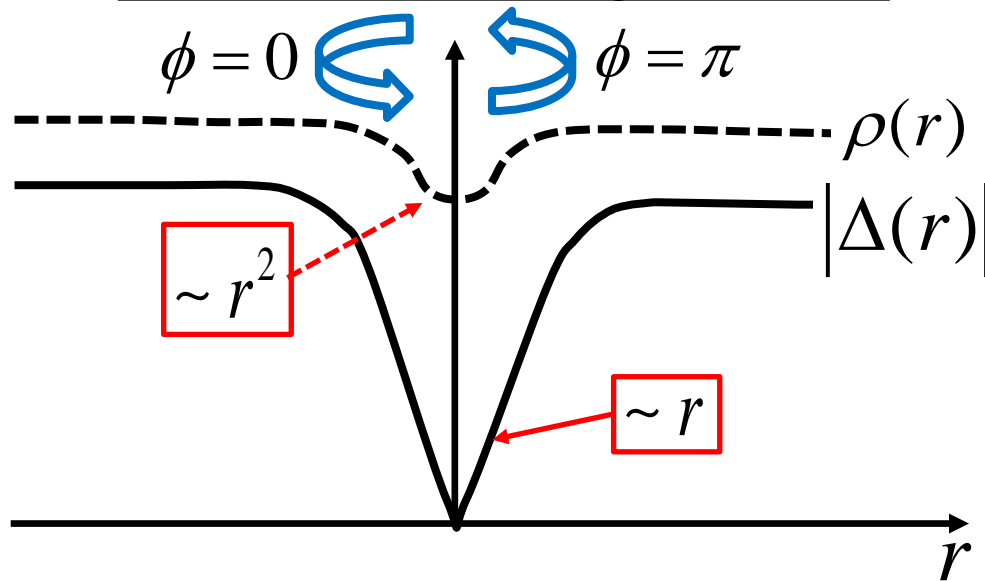
Pairing as a field

$$\Delta(\vec{r}, t) = |\Delta(\vec{r}, t)| e^{i\phi(\vec{r}, t)}$$

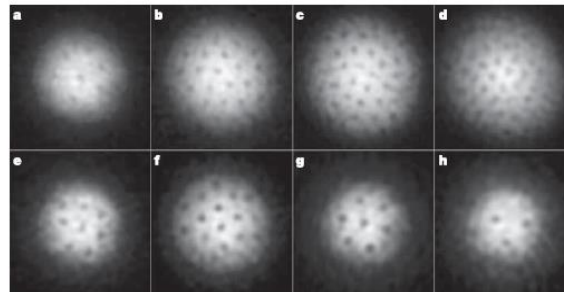
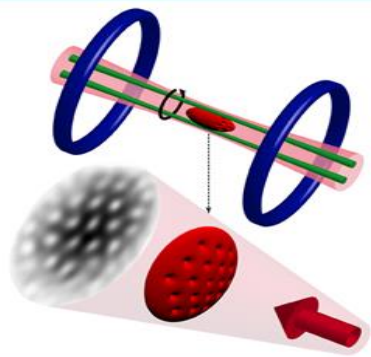
Both magnitude and phase may have a nontrivial spatial and time dependence.

Example of a nontrivial spatial dependence: *quantum vortex*

Vortex structure – section through the vortex core



Example of a topological excitation: magnitude of the pairing gap vanishes in the vortex core.



bosons

fermions

Experiments with ultracold Li-6 atoms: pictures of the vortex lattice.

Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) or 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the

magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (a), 766 G (b), 792 G (c), 812 G (d), 833 G (e), 843 G (f), 853 G (g) and 863 G (h). The field of view of each image is $880 \mu\text{m} \times 880 \mu\text{m}$.

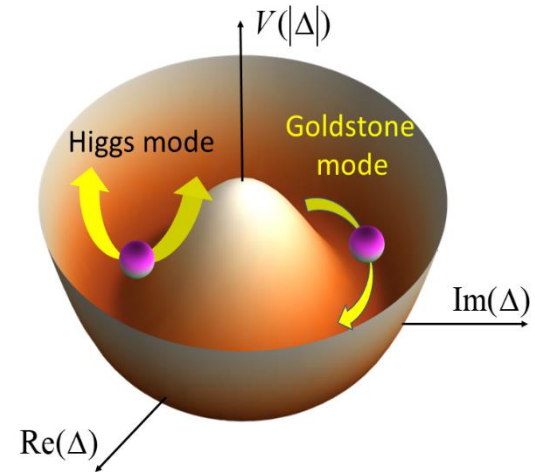
M.W. Zwierlein *et al.*,
Nature, 435, 1047 (2005)

$$\Delta(\vec{r}, t) = |\Delta(\vec{r}, t)| e^{i\phi(\vec{r}, t)}$$

Appearance of pairing field in Fermi systems is associated with U(1) symmetry breaking.

