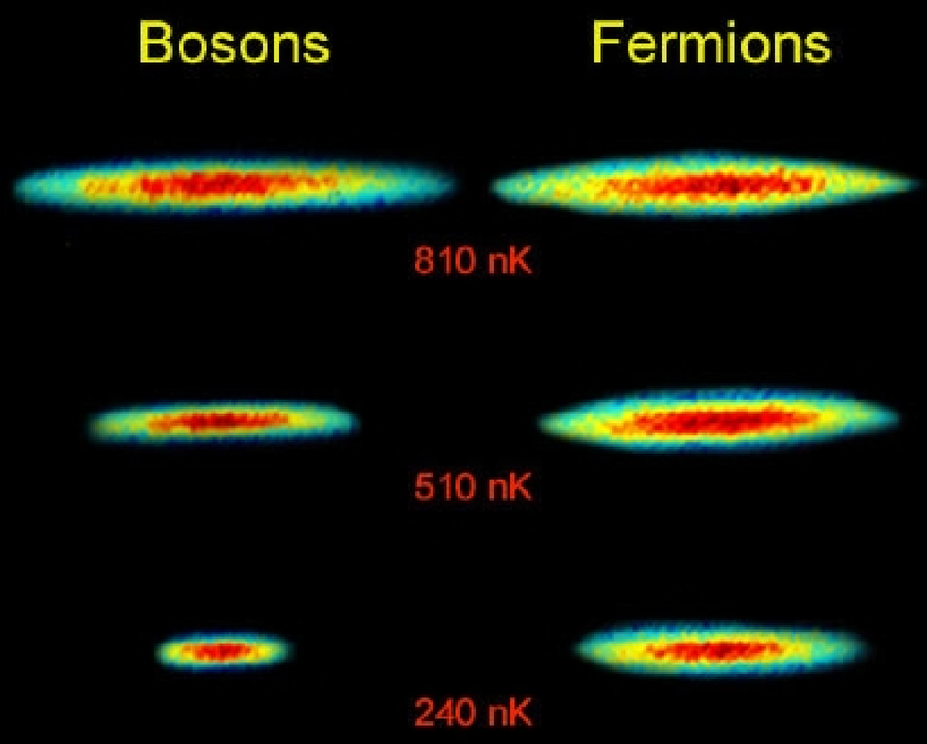
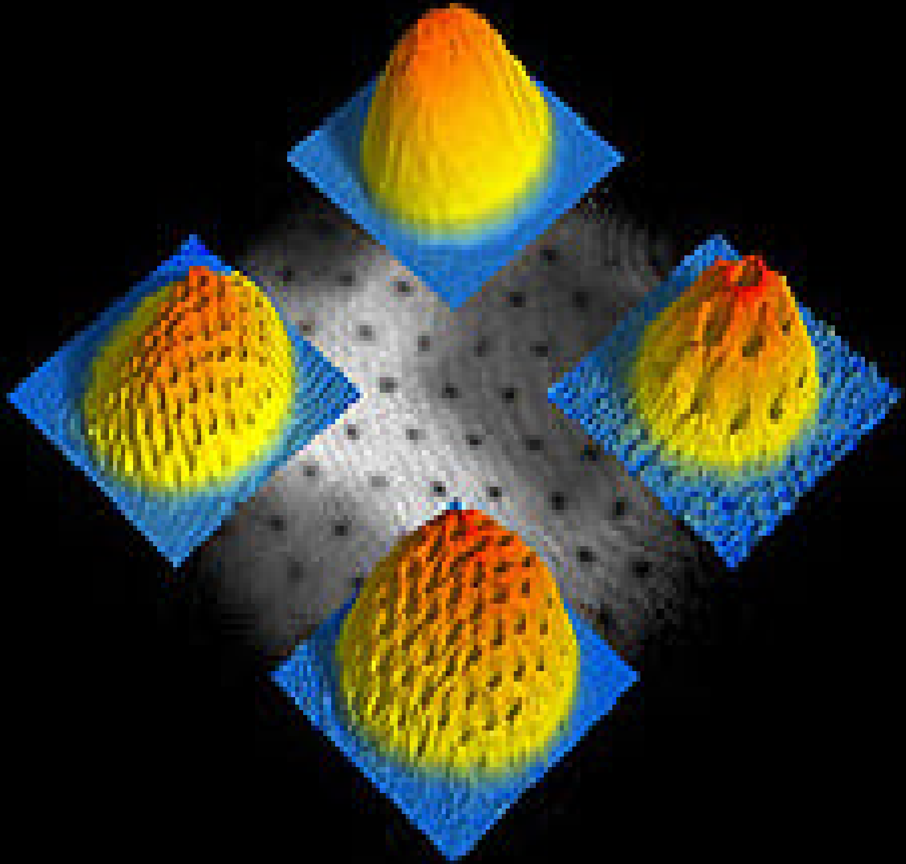
Surprising features of ultra cold atomic gases at BCS-BEC crossover





Surprising features of ultracold atomic gases at BCS-BEC crossover

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Collaborators:

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Gabriel Wlazłowski	(Warsaw)
Yongle Yu	(Seattle Lund Wuhan)

Outline

BCS-BEC crossover. Universality of the unitary regime.

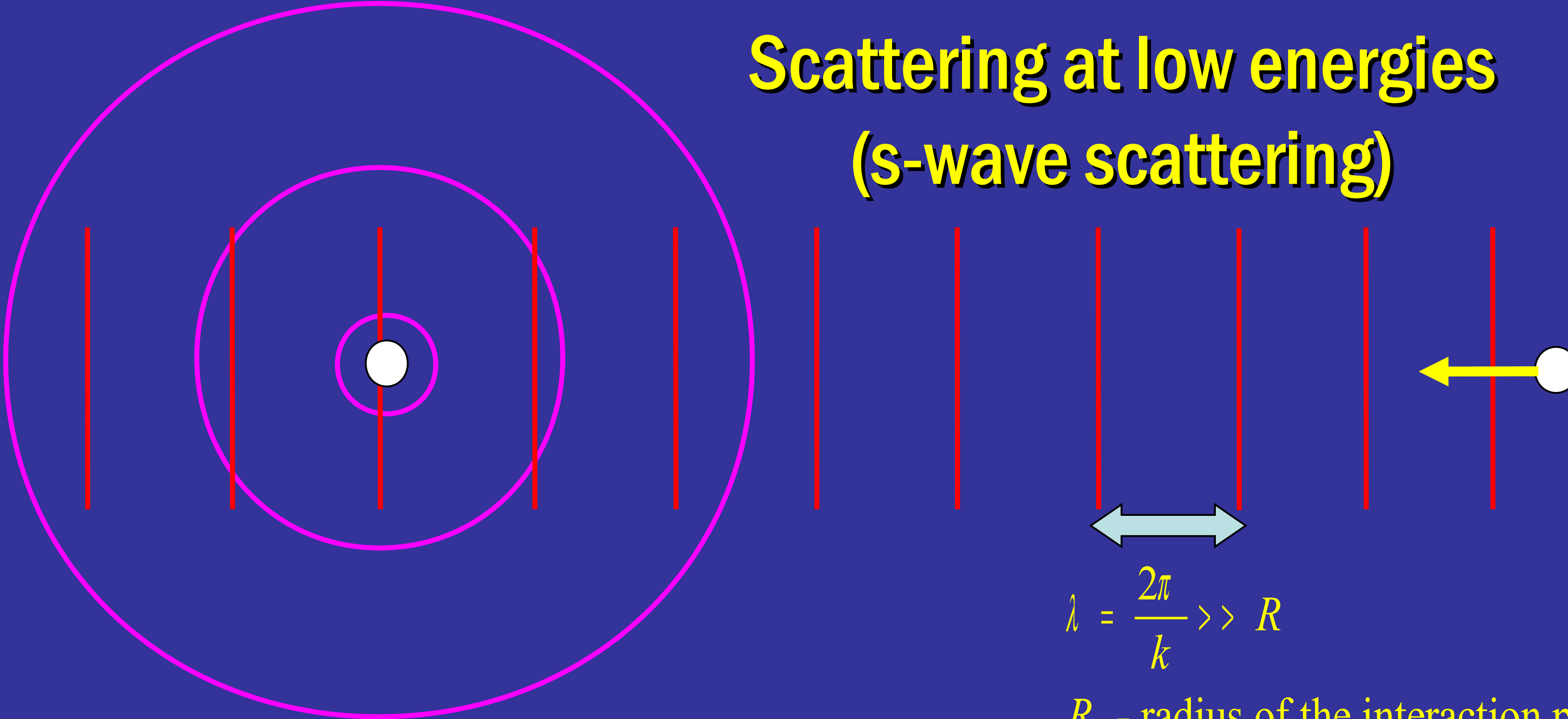
Physical realization of the unitary regime: ultra cold atomic gases.

Equation of state for the uniform Fermi gas in the unitary regime. Critical temperature. Experiment vs. Theory.

Unitary Fermi gas as a high-Tc superconductor: pseudogap phase

Nonequilibrium phenomena: generation and dynamics of superfluid vortices.

Scattering at low energies (s-wave scattering)



$\lambda = \frac{2\pi}{k} >> R$

R - radius of the interaction potential

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + f(k)\frac{e^{ikr}}{r}; \quad f(k) \text{ - scattering amplitude}$$

$$f(k) \overset{k\to 0}{=} \frac{1}{-ik - \frac{1}{a} + \frac{1}{2}r_0k^2},$$

a - scattering length, r_0 - effective range

If $k \to 0$ then the interaction is determined by the scattering length alone.

What is a unitary gas?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$n\, r_0^3 \ll 1$ $\quad n\, |a|^3 \gg 1$

n - particle density
a - scattering length
r_0 - effective range

$$\text{i.e. } r_0 \to 0,\, a \to \pm\infty$$

← **NONPERTURBATIVE REGIME**

System is dilute but strongly interacting!

