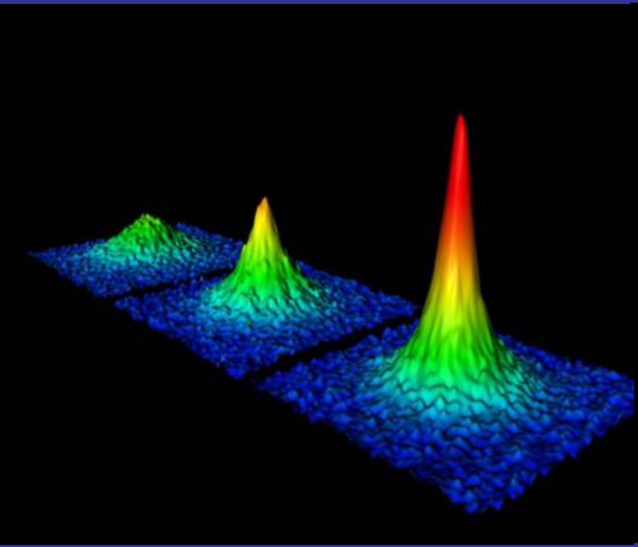
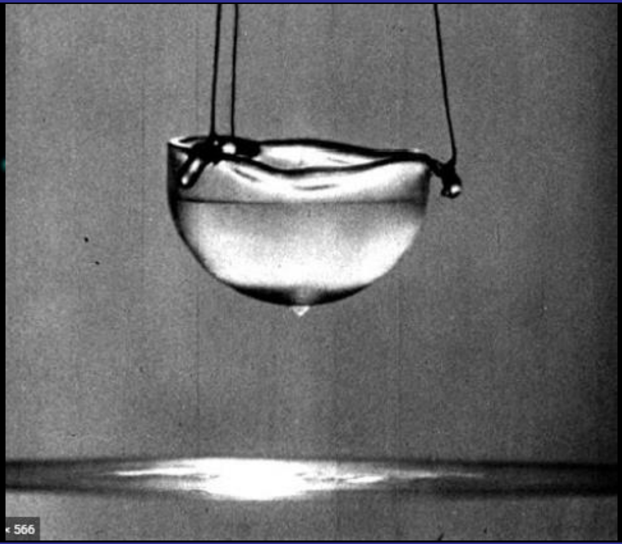
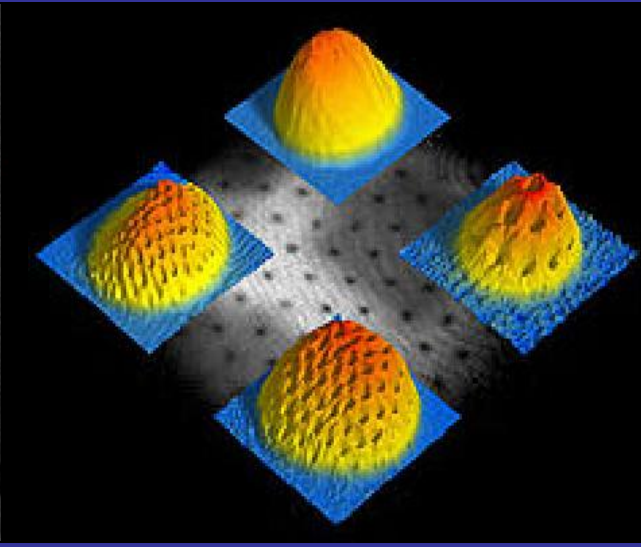
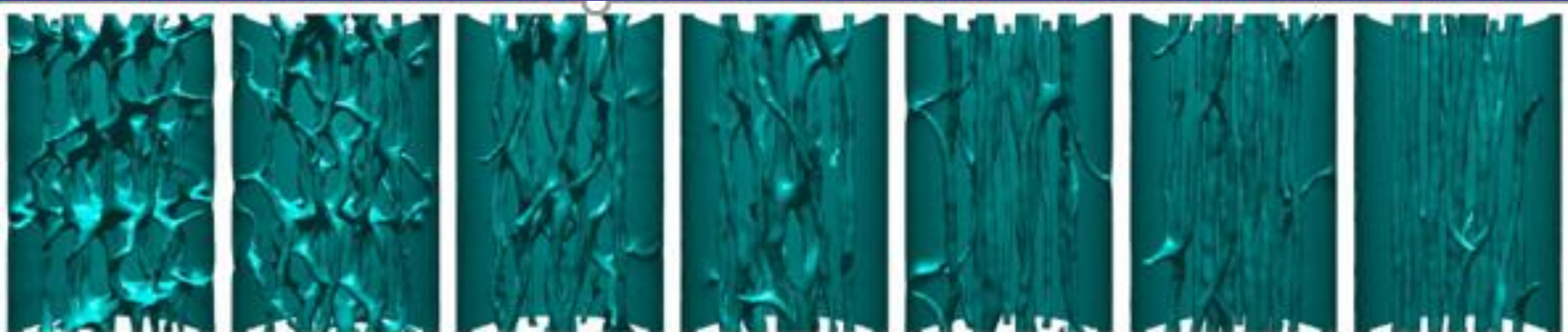
Exotic Structures in Superfluids



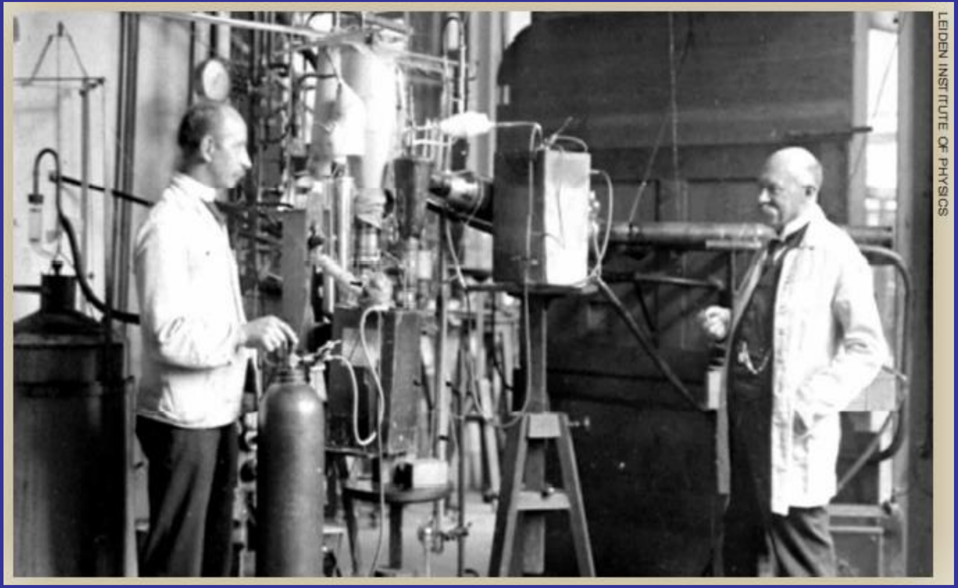




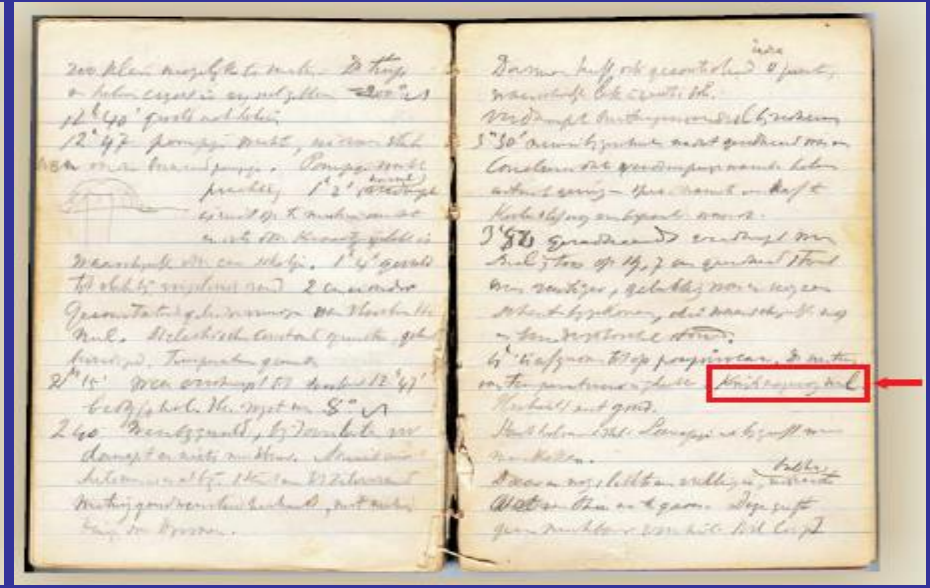
Piotr Magierski
(Warsaw University of Technology)



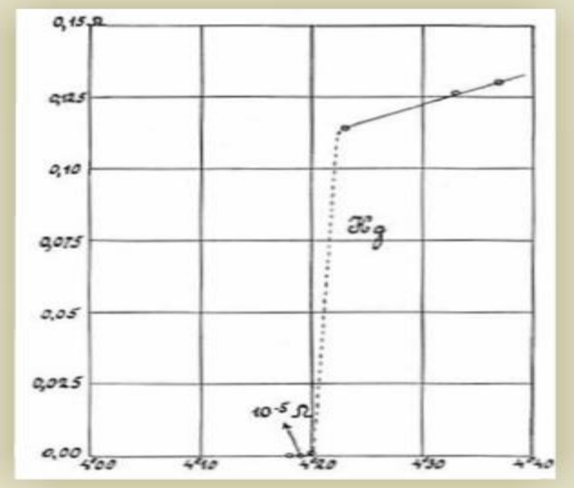
Low temperature physics at the beginning of XX century: Discovery of **superconductivity** and **superfluidity**



Heike Kamerlingh Onnes in his laboratory



Laboratory notebook, April 8, 1911
Underlined sentence:
„Mercury[`s resistivity] practically zero [at 3K]"
(from Boerhaave Museum)

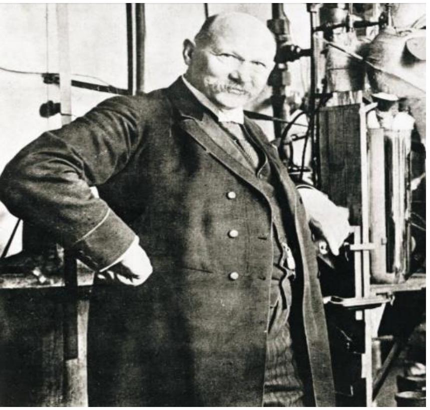


Original plot of resistance as a function of temperature

H. Kamerlingh Onnes, *Commun. Phys. Lab. Univ. Leiden. Suppl.* **29** (Nov. 1911).

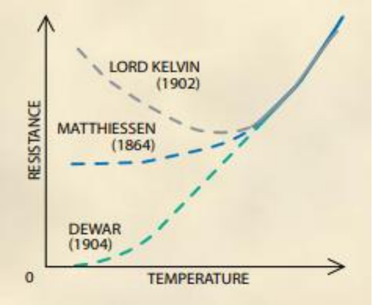
At the temperature of 4.25K resistance dropped suddenly from 0.1Ω to 10^{-6} Ω!