There are two characteristic modes associated with the field $\Delta(\vec{r}, t)$

- 1) **Nambu-Goldstone mode** explores the degree of freedom associated with the phase: $\phi(\vec{r}, t)$
- 2) **Higgs mode** explores the degree of freedom associated with the magnitude: $|\Delta(\vec{r}, t)|$



What's the difference between pairing correlations and existence of superfluid phase?

- Superfluid phase exists if the *off-diagonal long range order* is present:

$$\lim_{|r-r'|\rightarrow\infty} \langle \psi_{\uparrow}^{\dagger}(r) \psi_{\downarrow}^{\dagger}(r') \psi_{\downarrow}(r') \psi_{\uparrow}(r) \rangle \neq 0$$

C.N. Yang, Rev. Mod. Phys. 34, 694 (1962)

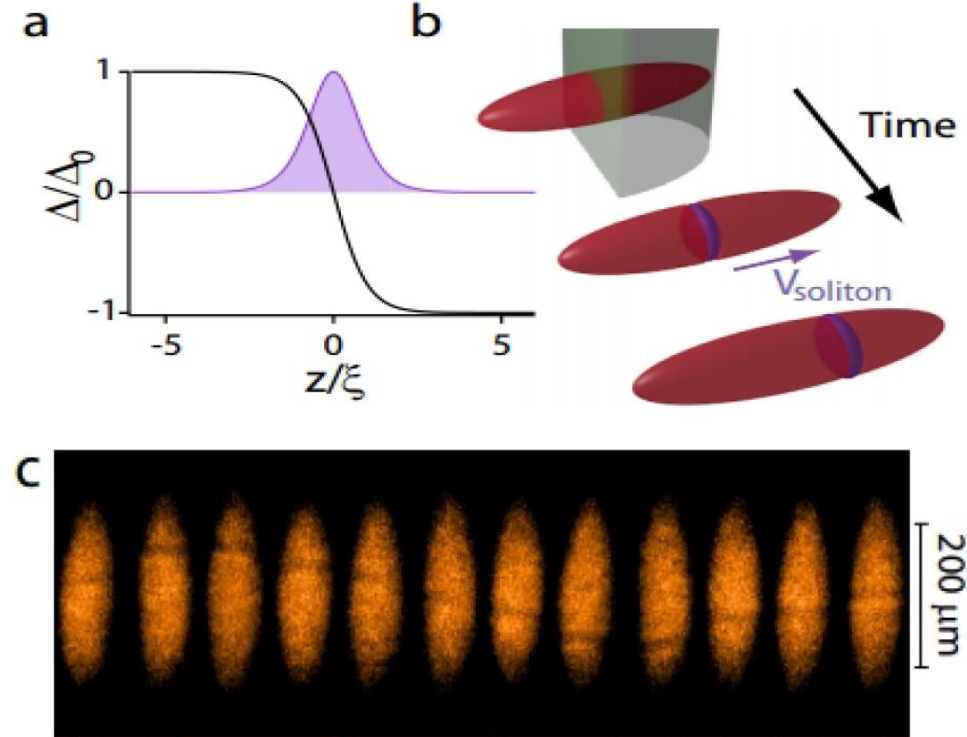
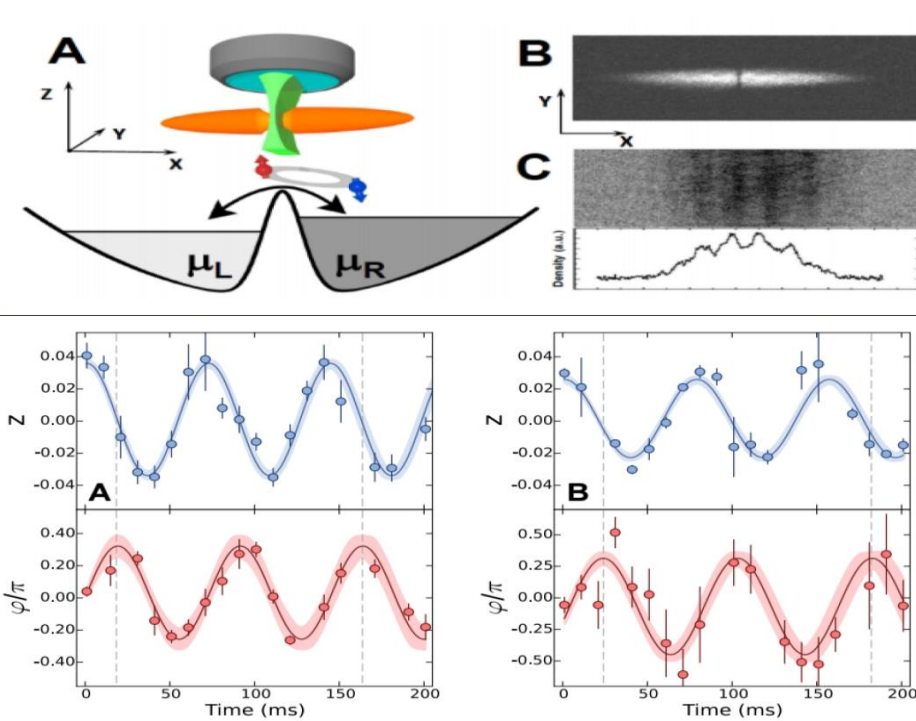
- This limit is unreachable in atomic nuclei due to their finite size. Therefore it is more convenient to look instead for the manifestations of the phase: $\Delta(\vec{r}, t) = |\Delta(\vec{r}, t)| e^{i\phi(\vec{r}, t)}$

Note: whenever I mention theory I mean: time dependent HFB (TDHFB) or time dependent Density Functional Theory (TDDFT) with local pairing field.