UNIVERSALITY: $E = \xi_0 E_{FG}$

AT FINITE TEMPERATURE: $E(T) = \xi\left(\frac{T}{\varepsilon_F}\right) E_{FG}, \quad \xi(0) = \xi_0$

Thermodynamics of the unitary Fermi gas

$$\text{ENERGY:}\quad E(x)=\frac{3}{5}\xi(x)\varepsilon_F N;\quad x=\frac{T}{\varepsilon_F}$$

$$C_V=T\frac{\partial S}{\partial T}=\frac{\partial E}{\partial T}=\frac{3}{5}N\xi'(x)\Rightarrow S(x)=\frac{3}{5}N\int_0^x\frac{\xi'(y)}{y}dy$$

$$\text{ENTROPY/PARTICLE:}\quad \sigma(x)=\frac{S(x)}{N}=\frac{3}{5}\int_0^x\frac{\xi'(y)}{y}dy$$

$$\text{FREE ENERGY:}\ F=E-TS=\frac{3}{5}\varphi(x)\varepsilon_F N$$

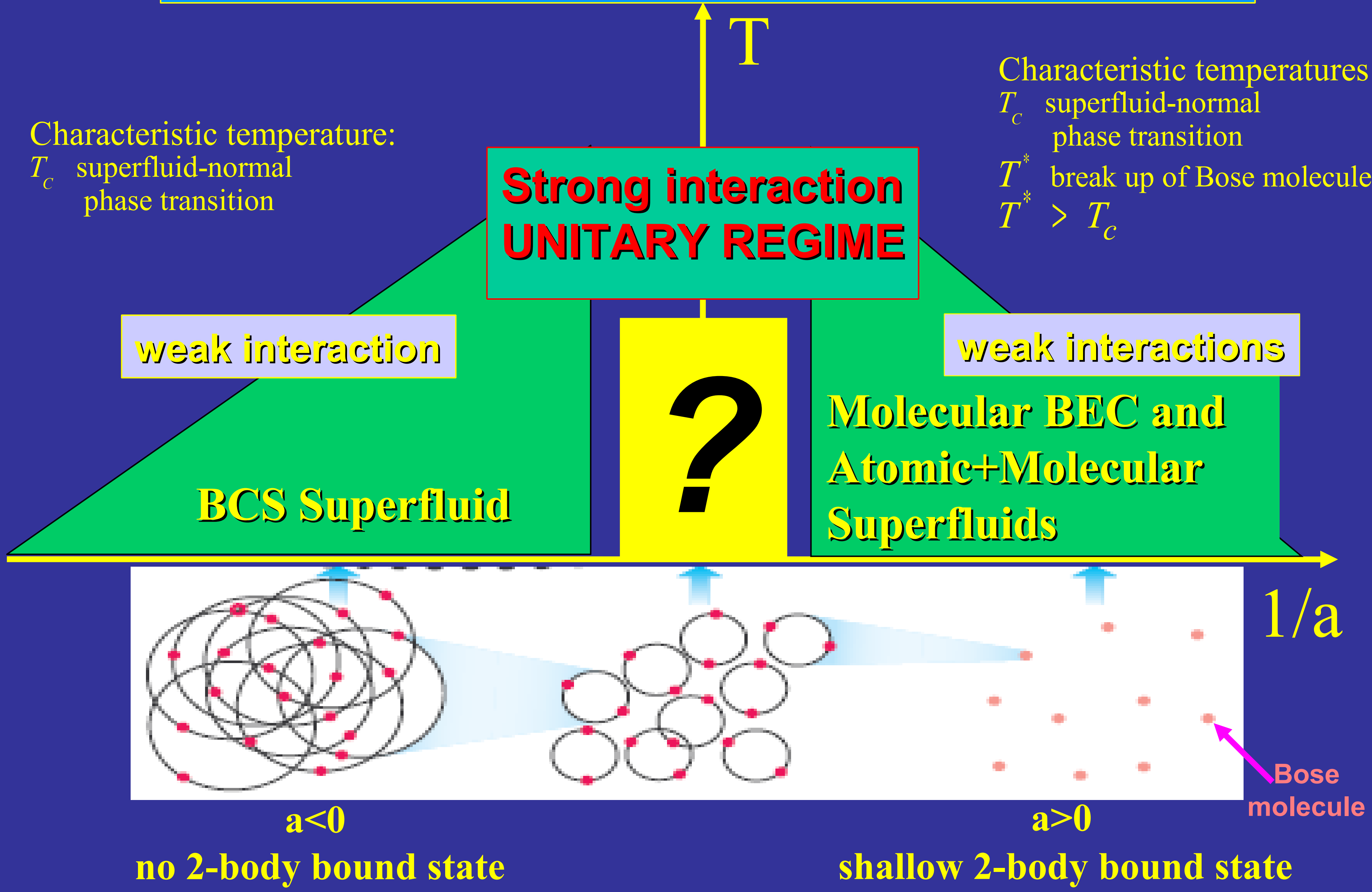
$$\varphi(x)=\xi(x)-x\sigma(x)$$

$$\text{PRESSURE:}\quad P=-\frac{\partial E}{\partial V}=\frac{2}{5}\xi(x)\varepsilon_F\frac{N}{V}$$

$$PV=\frac{2}{3}E$$

Note the similarity to the ideal Fermi gas

Expected phases of a two species dilute Fermi system
BCS-BEC crossover



Bertsch's Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

Why? Besides pure theoretical curiosity, this problem is relevant to neutron stars!

In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not! A number of people argued that under such conditions Fermionic matter is unstable.

- ***systems of bosons are unstable***
- ***systems of three or more fermion species are unstable***

- **Baker (LANL, winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)**
- **Carlson *et al* (2003) Fixed-Node Green Function Monte Carlo and Astrakharchik *et al* (2004) FN-DMC provided the best theoretical estimates for the ground state energy of such systems.**

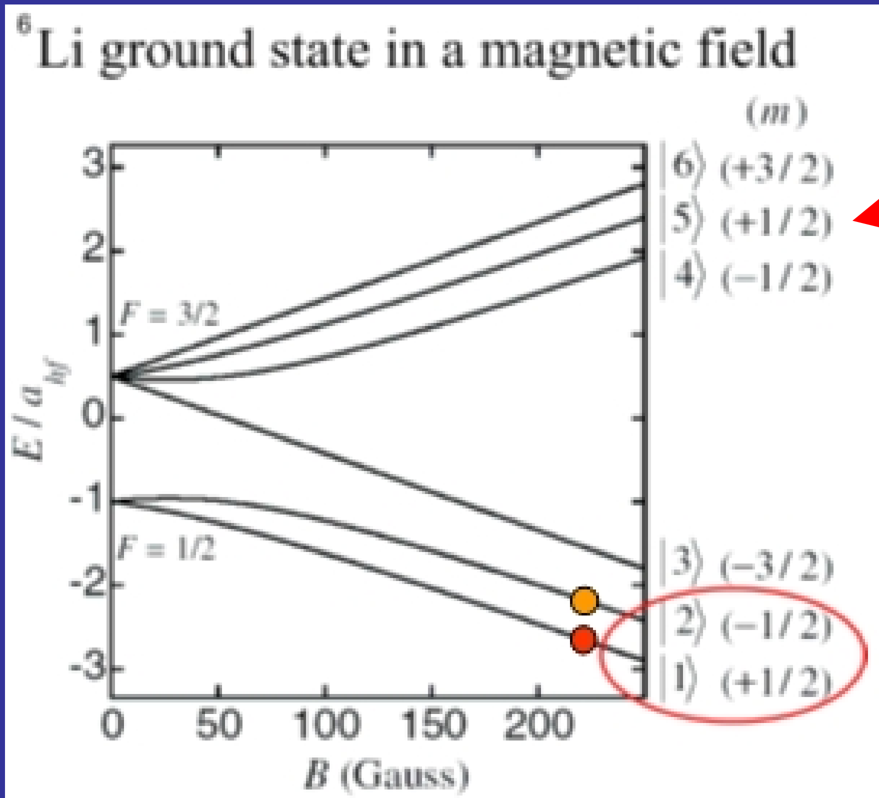
Carlson *et al* (2003) have also shown that the system has <u>a huge pairing gap</u> !

$$E_{gs} = \frac{3}{5}\varepsilon_F N \times \xi \qquad \Delta = \varepsilon_F \times \zeta$$

$$\xi = 0.40(1), \qquad \zeta = 0.50(1)$$

- **Thomas' Duke group (2002) demonstrated experimentally that such systems are (meta)stable.**

One fermionic atom in magnetic field



$$|F\ m_F\rangle$$

$$F = I + J\ ;\ J = L + S$$

Nuclear spin   Electronic spin

Two hypefine states are populated in the trap

Collision of two atoms: At low energies (low density of atoms) only L=0 (s-wave) scattering is effective.

- Due to the high diluteness atoms in the same hyperfine state do not interact with one another.
- Atoms in different hyperfine states experience interactions only in s-wave.