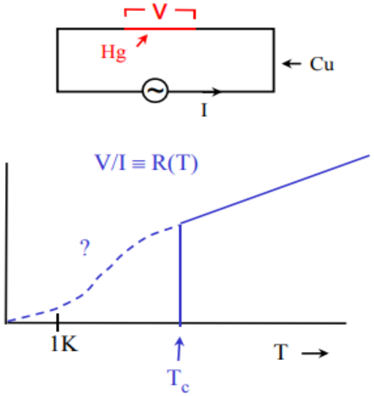
Important events in the history of superconductivity



Heike Kamerlingh Onnes (Leiden Institute of Physics)

Theoretical predictions before 1911





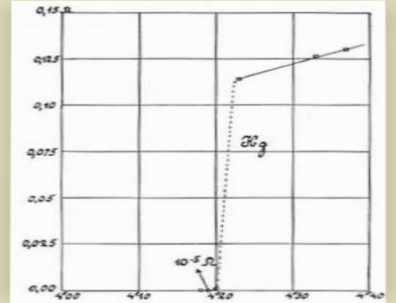
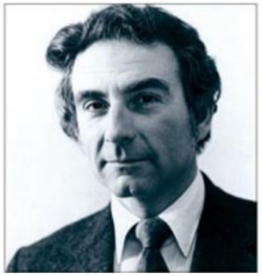


Figure 4. Historic plot of resistance (ohms) versus temperature (kelvin) for mercury from the 26 October 1911 experiment shows the superconducting transition at 4.20 K. Within 0.01 K, the resistance jumps from unmeasurably small (less than 10^{-6} Ω) to 0.1 Ω. (From ref. 9.)

J. Bardeen



L. Cooper

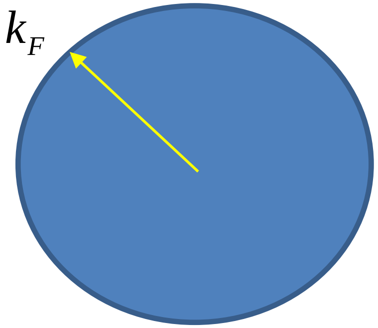
J.R. Schrieffer

- 1911 – Measurements of electric resistance of mercury as a function of temperature. Sudden drop of resistance at T=4.2K. The effect was dubbed as **superconductivity**.
- 1933 – Meissner-Ochsenfeld effect: Expulsion of magnetic flux from superconductor.
- 1935 – London phenomenological theory.
- 1950 – discovery of isotopic effect.
- 1950 - Ginzburg-Landau (GL) theory. (introduced certain complex function playing the role of the order parameter for superconductors)
- 1957 – **microscopic theory (BCS**- Bardeen, Cooper, Schrieffer)
- 1959 – L. Gorkov derives GL equations from BCS theory
- 1962 – Josephson effect (Josephson junction)
- 1986 – Discovery of high-Tc superconductivity (still mysterious)

It took about 50 years to formulate microscopic theory!

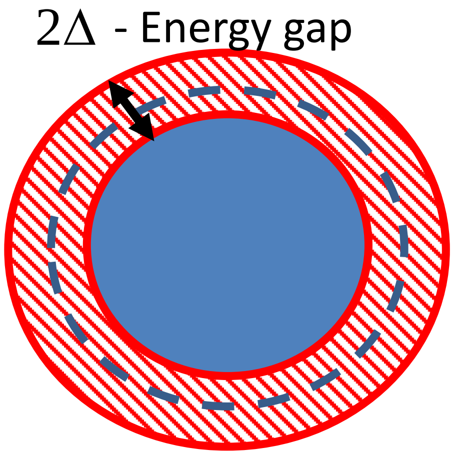
Cooper pairs

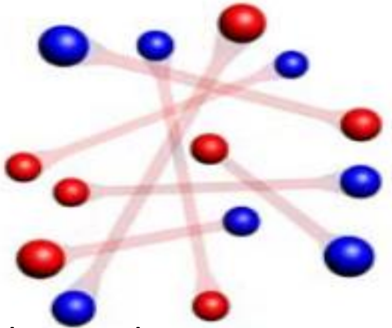
$$\varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$



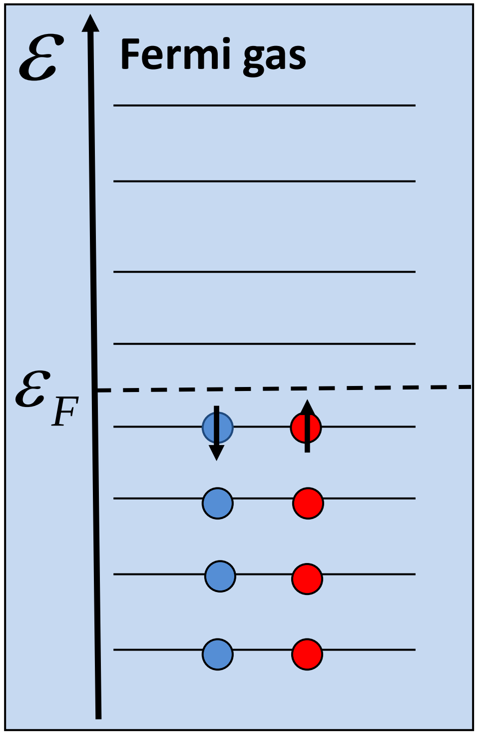
Fermi sphere

Attractive interaction





Correlations between pairs (Cooper)
of particles of opposite spins.



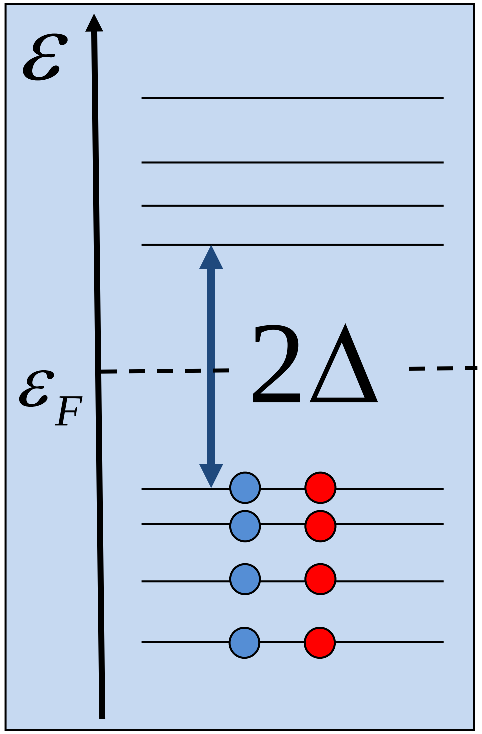
Energy of elementary excitation
(quasiparticle):

$$\boxed{E_{qp} = \sqrt{(\varepsilon - \varepsilon_F)^2 + |\Delta|^2} > |\Delta|}$$

BCS theory:

$$|BCS\rangle = \prod_k \left(u_k + v_k a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow}\right)|vacuum\rangle$$

$$\Delta = f(u_k, v_k)$$



Discovery of **superfluidity** occured somewhat later but was actually triggered by the discovery of superconductivity!

Namely, liquid helium was used in cryostat to cool down mercury sample:

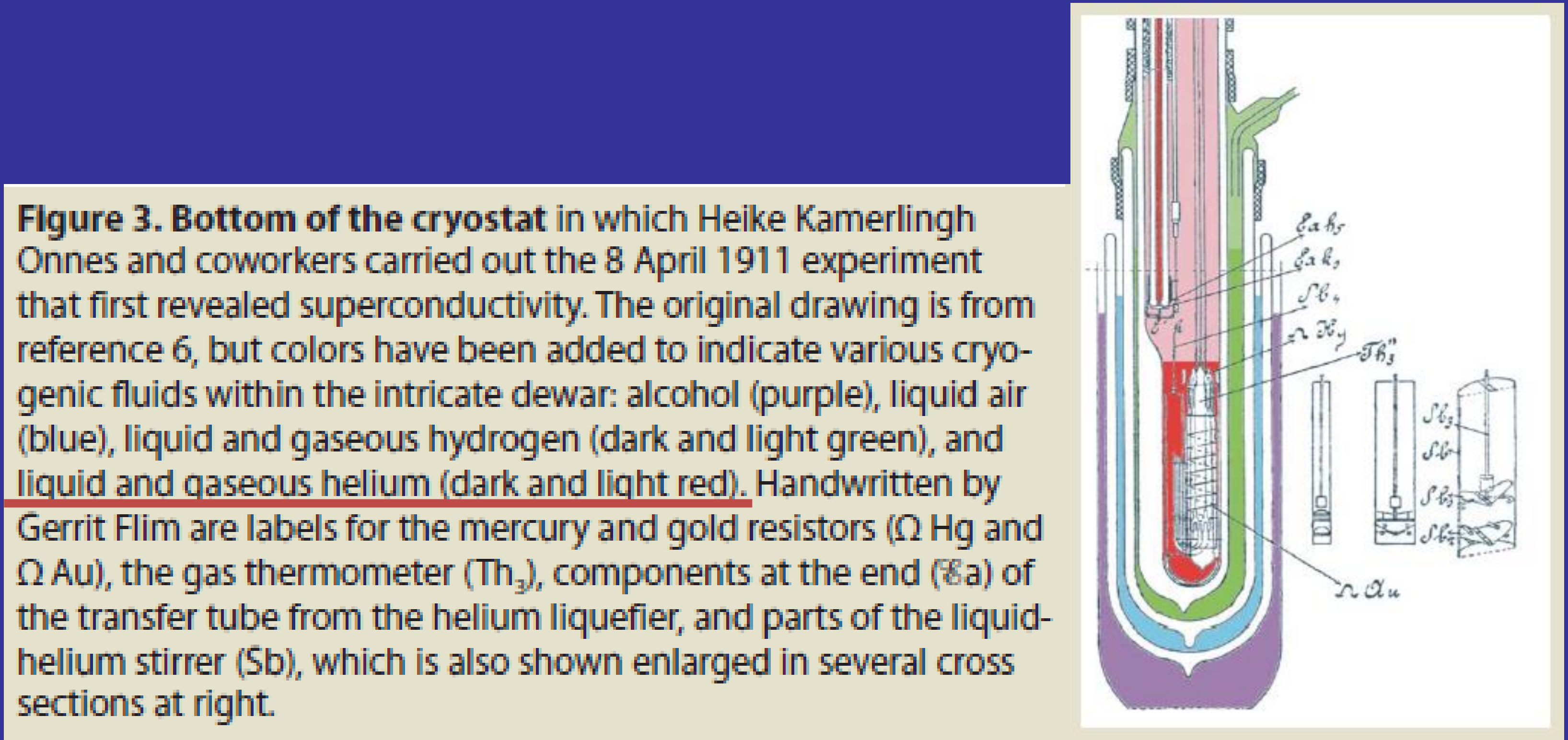


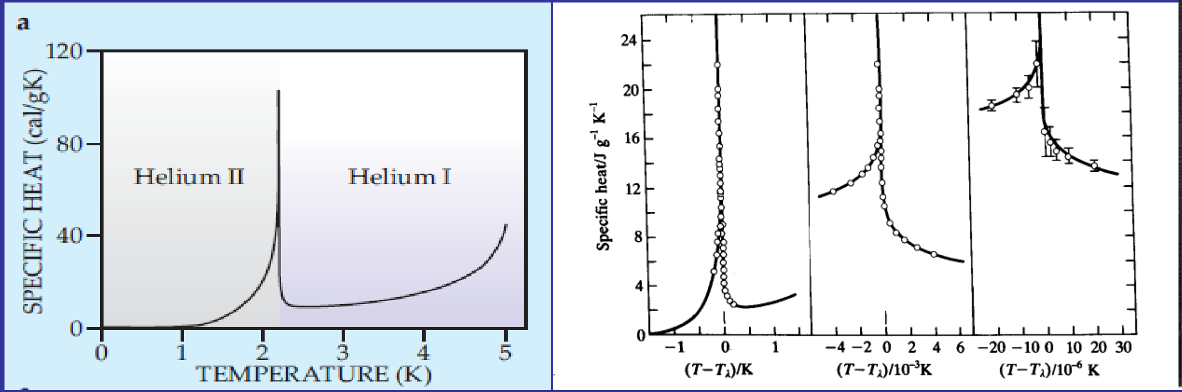
Figure 3. Bottom of the cryostat in which Heike Kamerlingh Onnes and coworkers carried out the 8 April 1911 experiment that first revealed superconductivity. The original drawing is from reference 6, but colors have been added to indicate various cryogenic fluids within the intricate dewar: alcohol (purple), liquid air (blue), liquid and gaseous hydrogen (dark and light green), and liquid and gaseous helium (dark and light red). Handwritten by Gerrit Flim are labels for the mercury and gold resistors (Ω Hg and Ω Au), the gas thermometer (Th$_3$), components at the end (Ɛa) of the transfer tube from the helium liquefier, and parts of the liquid-helium stirrer (Sb), which is also shown enlarged in several cross sections at right.

From Kamerlingh Onnes's laboratory notebook:

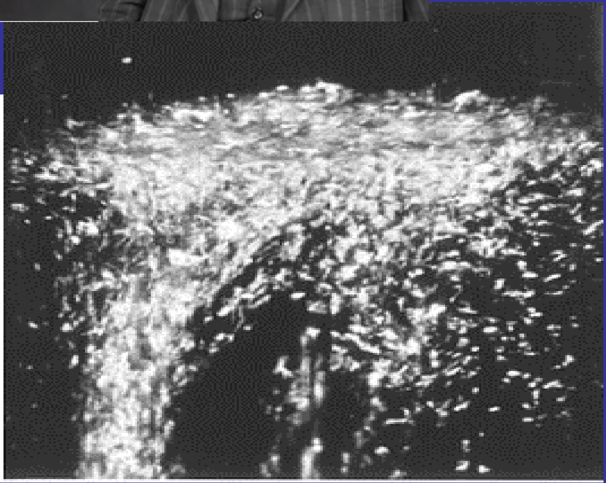
Dorsman [who had controlled and measured the temperatures] really had to hurry to make the observations.[...] Just before the lowest temperature [about 1.8 K] was reached, the boiling [of liquid helium] suddenly stopped and was replaced by evaporation in which the liquid visibly shrank. So, a remarkably strong evaporation at the surface."

Term „**helium II**" was used in papers:
M. Wolfke and W.H. Keesom,
Proc. Amsterdam 31, 81 (1927).
W.H. Keesom and M. Wolfke,
Leiden. Comm. 190b, (1927).

Measurements of specific heat of helium



lambda point and lambda temperature



Boiling of helium when passing through lambda point.

Two articles in Nature

Piotr Kapitza, „*Viscosity of liquid helium below the lambda point*", Nature 141, 74 (1938).

John F. Allen, Don Misener, „*Flow of liquid Helium-II*" Nature 141, 75 (1938).

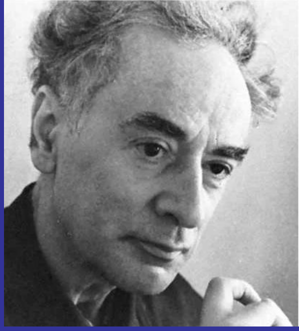
Kapitza: *From the measurements we can conclude that the viscosity of helium II is at least 1500 times smaller than that of helium I at normal pressure.*

*Kapitza introduced the term **superfluidity***

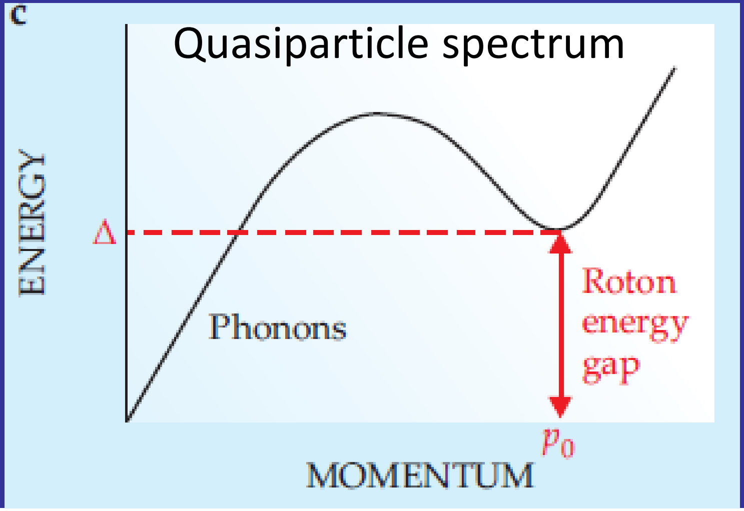
First theoretical approach: Two fluid model (1938)

L. Tisza

We treat the system as a mixture of two fluids:
superfluid and **normal** component.
Superfluid component has vanishing viscosity and entropy and disappears at lambda temperature. Superfluid component is formed by Bose-Einstein condensate (Tisza).
Normal component consists of excitations (quasiparticles) which fulfill certain dispersion relations (Landau). Normal component disappears at T=0.



L. Landau



Microscopic theory (1947): weakly interacting Bose gas

- N.Bogoliubov has shown that **weakly interacting Bose-Einstein condensate** possess excitation spectrum with **linear dispersion relation (Landau's phonons)** (Bogoliubov N.N., J. Phys. USSR 11, 23 (1947))
- **Rotons** are peculiar to Helium-II which is a strongly interacting system (ie. cannot be reproduced by Bogoliubov's theory)

Critical temperatures for superconductivity and superfluidity

- ✓ **Ultracold atomic gases:** **$T_c \approx 10^{-12} - 10^{-9}$ eV**
- ✓ **Liquid ^{3}He:** **$T_c \approx 10^{-7}$ eV**
- ✓ **Metals and alloys:** **$T_c \approx 10^{-3} - 10^{-2}$ eV**
- ✓ **Atomic nuclei and neutron stars:** **$T_c \approx 10^{5} - 10^{6}$ eV**
- **Color superconductivity (quarks) :** **$T_c \approx 10^{7} - 10^{8}$ eV**

(1 eV $\approx 10^4$ K)

Superfluidity and superconductivity

- **Requirement**: Bose-Einstein (BEC) condensation of interacting *bosons*.
- **Result**: linear dispersion relation
- **Consequence**: no viscosity (below certain flow velocity)
- **Theoretical description**:
  „Condensate wave function”

$$\Psi(\vec{r}) = \left|\Psi\left(\vec{r}\right)\right| e^{i\phi(\vec{r})}$$

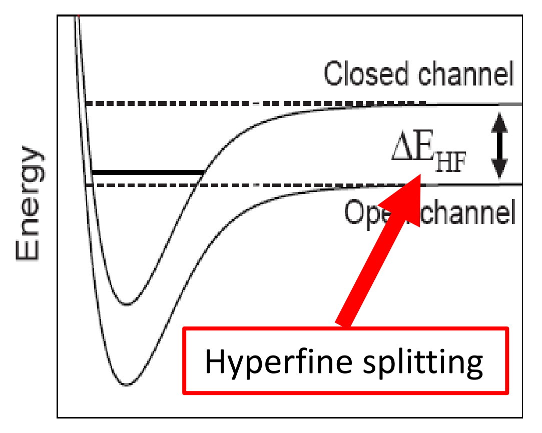
- **Requirement**: arbitrary weak attraction between *fermions.*
- **Result**: formation of Cooper pairs
- **Consequence**: no resistance
- **Theoretical description**:
  Field of Cooper pairs

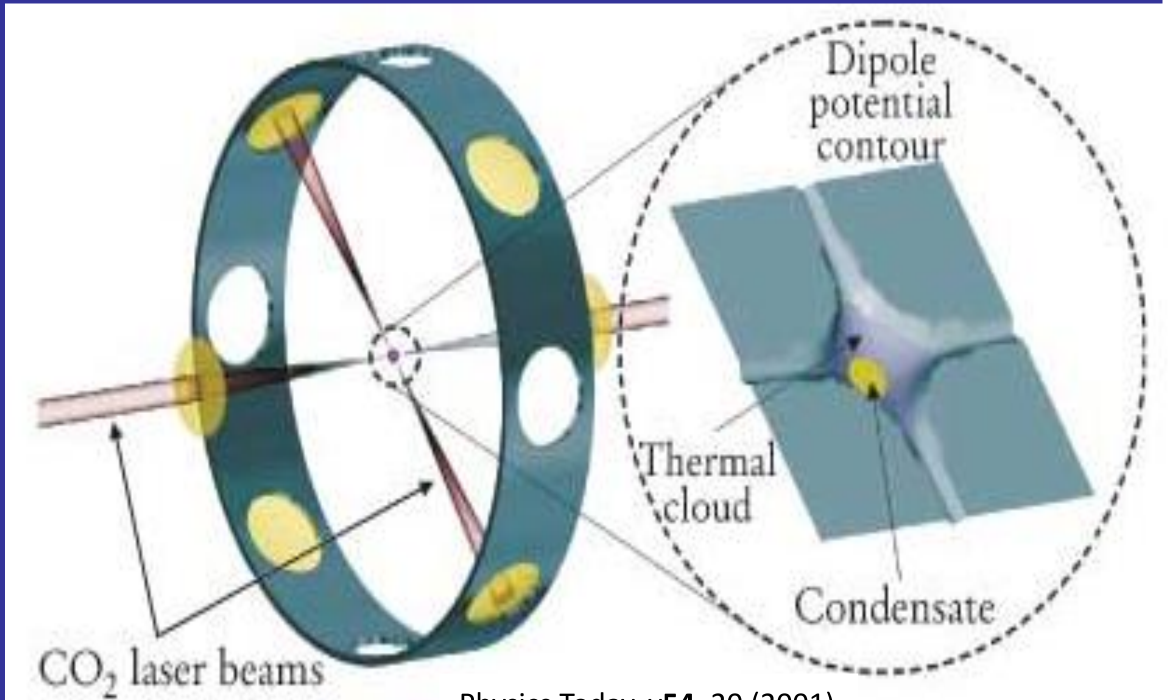
$$\Delta(\vec{r}) = \left|\Delta\left(\vec{r}\right)\right| e^{i\phi(\vec{r})}$$

Both phenomena are actually like two sides of the same coin!