Two regimes for phase-induced effects in fermionic superfluids

Weak coupling (weak link)

Strong coupling



Observation of **AC Josephson effect** between two 6Li atomic clouds.

G. Valtolina et al., *Science* 350, 1505 (2015).

Creation of a „heavy soliton“ after merging two superfluid atomic clouds.

T. Yefsah et al., *Nature* 499, 426 (2013);

M.J.H. Ku et al., *Phys. Rev. Lett.* 116, 045304 (2016)

Superflow is accompanied with creation of topological excitations (vortices) leading to energy dissipation.

„Heavy soliton“ decays through the unique sequence of topological excitations.

G. Wlazłowski, K. Xhani, M. Tylutki, N.P. Proukakis, P. Magierski, *Phys. Rev. Lett.* **130**, 023003 (2023)

G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. Magierski, *Phys. Rev. Lett.* **120**, 253002 (2018)

Nuclear systems

Some evidence for a nuclear **DC Josephson effect** has been gathered over the years, following ideas presented in papers:

V.I. Gol'danskii, A.I. Larkin, JETP 26, 617 (1968), K. Dietrich, Phys. Lett. 32B 428 (1970)

Experimental evidence of enhanced nucleon pair transfer reported eg. in:

M.C. Mermaz, Phys. Rev. C36 1192, (1987), M.C. Mermaz, M. Girod, Phys. Rev. C53 1819 (1996)

Surprisingly evidence for AC Josephson effect has also been found

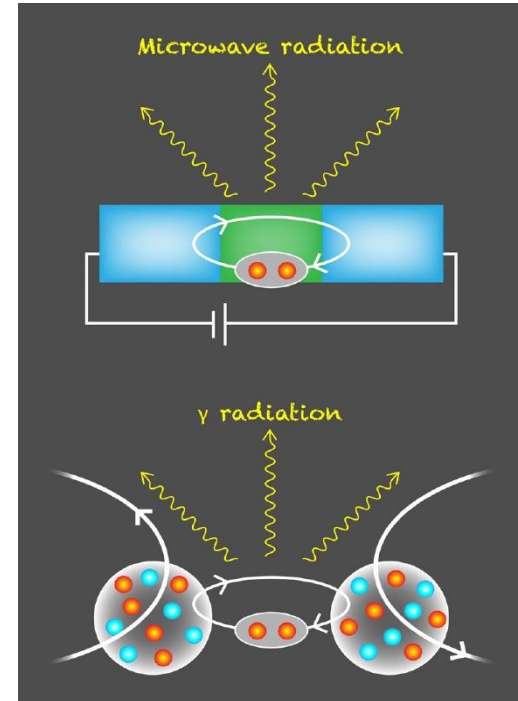
G.Potel, F.Barranco, E.Vigezzi, R.A. Broglia, "Quantum entanglement in nuclear Cooper-pair tunneling with gamma rays," Phys.Rev. C103, L021601 (2021)

R. Broglia, F. Barranco, G. Potel, E. Vigezzi

„Transient Weak Links between Superconducting Nuclei: Coherence Length”

Nuclear Physics News 31, 25 (2021)

(see next talk by Gregory Potel)



From P. Magierski, *Physics* 14 (2021) 27.

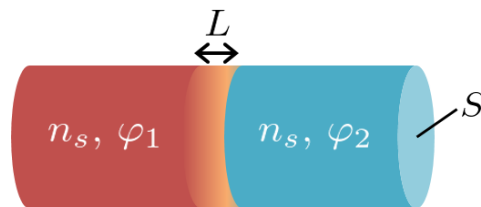
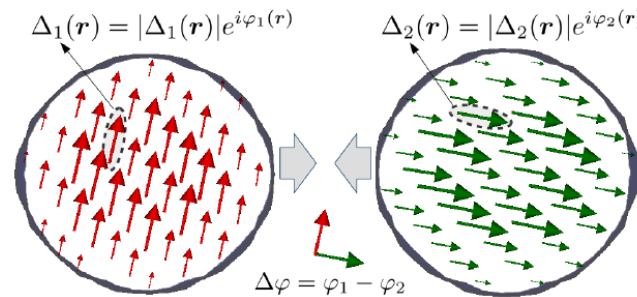
„Heavy soliton” creation in nuclear collision

Collisions of superfluid nuclei having different phases of the pairing fields

The main questions are:

- how a possible solitonic structure can be manifested in nuclear system?
- what observable effect it may have on heavy ion reaction:
kinetic energy distribution of fragments, capture cross section, etc.?