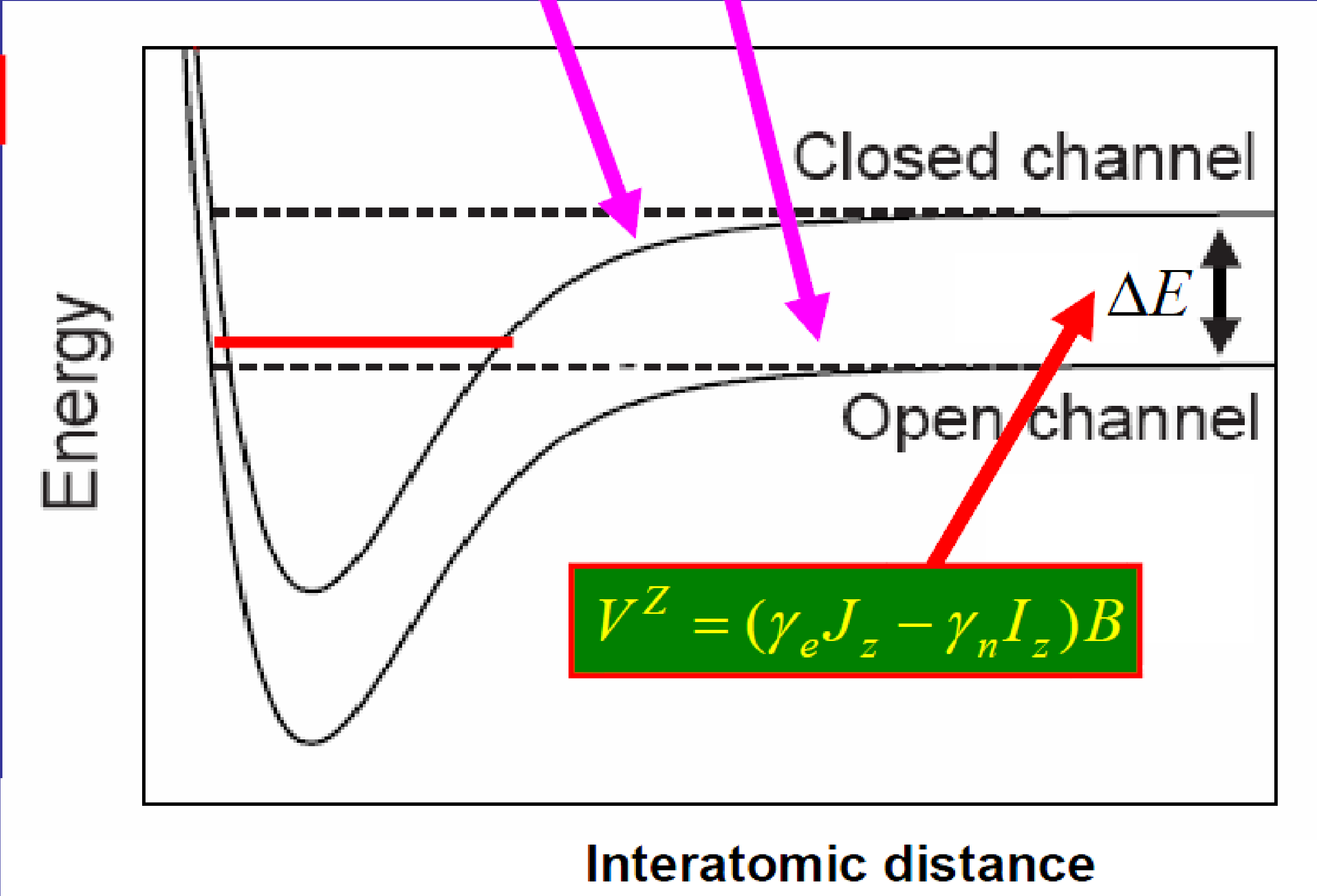
Effective Hamiltonian of an atom-atom system

$$H = \frac{\vec{p}^{\,2}}{2\mu} + \sum_{i=1}^{2} (V_i^{hf} + V_i^{Z}) + V_0(\vec{r})P_0 + V_1(\vec{r})P_1 + \ldots$$

$$V^{hf} = \frac{a_{hf}}{\hbar^2}\vec{I}\cdot\vec{J}, \;\; V_i - \text{Coulomb term}$$

Tiesinga, Verhaar, Stoof, Phys. Rev. A47, 4114 (1993)

Channel coupling



One open channel with one resonant bound state (s-wave scattering)

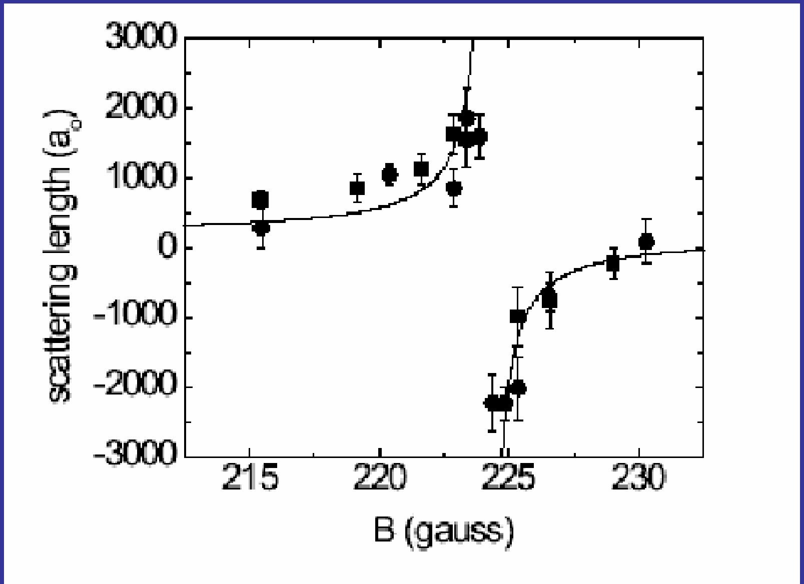
$$S(k)=S^{bg}(k)\left(1-\frac{2ik|g|^2}{-\frac{4\pi\hbar^2}{m}\left(\nu-\frac{\hbar^2k^2}{m}\right)+ik|g|^2}\right)$$

$$S^{bg}(k)=e^{-2ika_{bg}},\quad \nu=\varepsilon_b-\varepsilon_0,\quad |g|^2=\left|\left\langle\chi^+\middle|V^{hf}\middle|\phi_b\right\rangle\right|^2/k$$

A.J. Moerdijk et al.
Phys. Rev. A51 (1995)4852

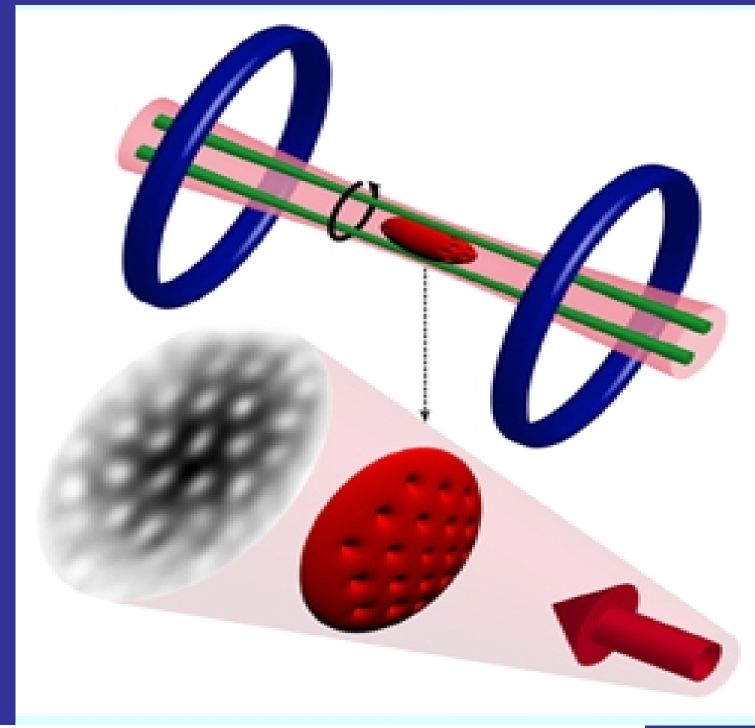
$$a=a_{bg}-\frac{m}{4\pi\hbar^2}\frac{|g|^2}{\nu}$$

$$\nu\sim B\Rightarrow a=a_{bg}\left(1-\frac{\Delta B}{B-B_0}\right)$$



Regal and Jin, PRL 90, 230404 (2003)

Evidence for fermionic superfluidity: vortices!

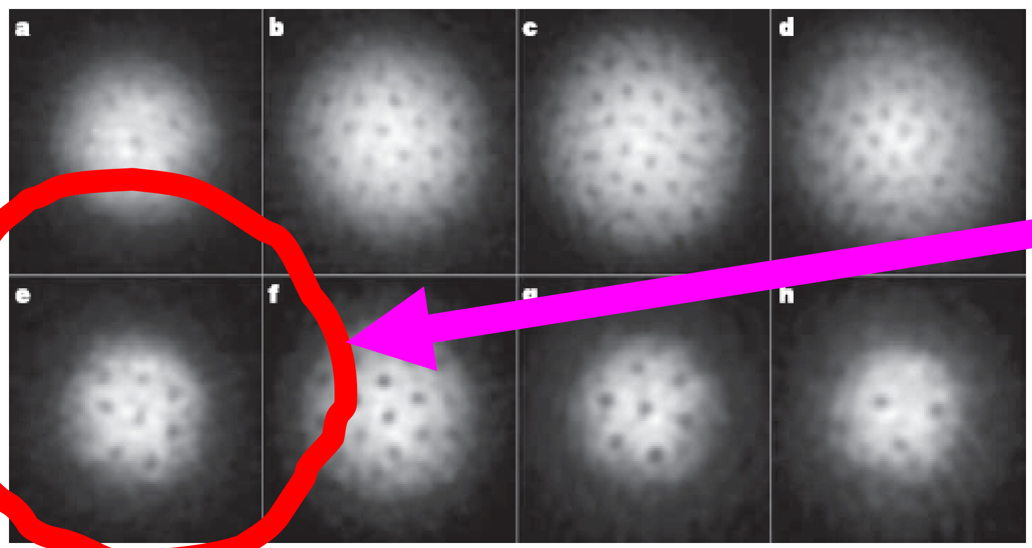


system of fermionic ^{6}Li atoms

Feshbach resonance: B=834G

BEC side: a>0

BCS side: a<0



UNITARY REGIME

M.W. Zwierlein *et al*., Nature, 435, 1047 (2005)

Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b**–**h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**), 812 G (**d**), 833 G (**e**), 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of view of each image is 880 μm × 880 μm.