In dilute, ultracold atomic systems experimenters can control nowadays almost anything:

- The number of atoms in the trap: typically about 10^5-10^6 atoms divided 50-50 among the lowest two hyperfine states.
- The density of atoms
- Mixtures of various atoms
- The temperature of the atomic cloud
- **The strength of this interaction is fully tunable!**



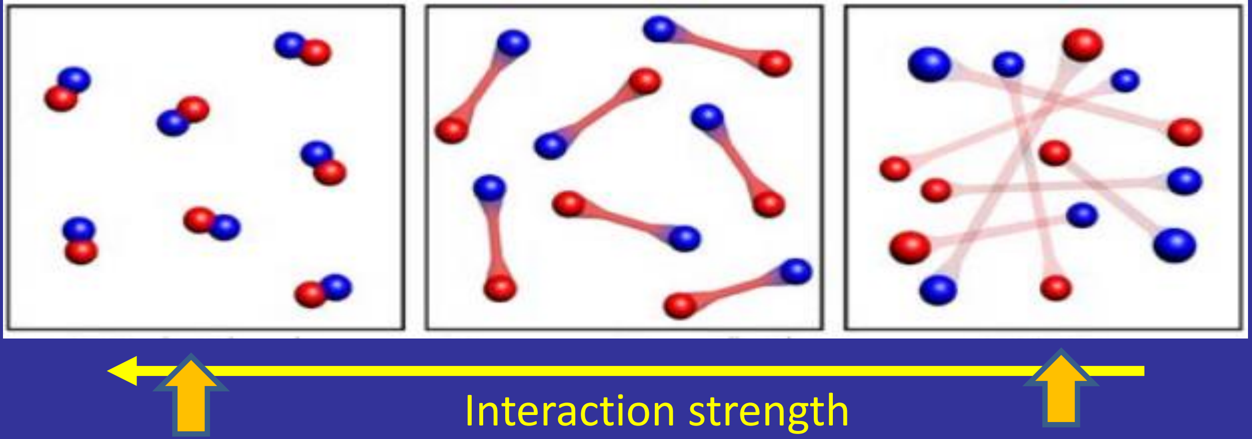


Physics Today, v**54**, 20 (2001)

Who does experiments?

- **Jin's group at Boulder**
- **Grimm's group in Innsbruck**
- **Thomas' group at Duke**
- **Ketterle's group at MIT**
- **Salomon's group in Paris**
- **Hulet's group at Rice**

BCS – BEC crossover



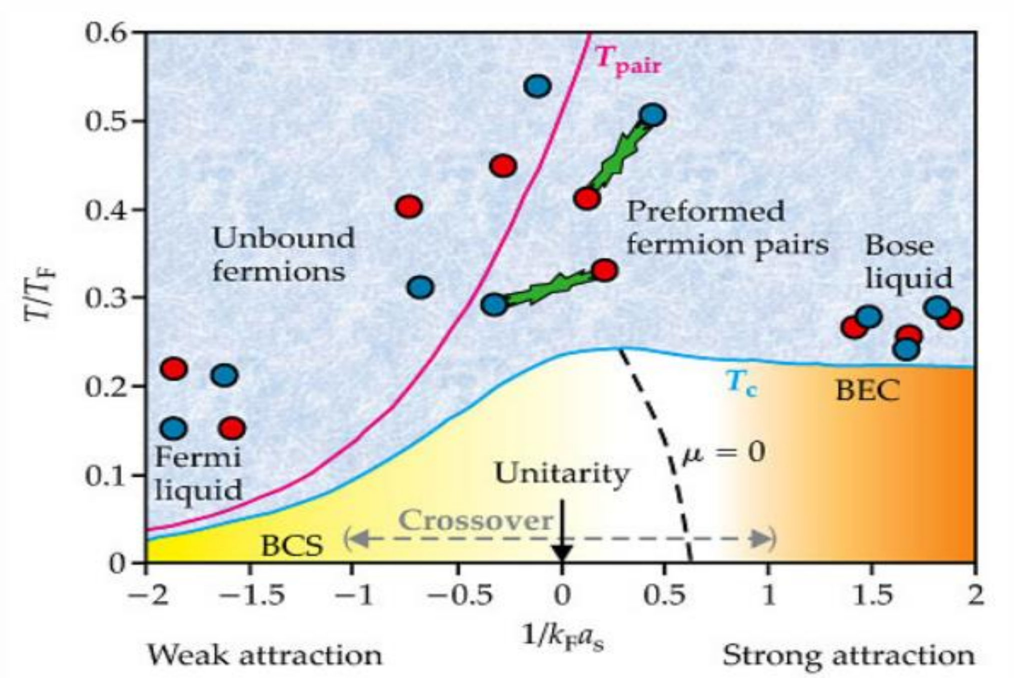
Interaction strength

Bose-Einstein condensate (BEC)
(bound fermion pairs = bosons)

Superconductor
(Cooper pairs)

No phase transition between BCS regimes and BEC regime!

Eagles (1969), Leggett (1980)



From Sa de Melo, Physics Today (2008)

Note that the existence of a condensate implies that the flow of superfluid component is **irrotational**:

$$\nabla \times \vec{v} = 0$$

Since:

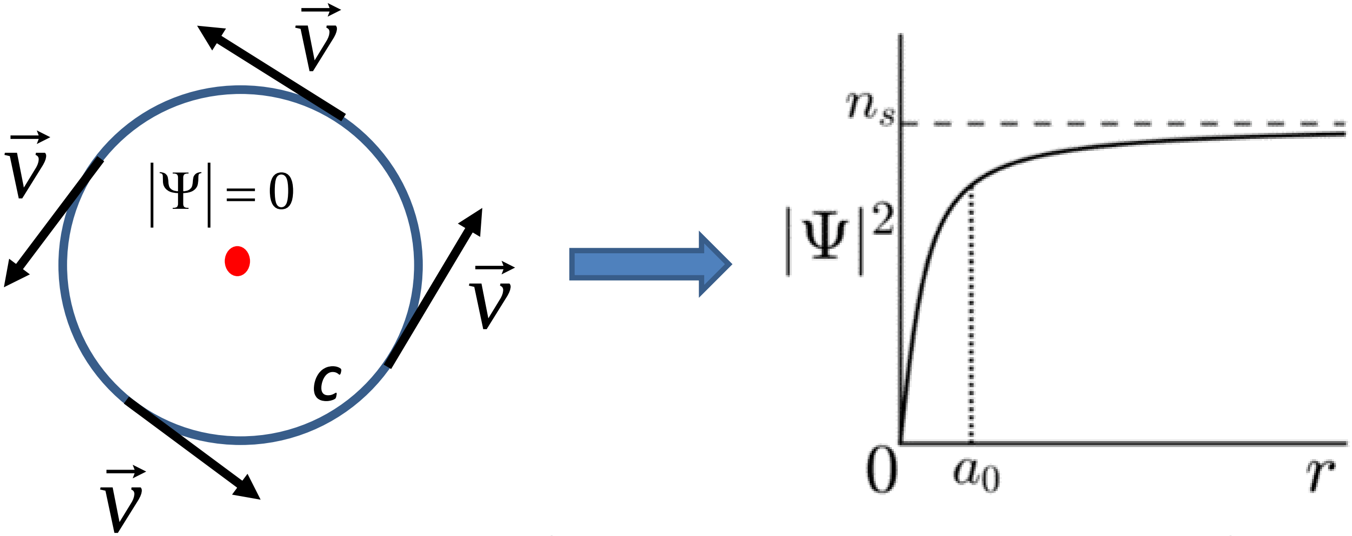
$$\Psi(\vec{r},t) = \left|\Psi(\vec{r},t)\right| \exp[i\varphi(\vec{r},t)]$$

$$\vec{v} \propto \nabla \varphi$$

$$\oint_C \vec{v} \cdot d\vec{l} = \frac{2\pi\hbar}{m} n \quad , \quad n = 0, \pm 1, \pm 2, \ldots$$

It also implies a certain quantization condition for circulation of a superfluid component.

<u>Quantum vortex – topological excitation</u>

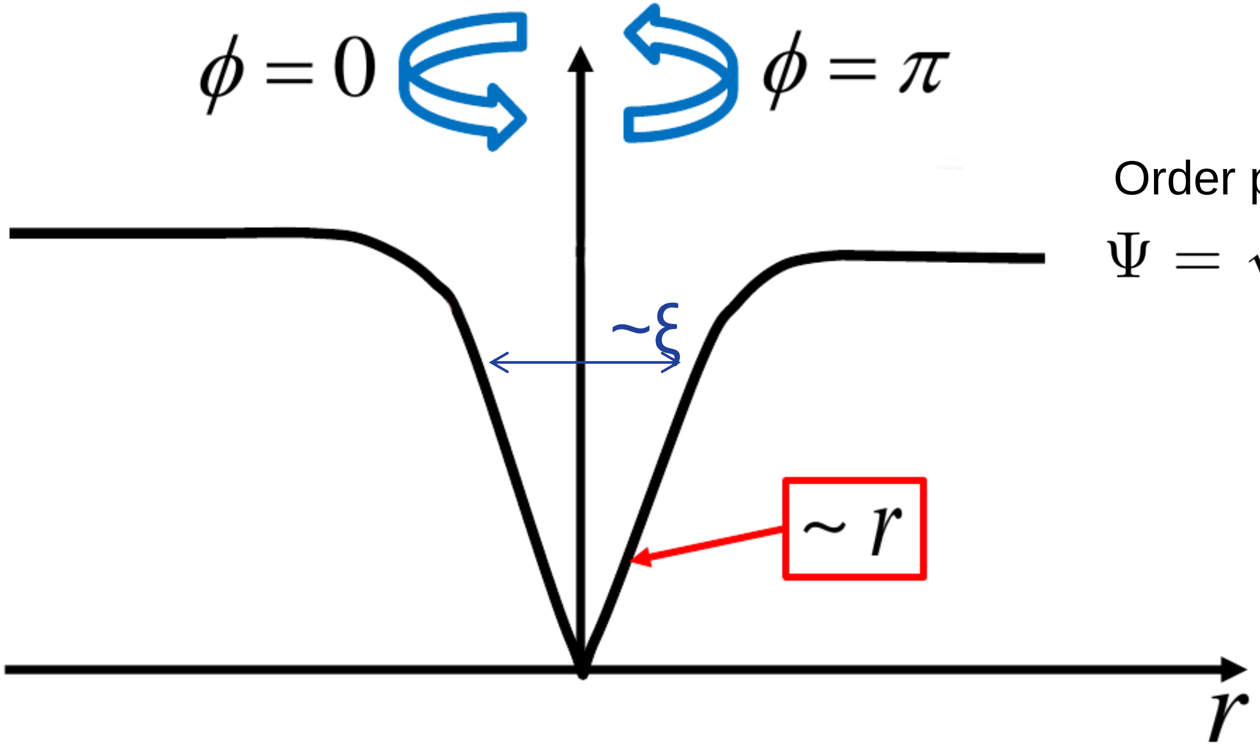


Below **critical velocity** the generation of quantum vortices is the only way for energy dissipation and generation of angular momentum in superfluid flow.