Clearly, we cannot control phases of the pairing field in nuclear experiments and the possible signal need to be extracted after averaging over the phase difference.

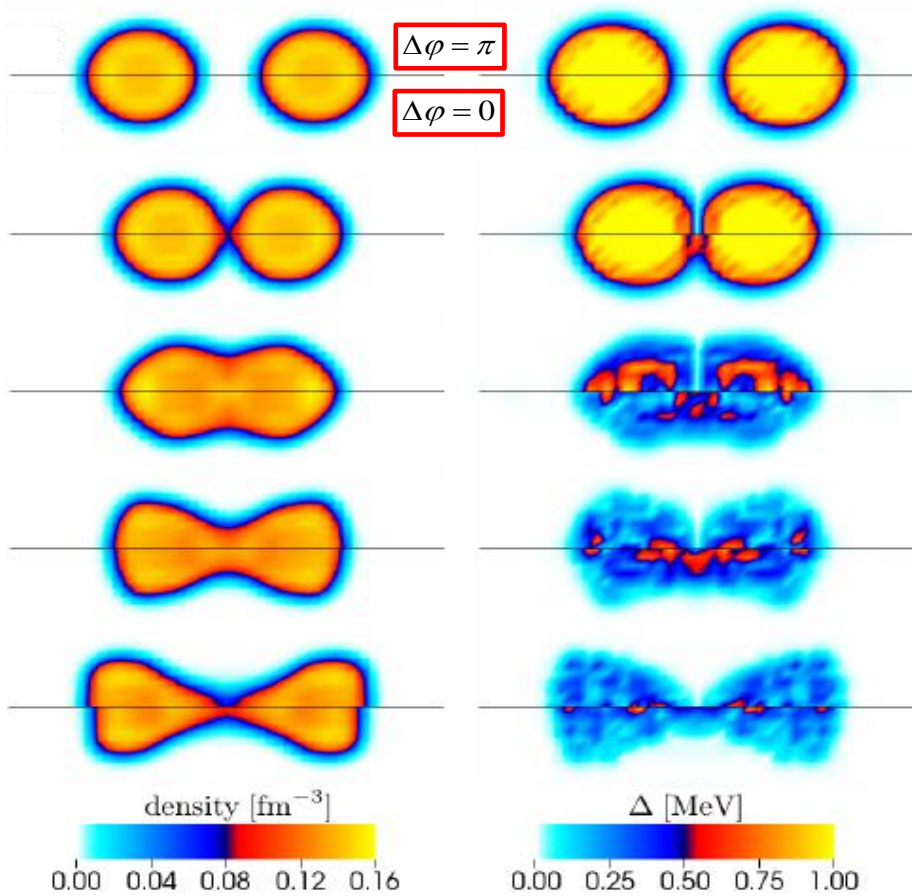
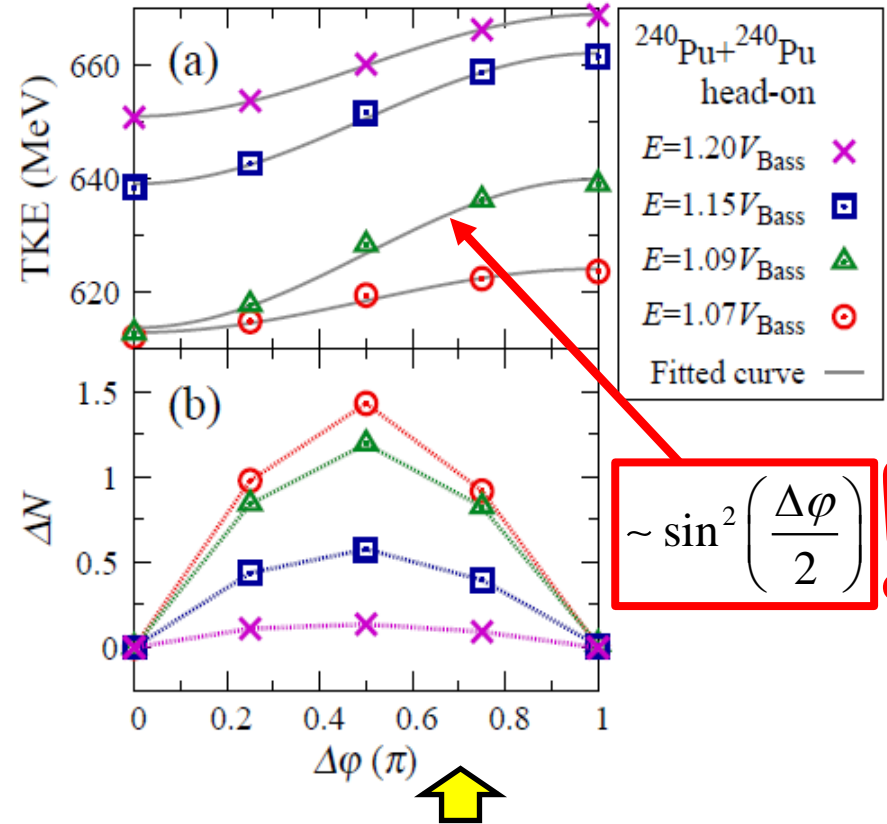