Superconductivity and superfluidity in Fermi systems

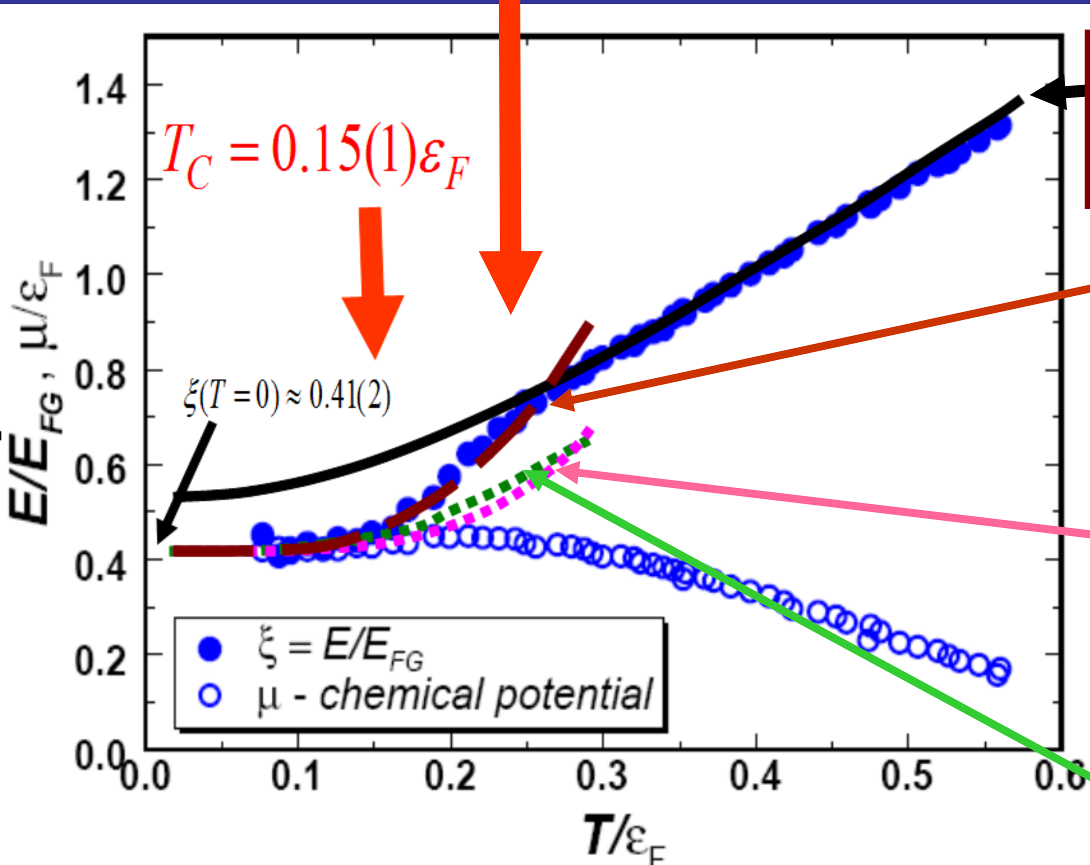
20 orders of magnitude over a century of (low temperature) physics

- ✓ **Dilute atomic Fermi gases** $T_c \approx 10^{-12} - 10^{-9}$ eV
- ✓ **Liquid ^{3}He** $T_c \approx 10^{-7}$ eV
- ✓ **Metals, composite materials** $T_c \approx 10^{-3} - 10^{-2}$ eV
- ✓ **Nuclei, neutron stars** $T_c \approx 10^{5} - 10^{6}$ eV
- **QCD color superconductivity** $T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV $\approx 10^4$ K)

a = ±∞

Deviation from Normal Fermi Gas



Normal Fermi Gas
(with vertical offset, solid line)

Bogoliubov-Anderson phonons and quasiparticle contribution
(dashed line)

Bogoliubov-Anderson phonons contribution only (dotted line)

Quasi-particle contribution only
(dotted line)

$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}}\exp\left(-\frac{\Delta}{T}\right)$$

$$\Delta = \left(\frac{2}{e}\right)^{7/3} \varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N \, \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.41$$

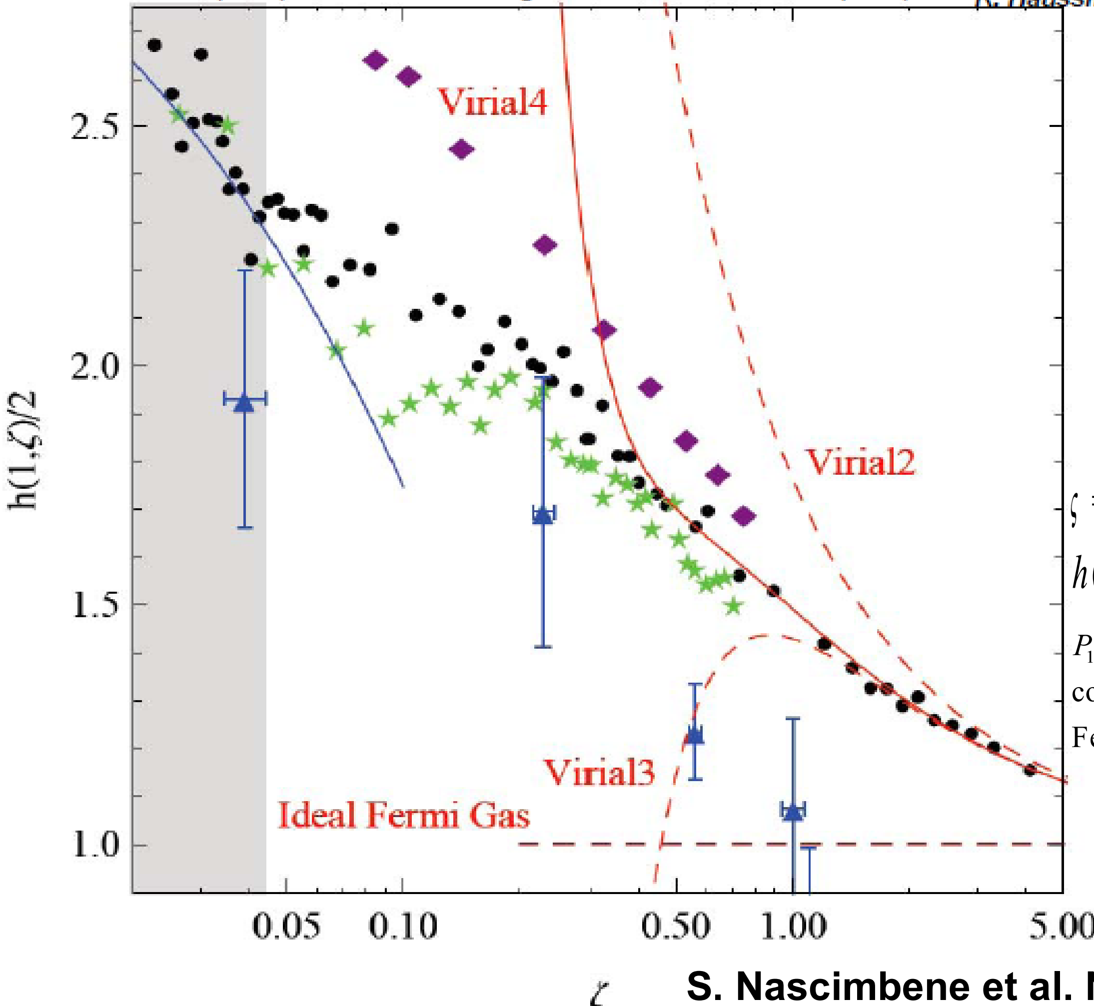
A. Bulgac, J.E. Drut, P. Magierski, PRL96,090404(2006)

Comparison with Many-Body Theories (1)