Anatomy of the vortex core

BOSONS:

Vortex structure: Bose gas



Order parameter:

$$\Psi = \sqrt{\rho(r)}e^{i\phi}$$

$$\boldsymbol{v}_s = \frac{\hbar}{M}\boldsymbol{\nabla}\phi$$

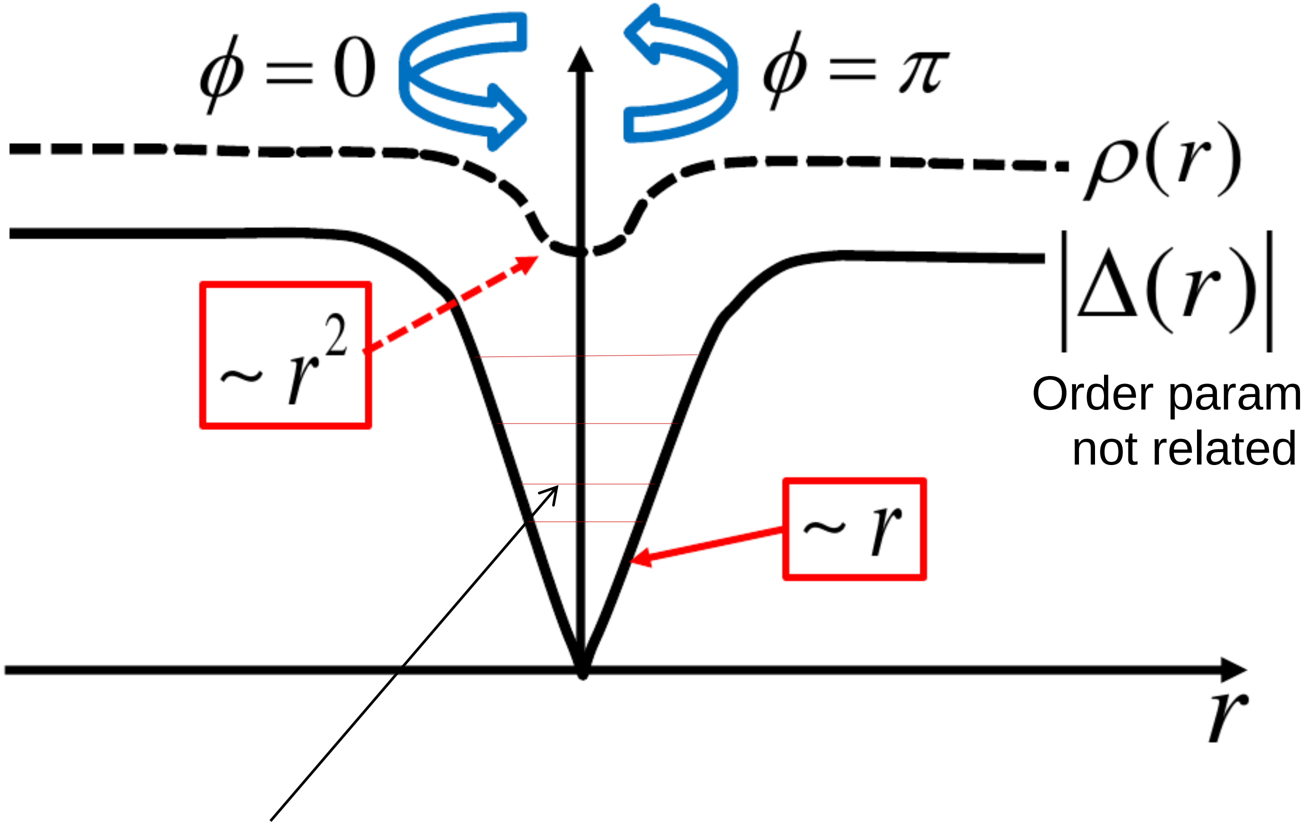
$$\kappa = \oint d\boldsymbol{l}\cdot\boldsymbol{v}_s = \frac{h}{M}$$

At T=0 the core is empty

FERMIONS: $\Delta(\vec{r},t)=\left|\Delta(\vec{r},t)\right|e^{i\phi(\vec{r},t)}$

The same type of topological excitation can be realized!

Vortex structure: Fermi gas.



Order parameter:
not related directly to density

Andreev states affect the density distribution inside the core.
The core is not empty!

Quantum vortices in Helium-II

Above lambda point Below lambda point

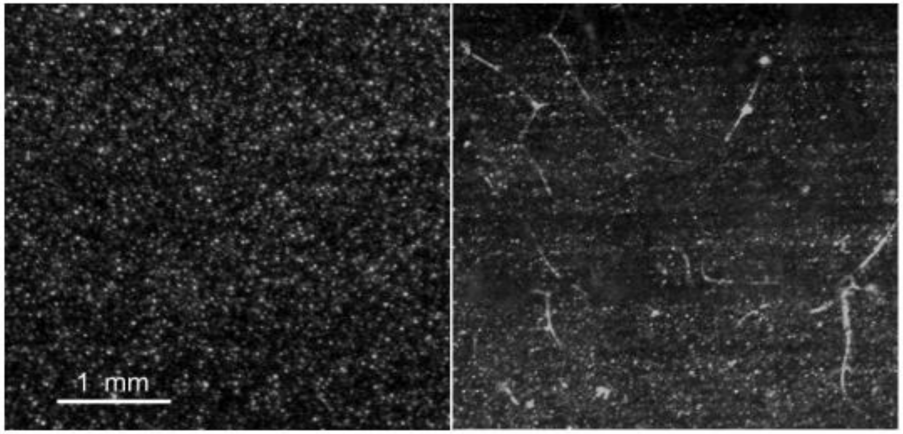
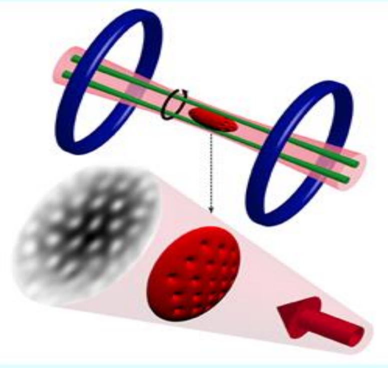
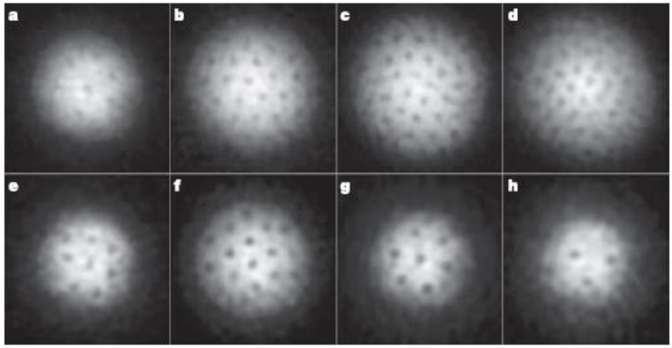


Figure 1: Images above and below the transition. For temperatures slightly above the lambda transition (left), hydrogen particles are randomly dispersed and make it possible to track the flow of the liquid helium. Below the transition (right), some fraction of the particles become trapped on lines in the flow. The particles are illuminated with a laser sheet; these are side views. Our data show that the lines thus observed are quantized superfluid vortices.