$$\Delta\varphi (\equiv \varphi_1 - \varphi_2)$$

From Ginzburg-Landau (G-L) approach:

$$E_j = \frac{S}{L} \frac{\hbar^2}{2m} n_s \sin^2 \frac{\Delta\varphi}{2}$$

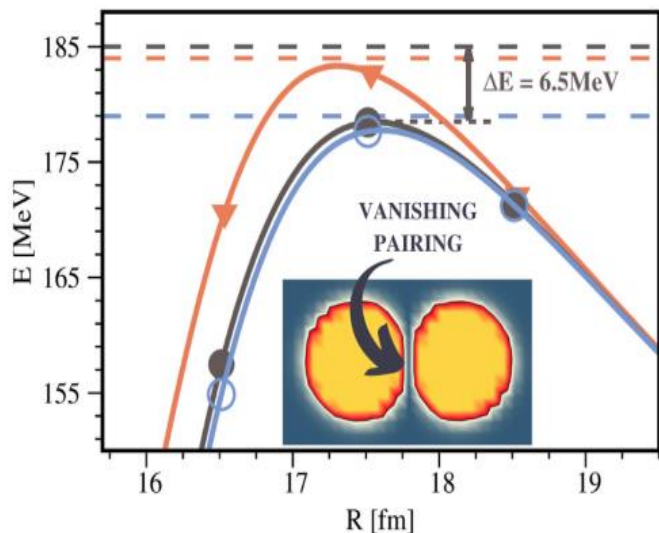
For typical values characteristic for two medium nuclei: $E_j \approx 30\text{MeV}$

$^{240}\text{Pu}+^{240}\text{Pu}$ Total kinetic energy of the fragments (TKE)

Average particle transfer between fragments.

Creation of the solitonic structure between colliding nuclei prevents energy transfer to internal degrees of freedom and consequently enhances the kinetic energy of outgoing fragments.

Surprisingly, the gauge angle dependence from the G-L approach is perfectly well reproduced in the kinetic energies of outgoing fragments!



Dynamic nature of the effect:

Solid lines: static barrier between two nuclei (with pairing included):

90Zr+90Zr - brown

96Zr+96Zr - black (0-phase diff.) and

blue (Pi-phase diff.)

Static barriers are practically insensitive to the phase difference of pairing fields.

Dashed lines: Actual threshold for capture obtained in dynamic calculations.

Hence ΔE measures the additional energy which has to be added to the system to merge nuclei.

TABLE I: The minimum energies needed for capture in $^{90}\text{Zr}+^{90}\text{Zr}$ and $^{96}\text{Zr}+^{96}\text{Zr}$ for the case of $\Delta\phi = 0$ [$E_{\text{thresh}}(0)$] and $\Delta\phi = \pi$ [$E_{\text{thresh}}(\pi)$]. The energy difference between the two cases is shown in the last column. The average pairing gap $\bar{\Delta}_i$ is defined by Eq. (4).

	$\bar{\Delta}_q$ (MeV)	$E_{\text{thresh}}(0)$ (MeV)	$E_{\text{thresh}}(\pi)$ (MeV)	ΔE_s
^{90}Zr	$\bar{\Delta}_n = 0.00$	184	184	0
	$\bar{\Delta}_p = 0.09$			
^{96}Zr	$\bar{\Delta}_n = 1.98$	179	185	6
	$\bar{\Delta}_p = 0.32$			
	$\bar{\Delta}_n = 2.44$	178	187	9
	$\bar{\Delta}_p = 0.33$			
	$\bar{\Delta}_n = 2.94$			
$\bar{\Delta}_p = 0.34$				

Dependence of the additional energy on pairing gap in colliding nuclei

P. Magierski, A. Makowski, M. Barton, K. Sekizawa, G. Wlazłowski, Phys. Rev. C 105, 064602, (2022)

G. Scamps, Phys. Rev. C 97, 044611 (2018): **barrier fluctuations extracted from experimental data provide evidence that the effect exists.**

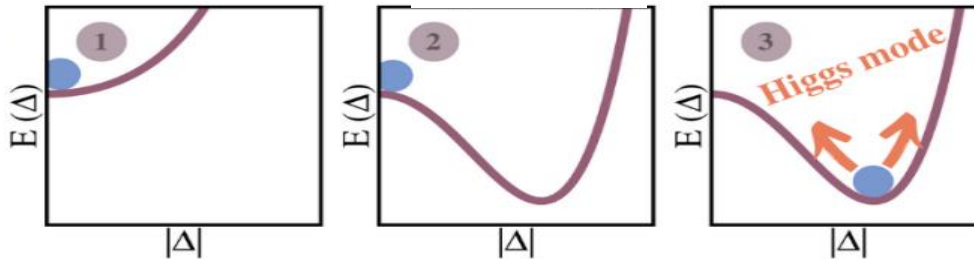
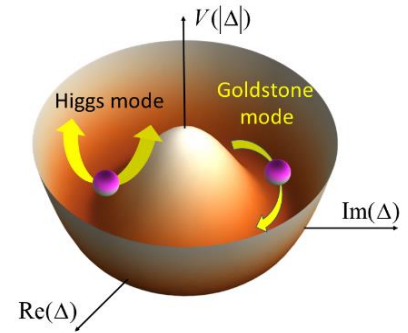
For details concerning dynamics and shape evolution see today's presentation of A. Makowski and poster