Diagram. MC — *E. Burovski et. al., PRL 96, 160402 (2006)*

QMC — *A. Bulgac et al., PRL 99, 120401 (2006)*

Diagram.+analytic — *R. Haussmann. et al., PRA 75, 023610 (2007)*



$\zeta = e^{-\mu\beta}$

$h(1,\zeta)/2 = P(\mu,T)/(2P_1(\mu,T))$

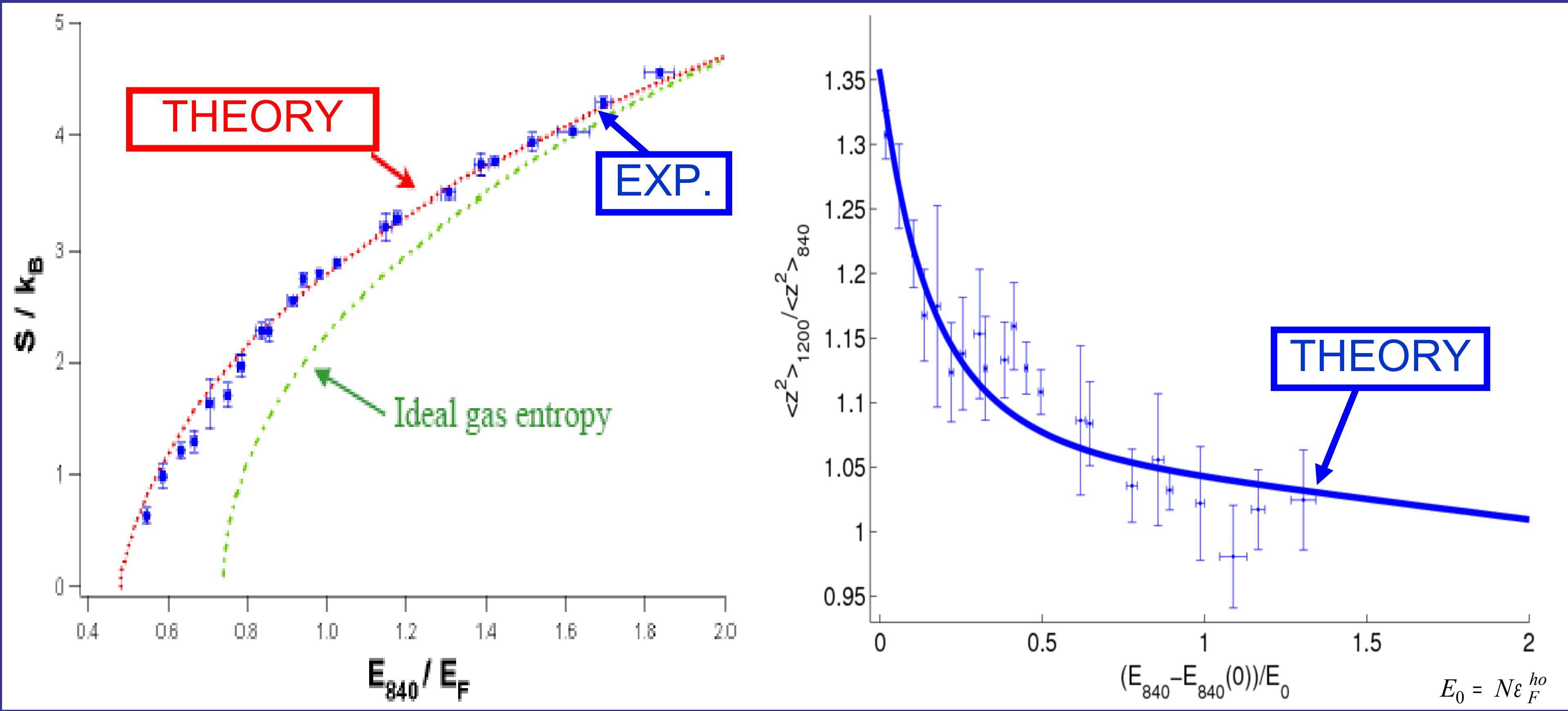
$P_1(\mu,T)$ - pressure of a single component of the noninteracting Fermi gas

S. Nascimbene et al. Nature 463, 1057 (2010)

From a talk given by C. Salomon, June 2nd, 2010, Saclay

Comparison with experiment

John Thomas' group at Duke University,
L.Luo, et al. Phys. Rev. Lett. 98, 080402, (2007)



$E_0 = N\varepsilon_F^{ho}$

Entropy as a function of energy (relative to the ground state) for the unitary Fermi gas in the harmonic trap.

Ratio of the mean square cloud size at B=1200G to its value at unitarity (B=840G) as a function of the energy. Experimental data are denoted by point with error bars.

$B = 1200G \; 1/k_F a \approx -0.75$

Theory: **Bulgac, Drut, and Magierski PRL 99, 120401 (2007)**

Superfluidity in ultra cold atomic gas

Eagles (1960), Leggett (1980), Nozieres and Schmitt-Rink (1985), Randeria *et al.* (1993),…

If $a<0$ at $T=0$ a Fermi system is a BCS superfluid

$$\Delta(T=0) = \alpha \frac{\hbar^2 k_F^2}{2m} e^{\left(\pi/2k_F a\right)}; \quad \frac{\Delta(T=0)}{T_C} \approx 1.7 \quad \text{if} \quad k_F|a| << 1; \quad \frac{\varepsilon_F}{\Delta} >> 1$$

If $|a|=\infty$ and $nr_0^3<<1$ a Fermi system is strongly coupled and its properties are universal (unitary regime). Carlson *et al.* PRL 91, 050401 (2003)

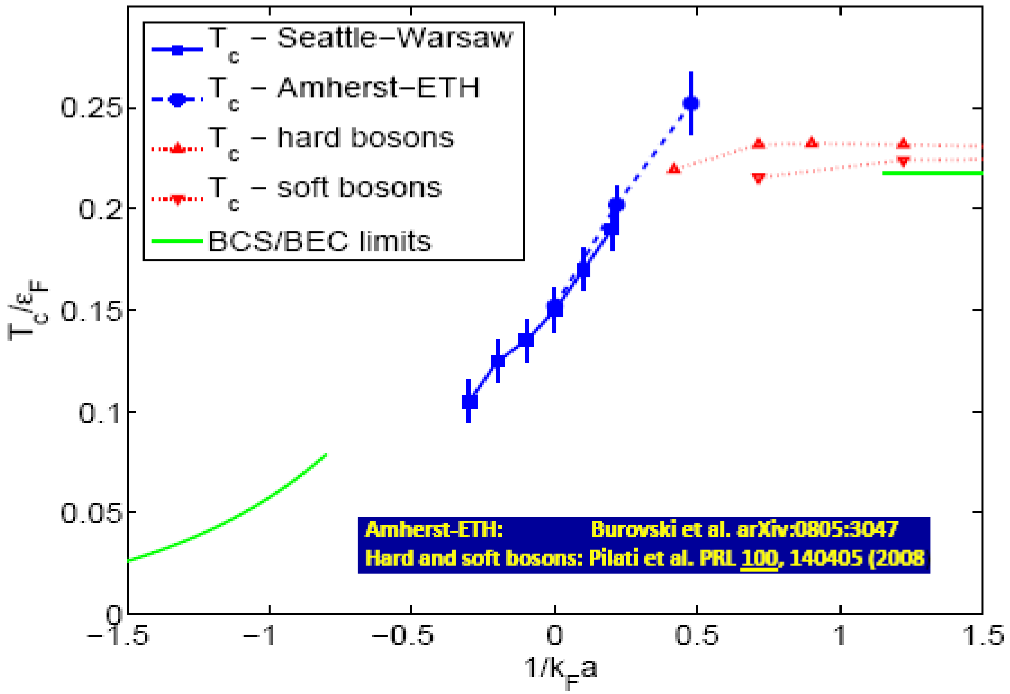
$$\Delta(T=0) = 0.50(1)\varepsilon_F; \quad \frac{\Delta(T=0)}{T_C} \approx 3.3 \ \text{(it is not a BCS superfluid!)}$$

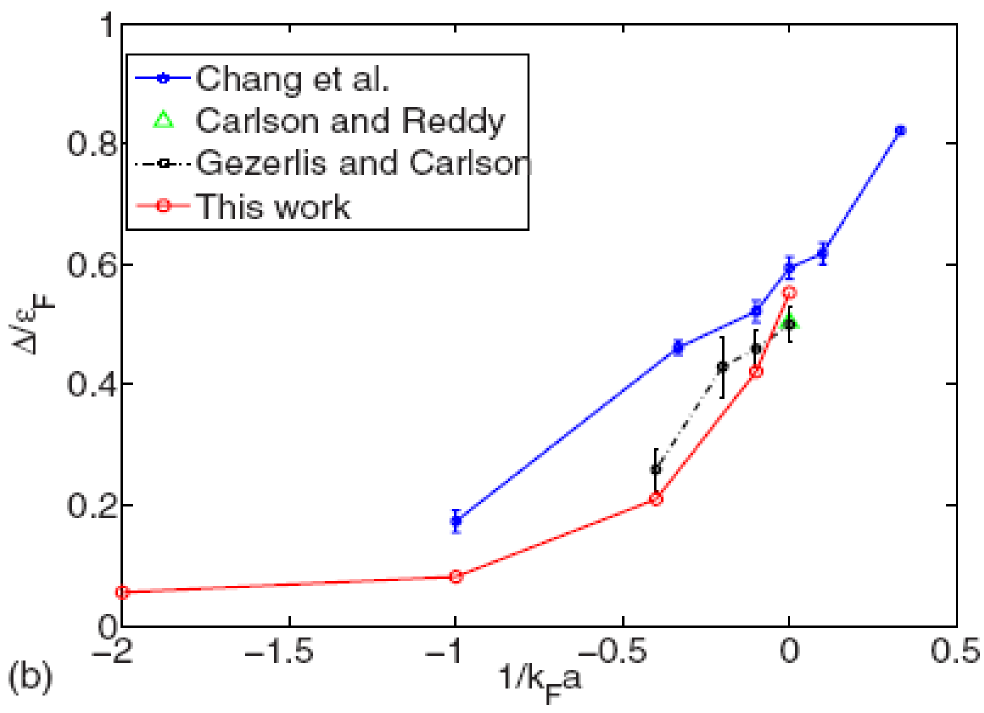
$$E_{normal} = 0.54 E_{FG}; \quad E_{\text{superfluid}} = 0.40 E_{FG}$$

If $a>0$ ($a>>r_0$) and $na^3<<1$ the system is a dilute BEC of tightly bound dimers

$$\varepsilon_b = -\frac{\hbar^2}{ma^2} \text{ - boson bounding energy}; \quad a_{bb} = 0.6\, a > 0 \text{ - effective boson-boson interaction}$$

$$T_C \approx 3.31 \frac{\hbar^2 n_b^{2/3}}{m}\left(1 + c(an_b^{1/3})\right) \text{ - Bose-Einstein condensation temp.}; \quad T^* \sim \frac{1}{a^2} \text{ - break up of Bose molecule}$$





Results in the vicinity of the unitary limit:
-Critical temperature
-Pairing gap at T=0

Note that
- at unitarity: $\Delta/\varepsilon_F \approx 0.5$
- for atomic nucleus: $\Delta/\varepsilon_F \approx 0.03$