Quantum vortices in ultracold atomic gas





bosons

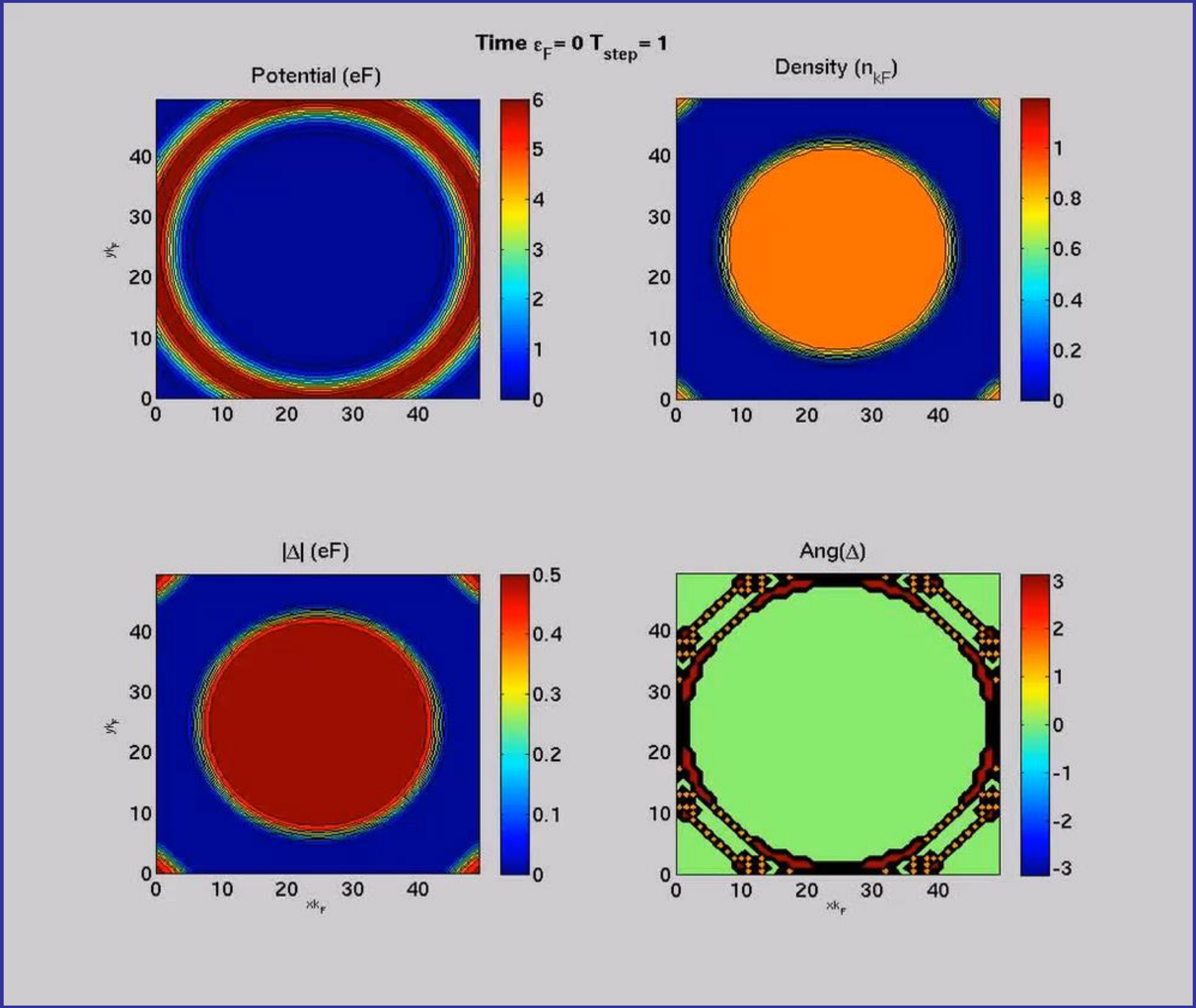
fermions

Experiment with Li-6 atoms

Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b**–**h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**), 812 G (**d**), 833 G (**e**), 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of view of each image is 880 μm × 880 μm.

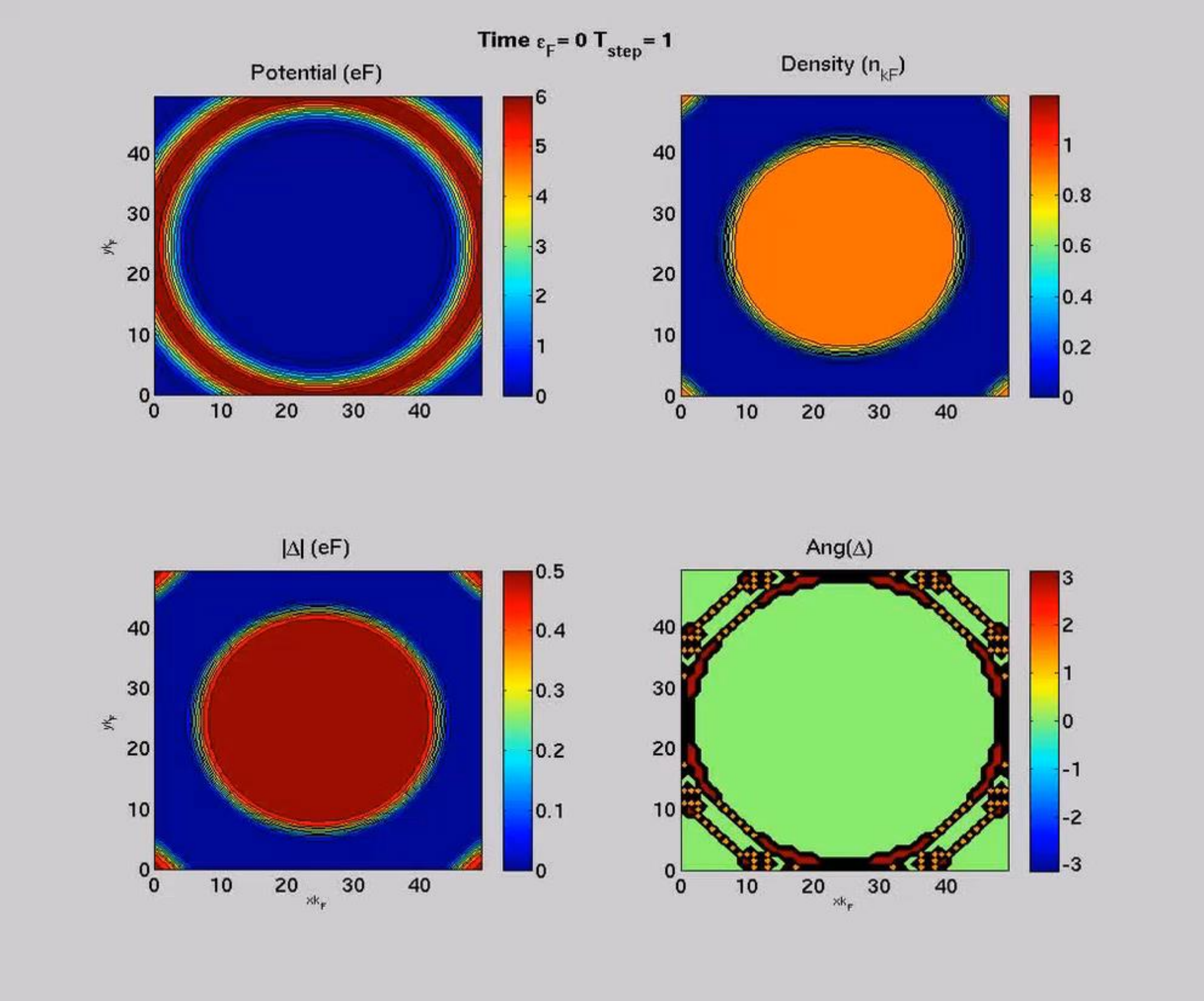
M.W. Zwierlein *et al*., Nature, 435, 1047 (2005)

Stirring the atomic cloud with stirring velocity **lower** than the critical velocity



Bulgac, Luo, Magierski, Roche, Yu, *Science* 332, 1288 (2011)

Stirring the atomic cloud with stirring velocity **exceeding** the critical velocity



Bulgac, Luo, Magierski, Roche, Yu, *Science* 332, 1288 (2011)

Road to quantum turbulence

Classical turbulence: *energy is transfered from large scales to small scales where it eventually dissipates.*

Kolmogorov spectrum: $E(k)=C\,\varepsilon^{2/3}\,k^{-5/3}$

E – kinetic energy per unit mass associated with the scale 1/k
ε - energy rate (per unit mass) transfered to the system at large scales.
k - wave number (from Fourier transformation of the velocity field).
C – dimensionless constant.

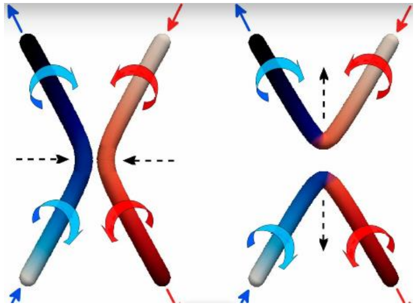
Superfluid turbulence (quantum turbulence): disordered set of quantized vortices. The friction between the superfluid and normal part of the fluid serves as a source of energy dissipation.

Problem: how the energy is dissipated in the superfluid system at small scales at T=0? - „pure" quantum turbulence

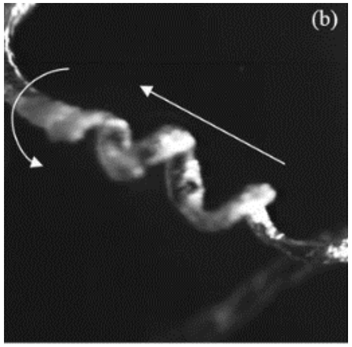
Possibility: vortex reconnections → Kelvin waves → phonon radiation