Pairing Higgs mode

Let's consider Fermi gas with schematic pairing interaction and coupling constant depending on time:

$$\hat{H} = \sum_k \varepsilon_k \hat{\psi}_k^+ \hat{\psi}_k - g(t) \sum_{k,l>0} \hat{\psi}_k^+ \hat{\psi}_{\bar{k}}^+ \hat{\psi}_{\bar{l}} \hat{\psi}_l$$

$g(t) = g_0 \theta(t)$ coupling constant is switched on withing time scale much shorter than \hbar/ε_F



As a result pairing becomes unstable and increases exponentially $\Delta(t) \propto e^{-i\zeta t} = e^{-i\omega t} e^{\gamma t}$

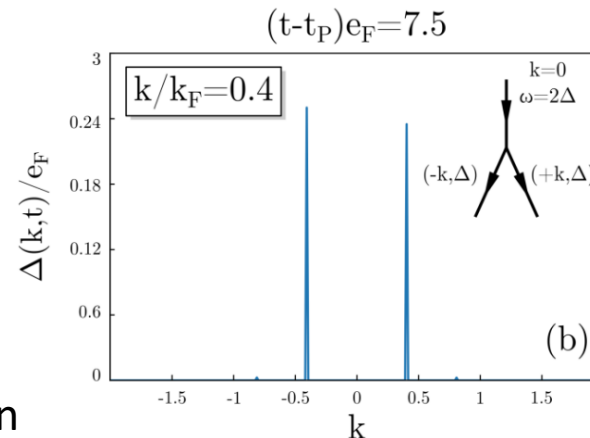
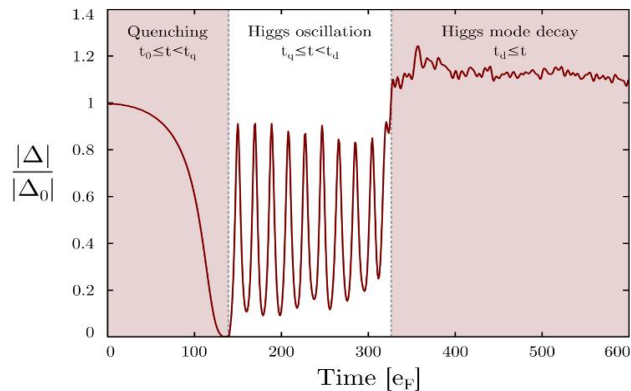
$$\frac{1}{g_0} = \sum_{k>0, \varepsilon_k > \mu} \frac{\tanh\left(\frac{\beta|\varepsilon_k - \mu|}{2}\right)}{2|\varepsilon_k - \mu| + \zeta} + \sum_{k>0, \varepsilon_k < \mu} \frac{\tanh\left(\frac{\beta|\varepsilon_k - \mu|}{2}\right)}{2|\varepsilon_k - \mu| - \zeta}$$

Time scale of growth and the period of subsequent oscillation is related to static value of pairing Δ_0 :

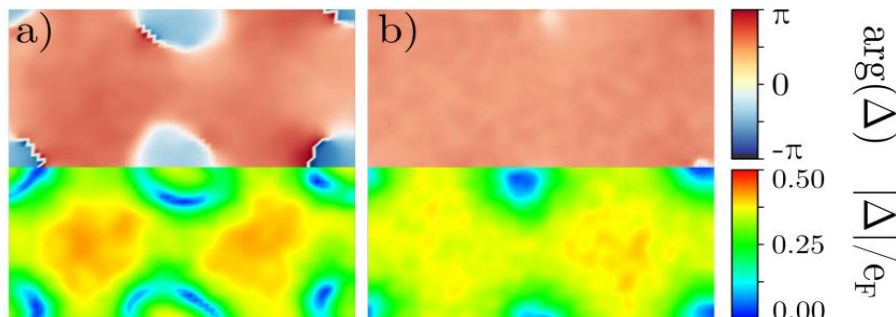
$$\tau = \frac{1}{\gamma} \approx \frac{\hbar}{\Delta_0}$$

Contrary to low-energy Goldstone modes Higgs modes are unstable and decay.