BCS theory predicts:

$$\Delta(T=0)/T_C \approx 1.7$$

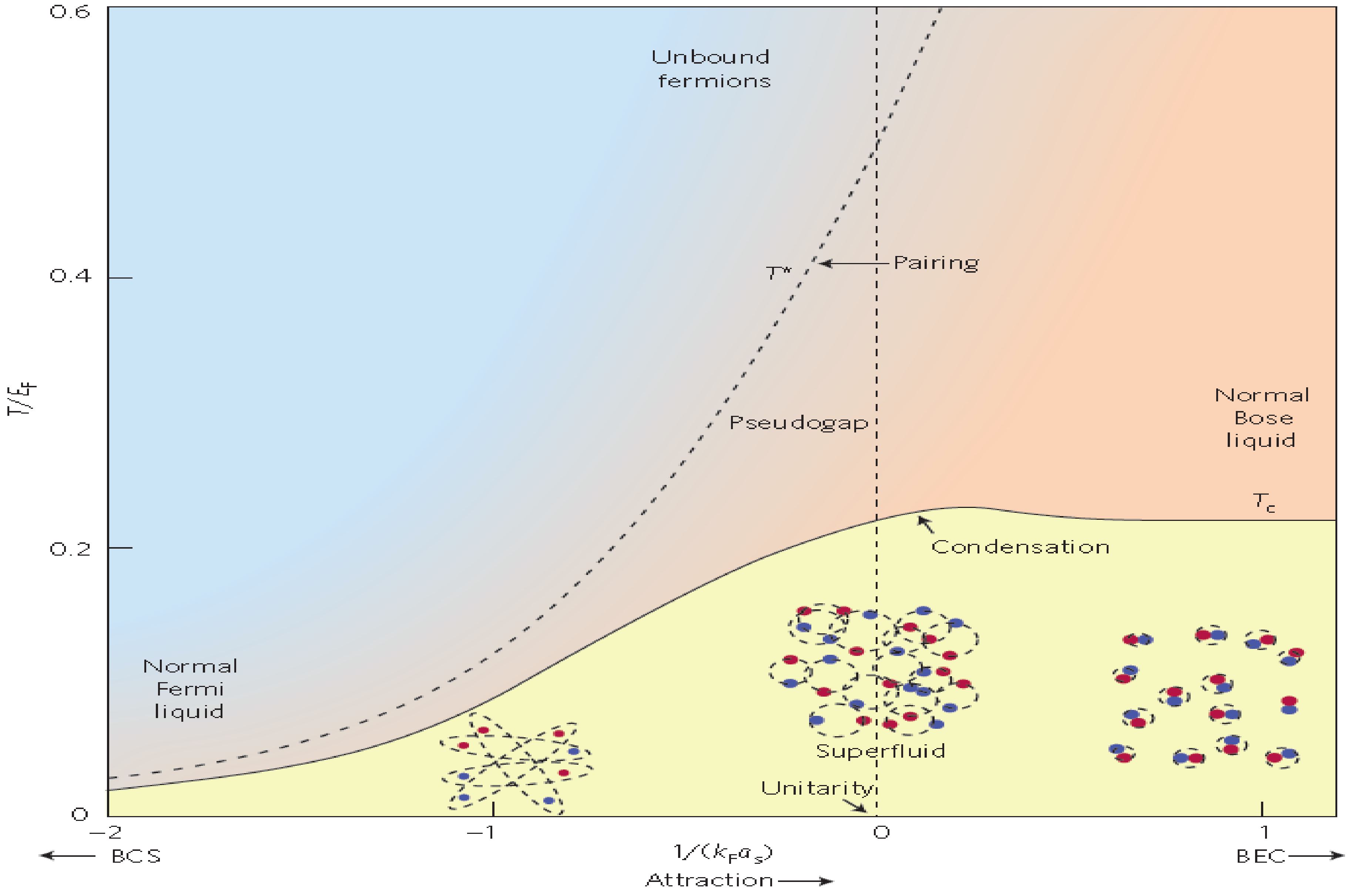
At unitarity:

$$\Delta(T=0)/T_C \approx 3.3$$

This is NOT a BCS superfluid!

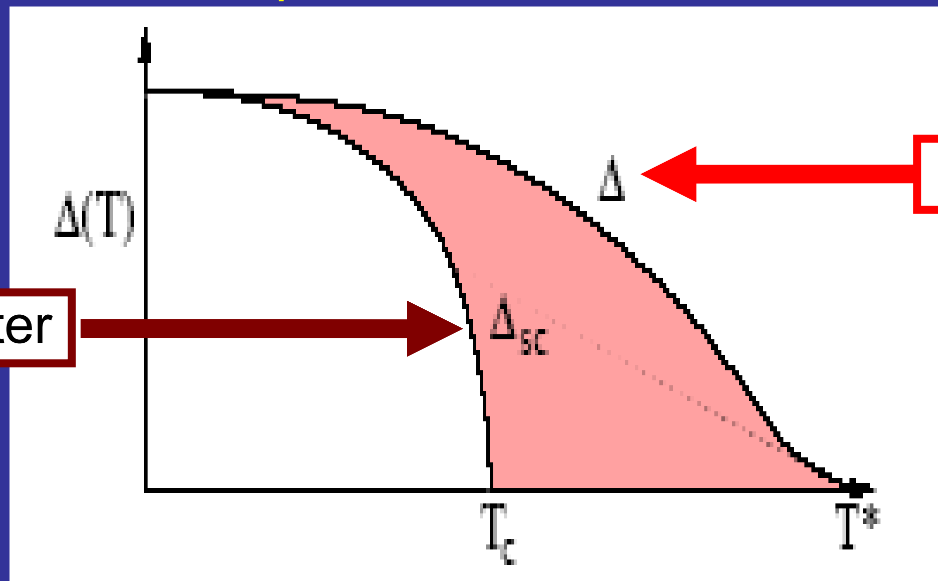
Bulgac, Drut, Magierski, PRA78, 023625(2008)

Nature of the superfluid-normal phase transition in the vicinity of the unitary regime



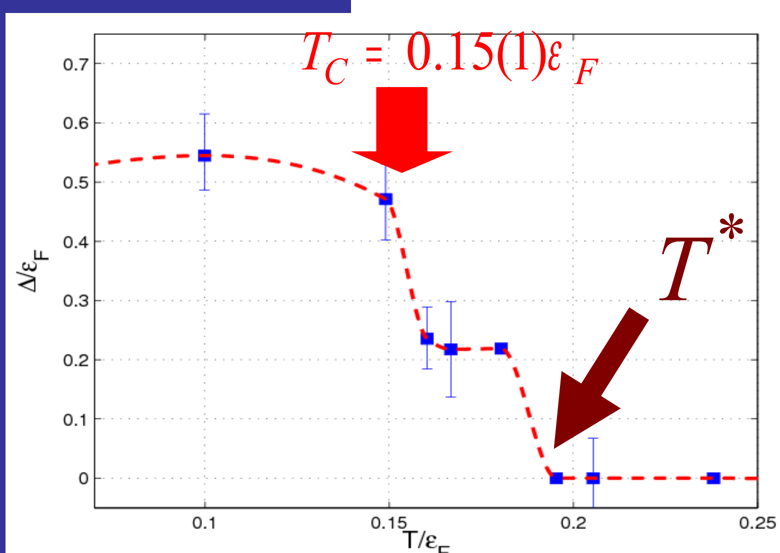
Pairing gap and pseudogap

Outside the BCS regime close to the unitary limit, but still before BEC, superconductivity/superfluidity emerge out of a very exotic, non-Fermi liquid normal state