Turbulence in superfluid systems

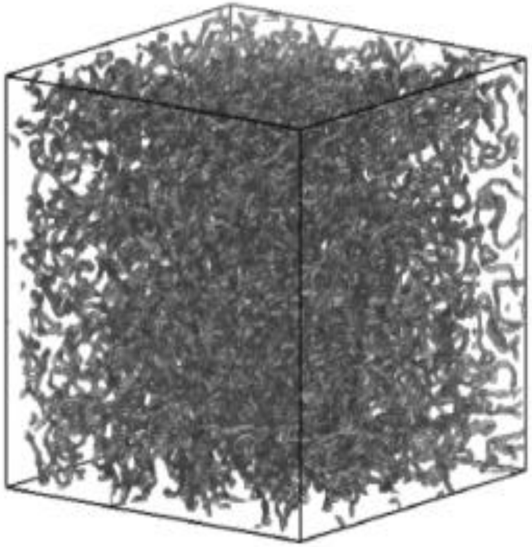
Vortex reconnection

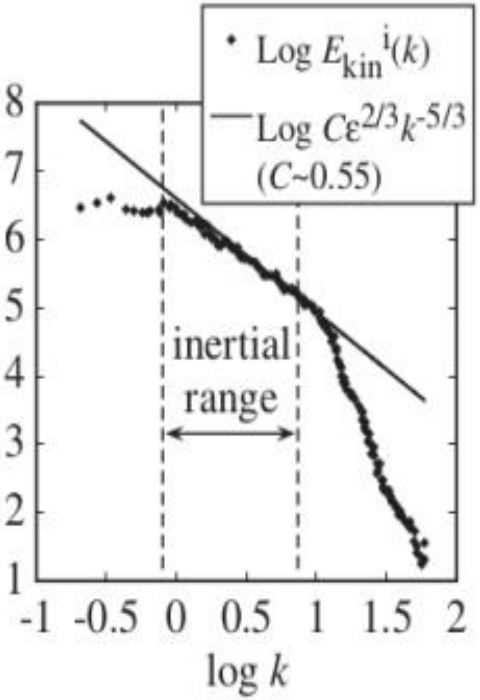


Kelvin wave



Turbulent state

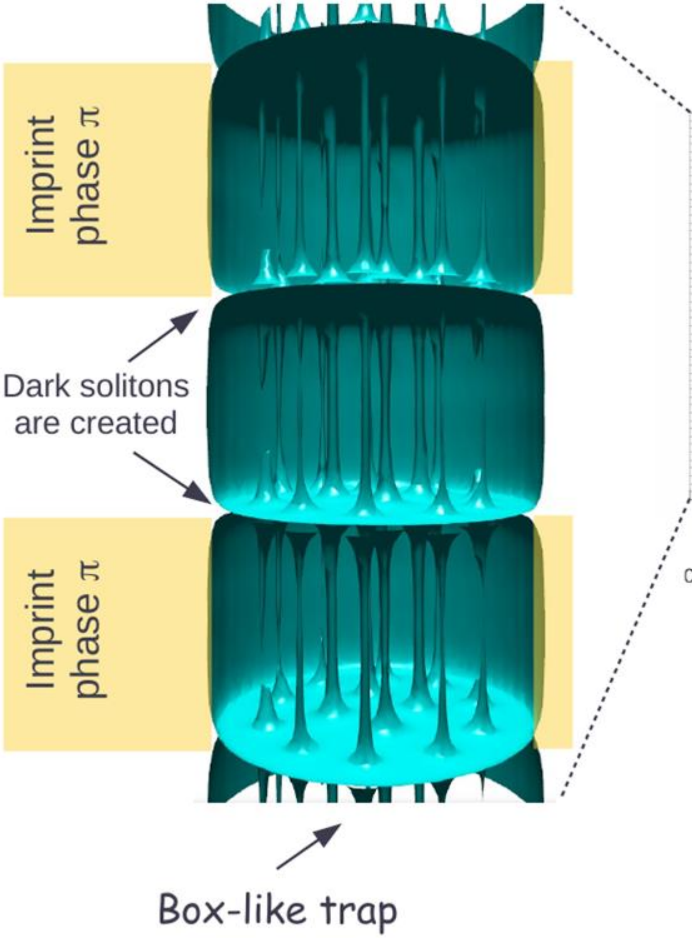


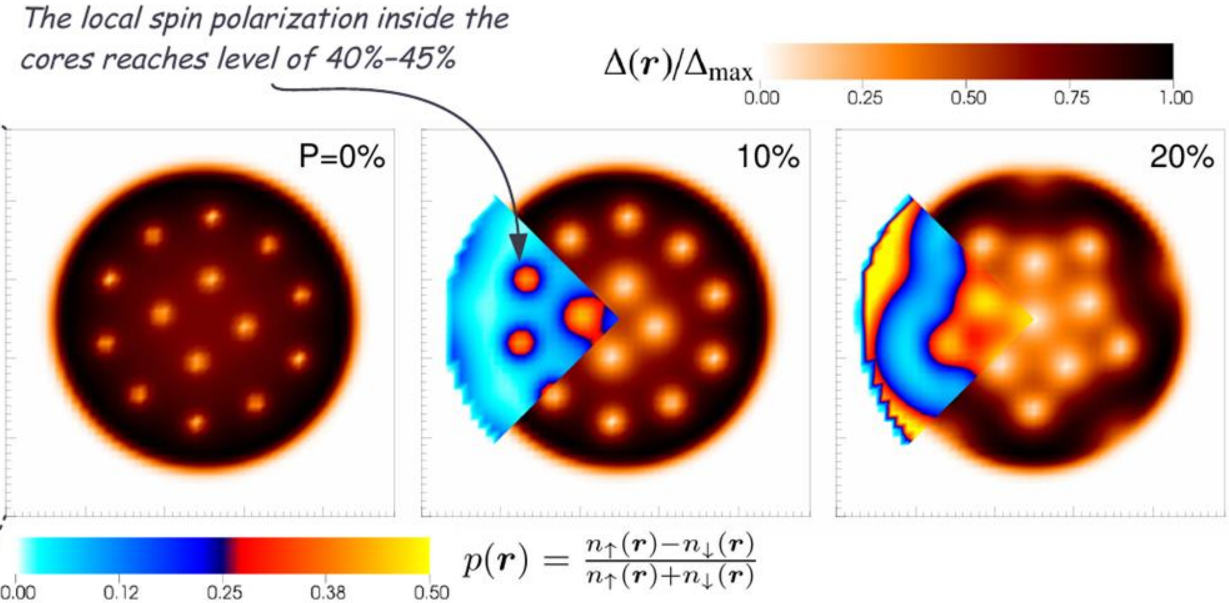


Towards quantum turbulence in fermionic gas

Problem 1: how to generate the turbulence?

→ Our suggestion: *imprint few dark solitons on existing vortex lattice*
→ *rotating turbulence* *(nonzero total angular momentum)*



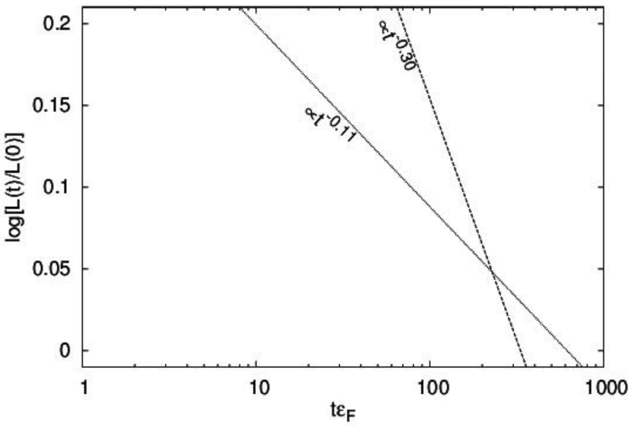


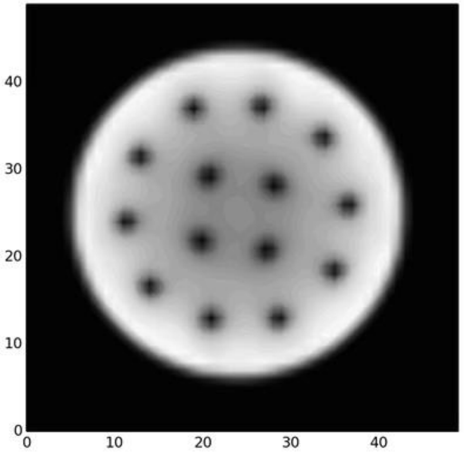
The local spin polarization inside the cores reaches level of 40%-45%

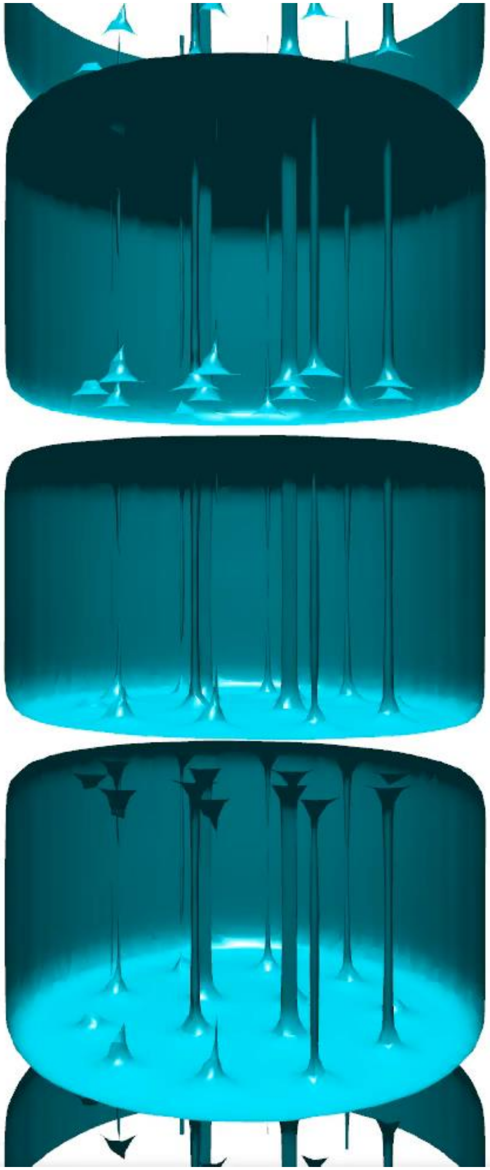
$$p(\boldsymbol{r}) = \frac{n_\uparrow(\boldsymbol{r}) - n_\downarrow(\boldsymbol{r})}{n_\uparrow(\boldsymbol{r}) + n_\downarrow(\boldsymbol{r})}$$

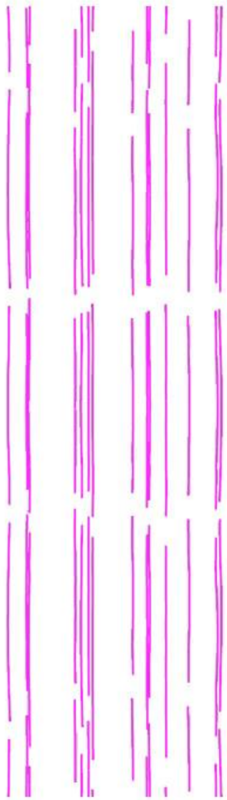
Interesting questions:

- *What are differences and similarities of turbulence in Fermi and Bose superfluids?*
- *What type of quantum turbulence is present in the fermionic systems?*
- *Turbulence in spin-imbalanced systems?*