M. Dzero, E. A. Yuzbashyan, and B. L. Altshuler, EPL 85, 20004 (2009)



Leading eventually to inhomogeneous pairing field distribution

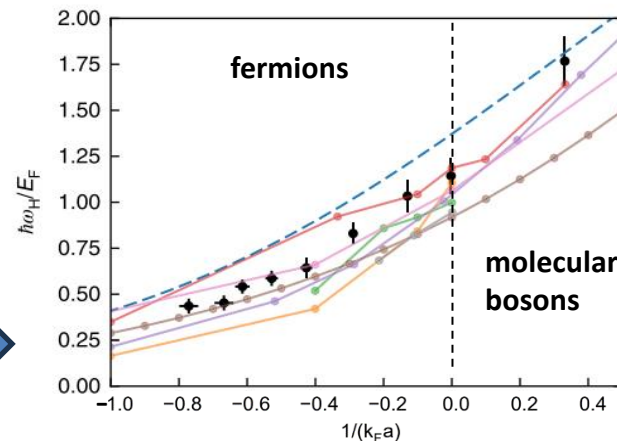


A. Barresi, A. Boulet, G. Wlazłowski, P. Magierski, Sci. Rep. 13, 11285 (2023)

In ultracold atomic gases one can induce Higgs mode by varying coupling constant.

A. Behrle et al.
Higgs mode in a strongly interacting fermionic Superfluid, Nature Physics **14**, 781 (2018).
Li-6 atoms in harmonic trap

Measured peak position of the energy absorption spectra (black dots) and theory predictions for Higgs mode.



Pairing instability in nuclear reaction

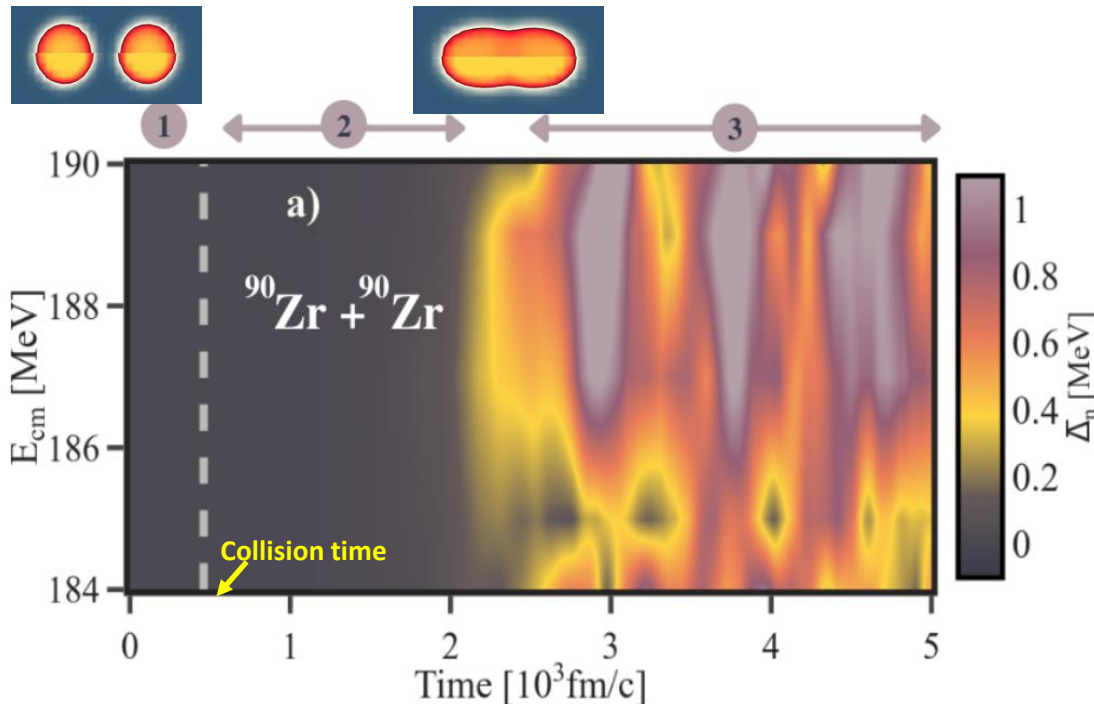
$$\Delta = \frac{8}{e^2} \varepsilon_F \exp\left(\frac{-2}{gN(\varepsilon_F)}\right) - \text{BCS formula – weak coupling limit}$$

ε_F - Fermi energy

g - Pairing coupling constant

$N(\varepsilon_F)$ - Density of states at the Fermi level

Although one cannot change coupling constant in atomic nuclei one may affect **density of states at the Fermi surface and consequently trigger pairing instability.**

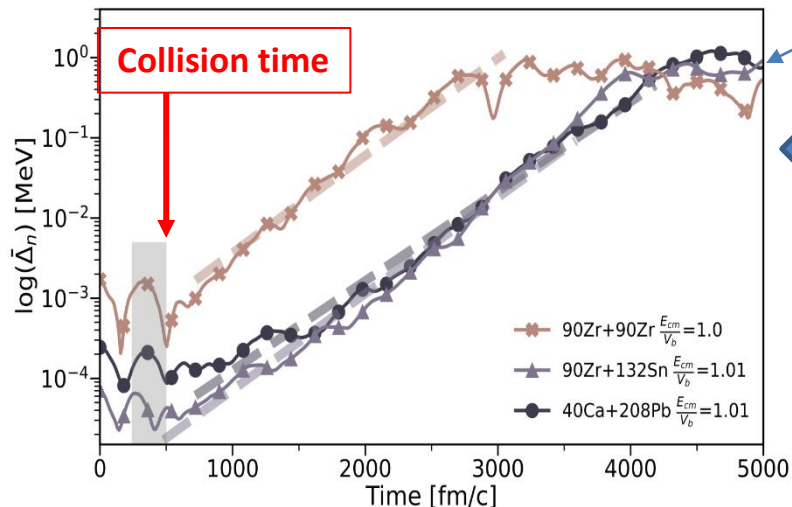


Collision of two neutron magic systems creates an elongated di-nuclear system.