pseudogap

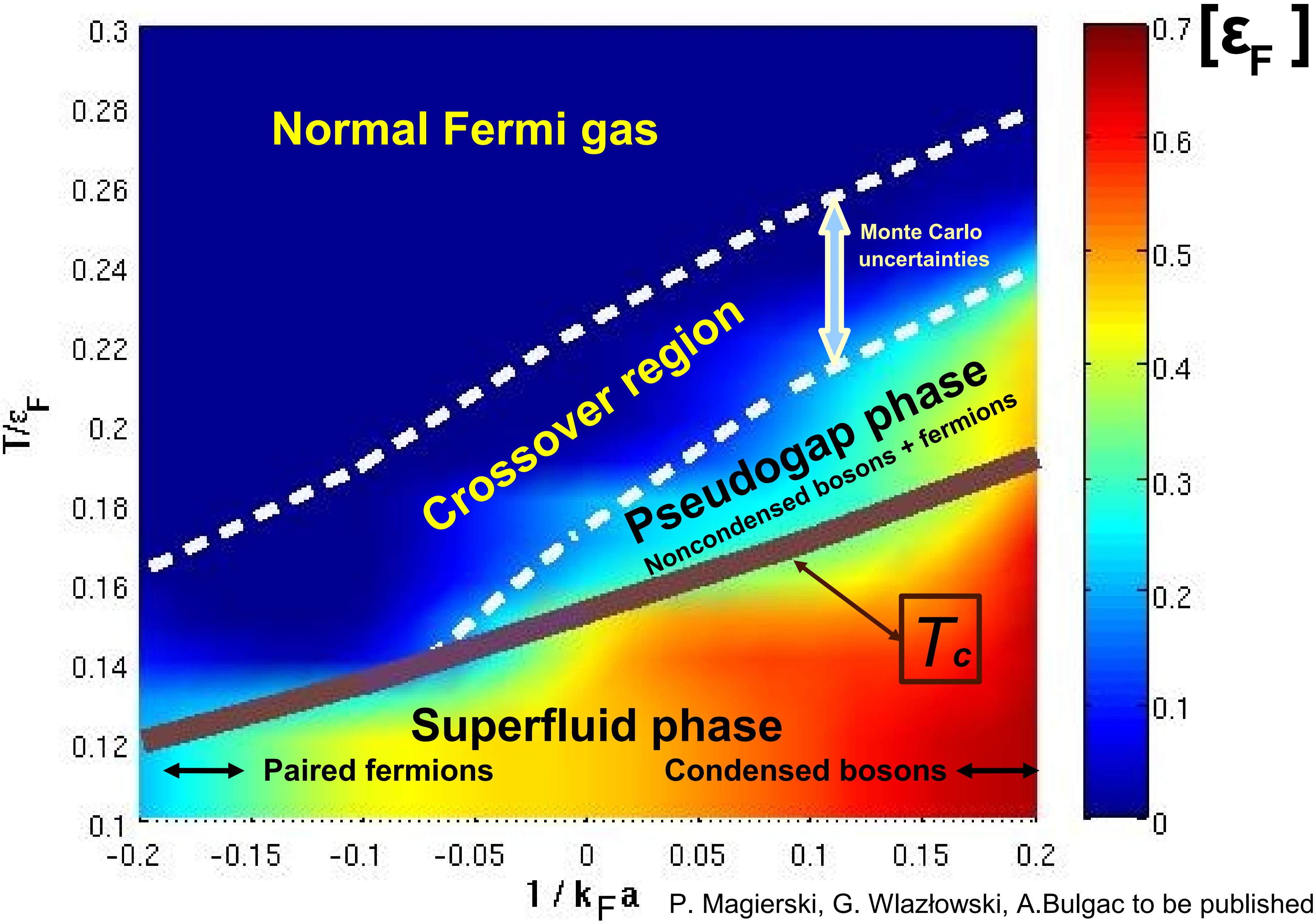
order parameter



Monte Carlo calculations at the unitary regime

The onset of superconductivity occurs in the presence of fermionic pairs!

Gap in the single particle fermionic spectrum from MC calcs.



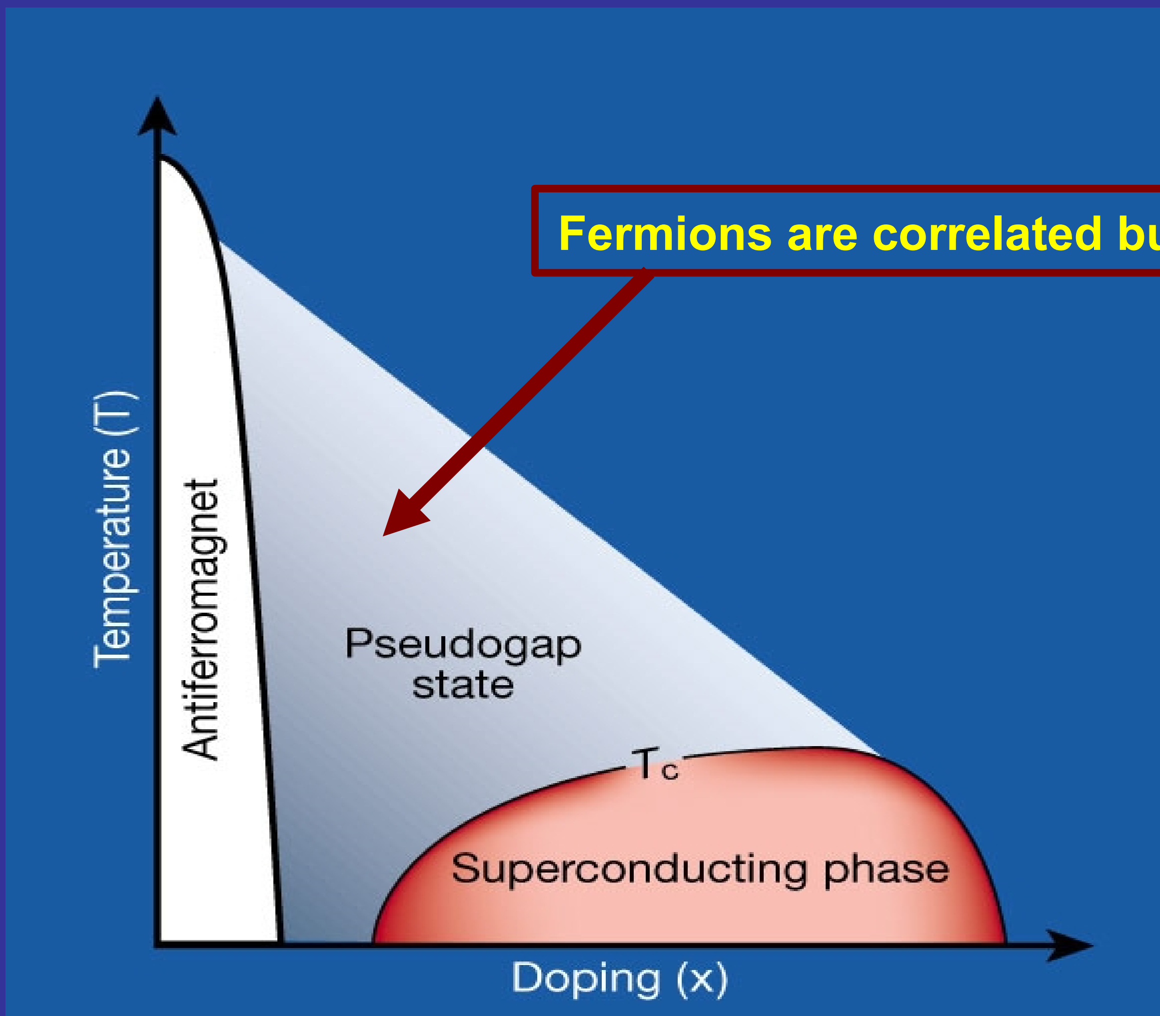
P. Magierski, G. Wlazłowski, A.Bulgac to be published

Unitary Fermi gas as a high-Tc superconductor

The situation in the ultracold atomic gases is somewhat similar to this encountered in high-temperature superconductors.

Despite of crucial differences between these two physical systems it seems as they share at least one common feature - the presence of the pseudogap phase.

Fermions are correlated but the long range is lost



Formalism for Time Dependent Phenomena

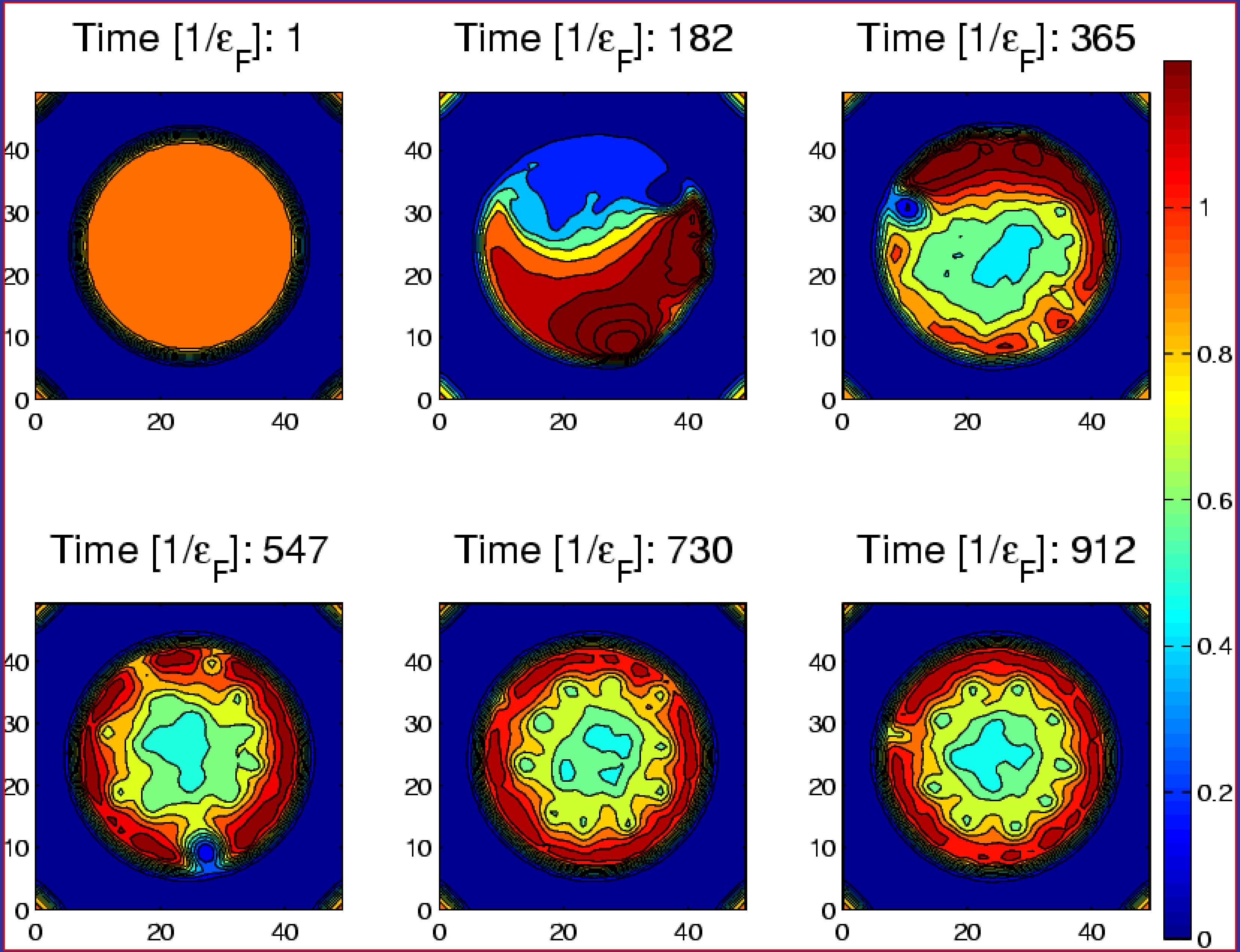
The time-dependent density functional theory is viewed in general as a reformulation of the exact quantum mechanical time evolution of a many-body system when only one-body properties are considered.

A.K. Rajagopal and J. Callaway, Phys. Rev. B 7, 1912 (1973)
V. Peuckert, J. Phys. C 11, 4945 (1978)
E. Runge and E.K.U. Gross, Phys. Rev. Lett. 52, 997 (1984)

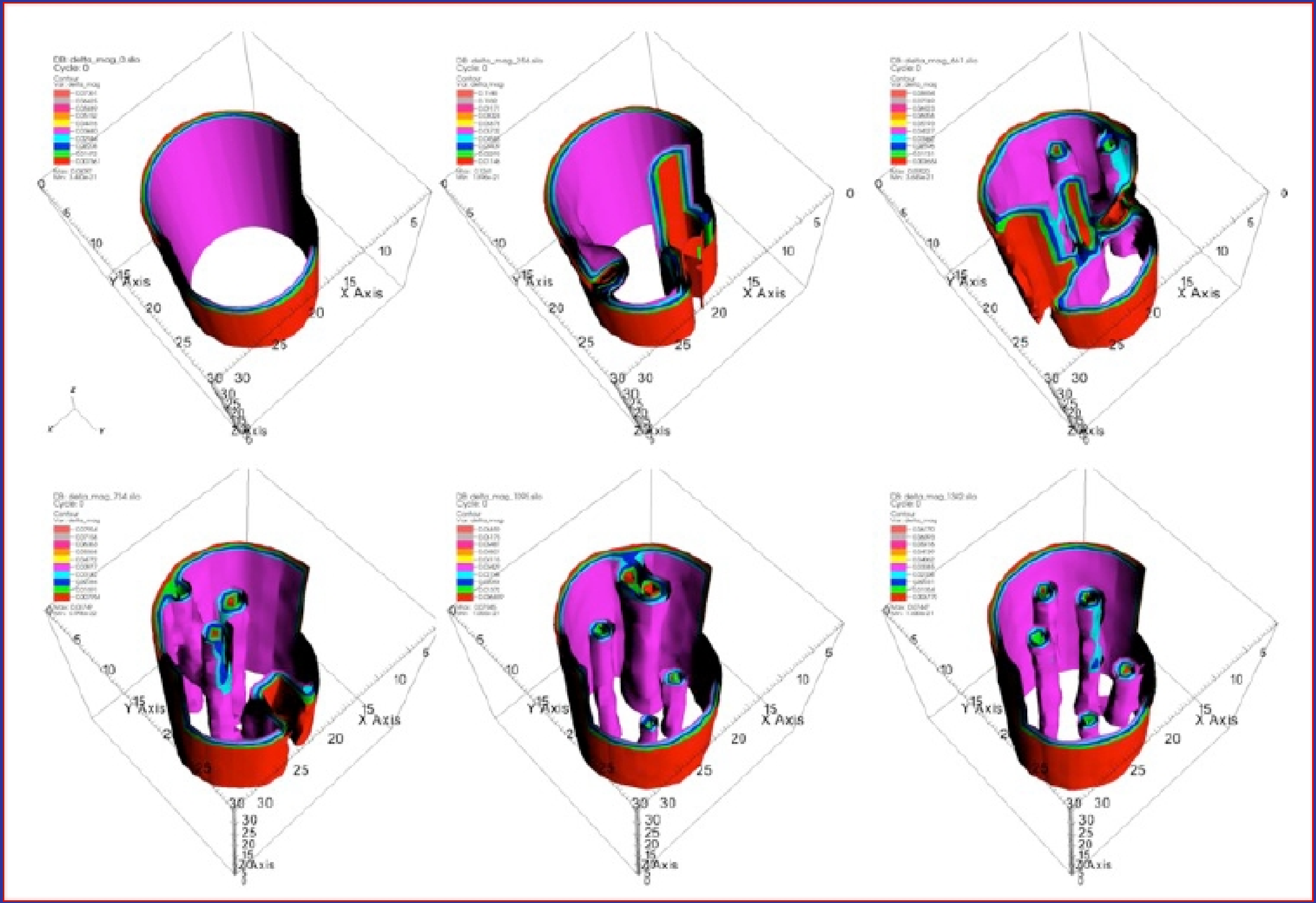
http://www.tddft.org

$$E(t) = \int d^3r \left[\varepsilon(n(\vec{r},t), \tau(\vec{r},t), \nu(\vec{r},t), \vec{j}(\vec{r},t)) + V_{ext}(\vec{r},t) n(\vec{r},t) + \dots \right]$$

$$\begin{cases} [h(\vec{r},t) + V_{ext}(\vec{r},t) - \mu] \mathrm{u}_i(\vec{r},t) + [\Delta(\vec{r},t) + \Delta_{ext}(\vec{r},t)] \mathrm{v}_i(\vec{r},t) = i\hbar \dfrac{\partial \mathrm{u}_i(\vec{r},t)}{\partial t} \\ [\Delta^*(\vec{r},t) + \Delta^*_{ext}(\vec{r},t)] \mathrm{u}_i(\vec{r},t) - [h(\vec{r},t) + V_{ext}(\vec{r},t) - \mu] \mathrm{v}_i(\vec{r},t) = i\hbar \dfrac{\partial \mathrm{v}_i(\vec{r},t)}{\partial t} \end{cases}$$



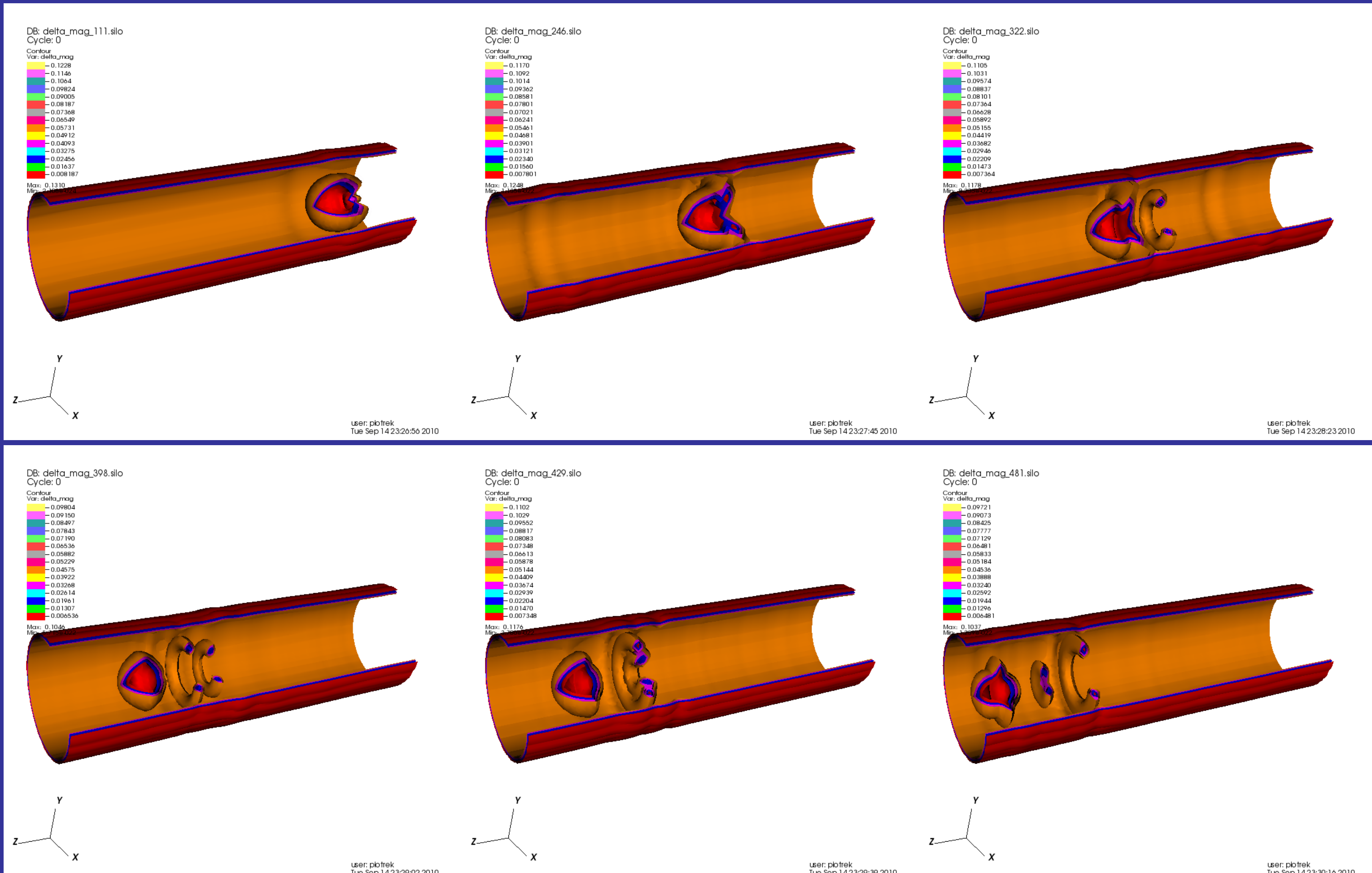
Density cut through a stirred unitary Fermi gas at various times.



Profile of the pairing gap of a stirred unitary Fermi gas at various times.

Exotic vortex topologies: dynamics of vortex rings

Heavy spherical object moving through the superfluid unitary Fermi gas



Applications to unitary gas and nuclei and how all this was implemented on JaguarPf (the largest supercomputer in the world)