Within 1500 fm/c pairing is enhanced in the system and reveals oscillations with frequency:

$$\Delta < \hbar\omega < 2\Delta$$

Interestingly the effect is generic and occurs for various collisions of magic nuclei.

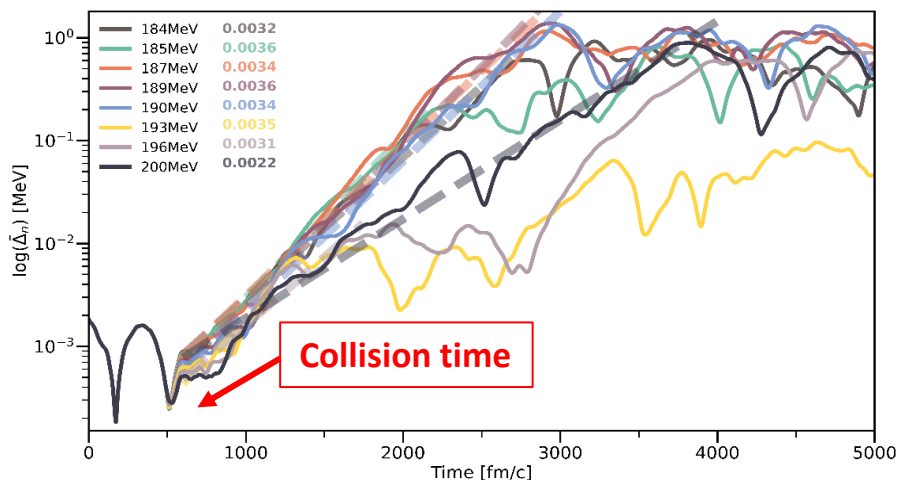


Exponential increase of pairing gap after collision indicating pairing instability in di-nuclear system. Time scale of pairing enhancement:

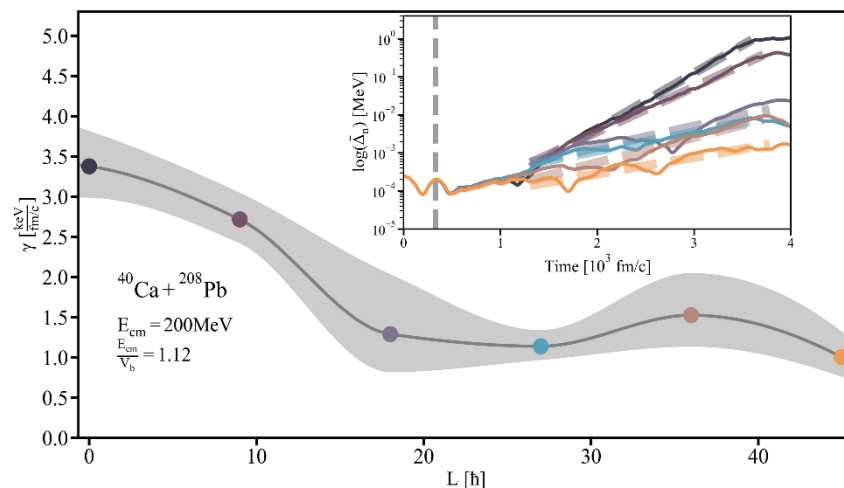
$$\tau \gg \frac{\hbar}{\Delta_0}$$

It occurs up to relatively high collision energies

$^{90}\text{Zr} + ^{90}\text{Zr}$ head-on collision as a function of E_{cm}



Non-central collision



The excitation energy of a compound system after merging exceeds **20-30 MeV**.

It corresponds to temperatures **close to critical temperature for superfluid-to-normal transition**.

Therefore it is unlikely that the system develops superfluid phase and it is rather nonequilibrium enhancement of pairing correlations.

See also today's presentation of A. Makowski and poster

Summary and open questions

- TDHFB provides evidence for nontrivial behavior of **pairing correlations in highly nonequilibrium conditions** which includes **solitonic excitations** (dynamic barrier modification for capture) and **pairing enhancement** as a result of collision.
- There is certain experimental evidence for **solitonic excitations**, although not easy to extract (G. Scamps, Phys. Rev. C C 97, 044611 (2018)).
- **Pairing enhancement** in collision of magic nuclei is a **generic feature of TDHFB** appearing in collisions of magic nuclei at energies close to the Coulomb barrier.
- Impact of **pairing enhancement** on dynamics is unknown and requires more theoretical effort: investigation of noncentral collisions, considerations of pairing correlations during subsequent stages of compound nucleus formation.