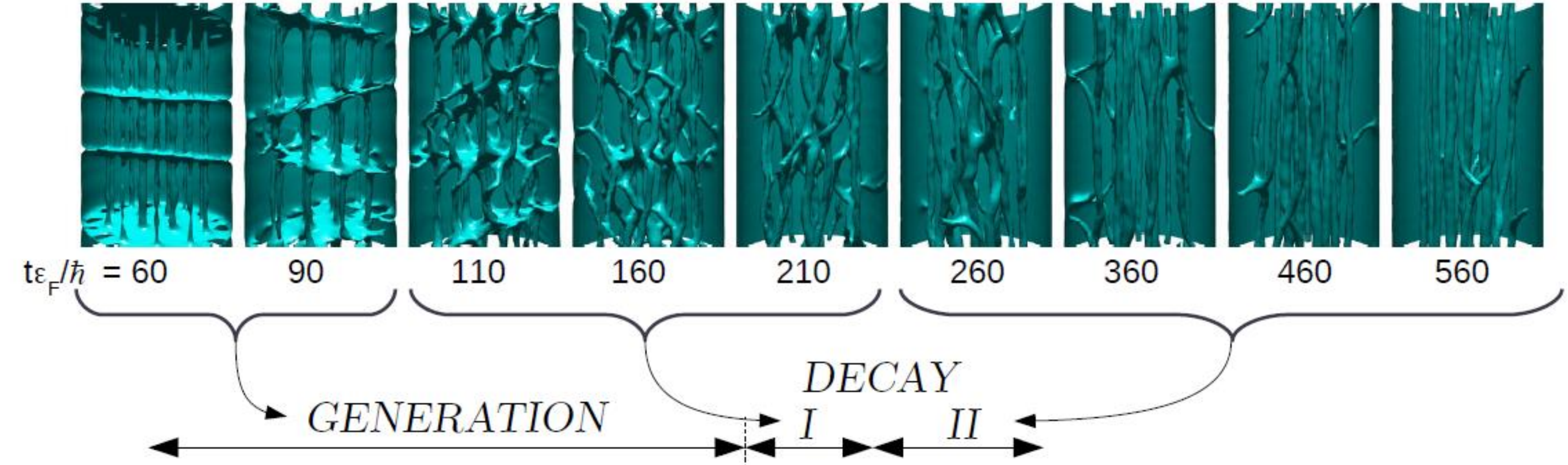


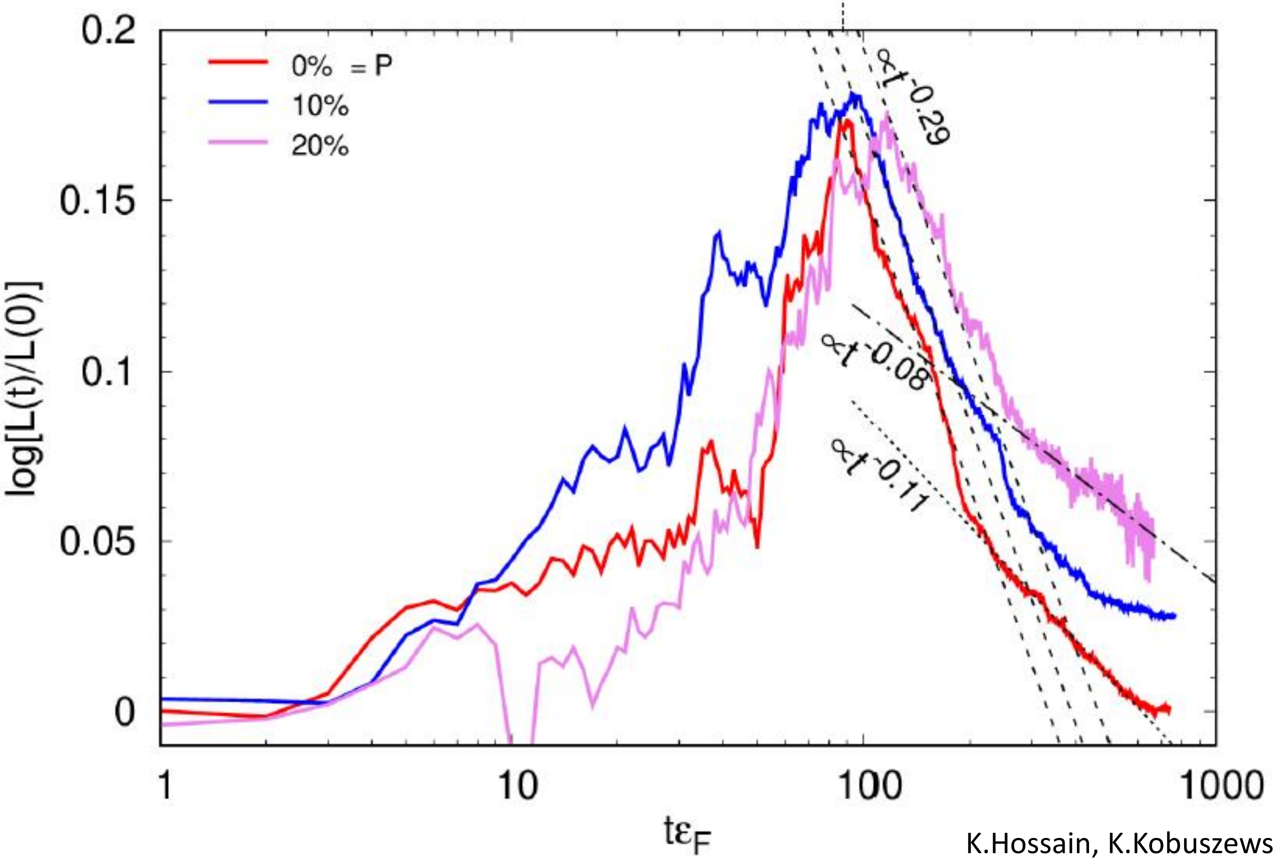






time = 0 $[\varepsilon_F^{-1}]$



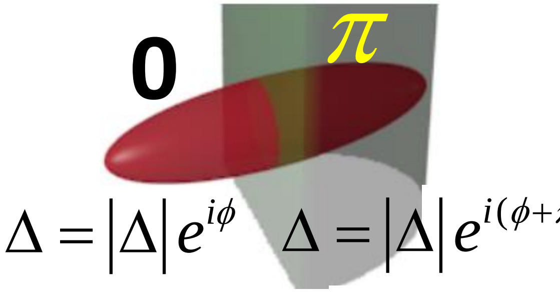


Qualitatively, the “turbulence” decay in a spin-imbalanced system exhibits the same properties as in spin-symmetric counterpart.

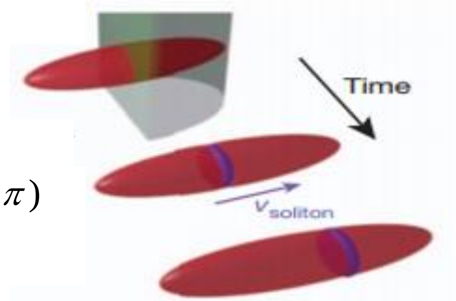
Decay rate of the first stage is almost insensitive to the spin-polarization of the system. It affects only the second stage of the decay process.

K.Hossain, K.Kobuszewski, M.M.Forbes, PM, K.Sekizawa, G.Wlazłowski
Phys. Rev. A 105, 013304 (2022)

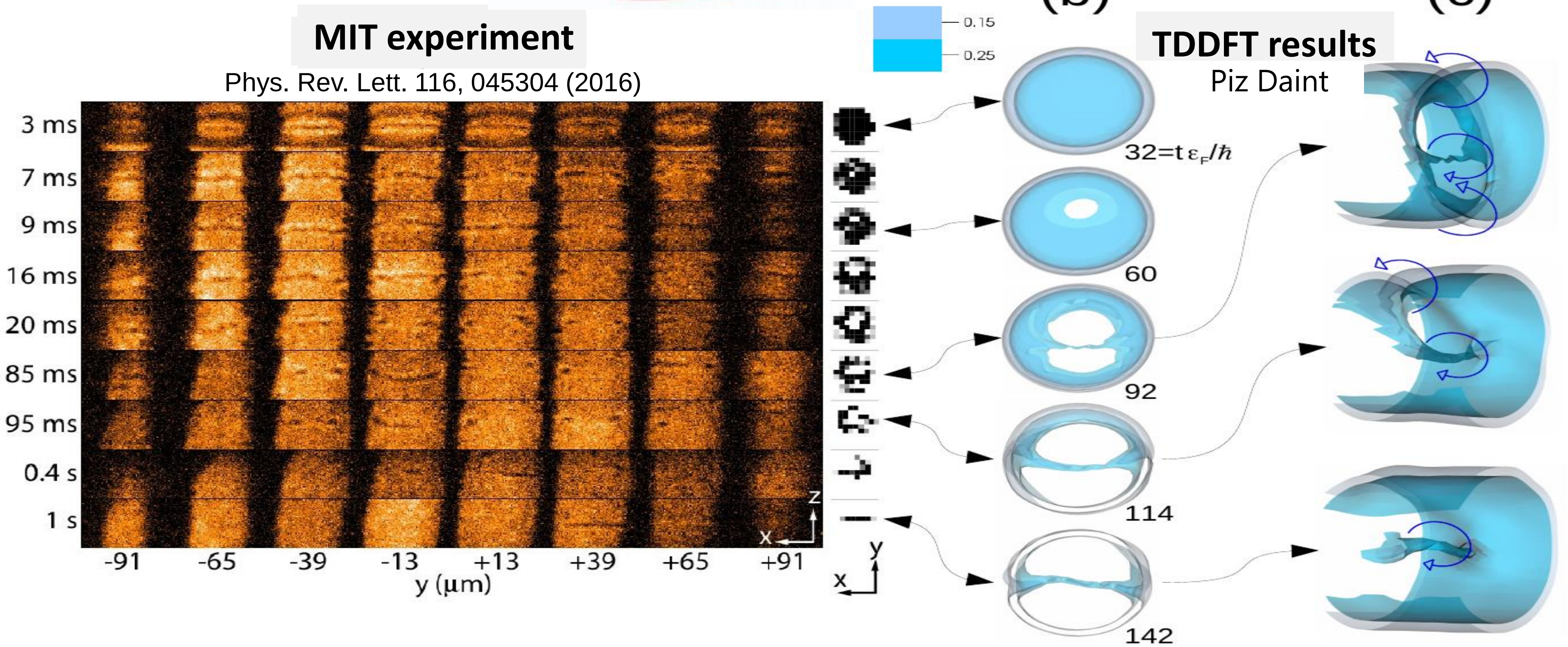
Soliton decay in superfluid



$$\Delta = |\Delta| e^{i\phi} \quad \Delta = |\Delta| e^{i(\phi+\pi)}$$



Series of MIT experiments:
Nature 499, 426 (2013);
PRL 113, 065301 (2014);
PRL 116, 045304 (2016);
→ observation of decay
of a dark soliton into a vortex line



MIT experiment

Phys. Rev. Lett. 116, 045304 (2016)

TDDFT results

Piz Daint

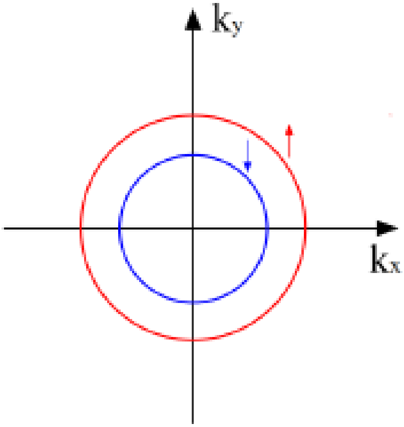
Decay of solitonic excitation (pairing nodal structure) generates a sequence of topological excitations: **dark soliton-> Phi soliton -> vortex ring -> vortex line** reproduced by TDDFT

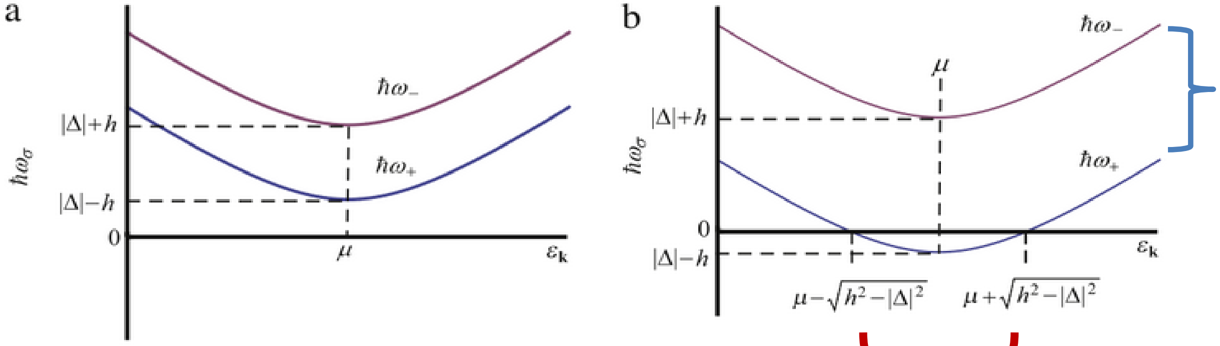
G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. M.,
Phys. Rev. Lett. **120**, 253002 (2018)

Pairing in spin imbalanced superfluids

Clogston-Chandrasekhar condition sets the limit for the chemical potential difference at which superfluidity is lost:

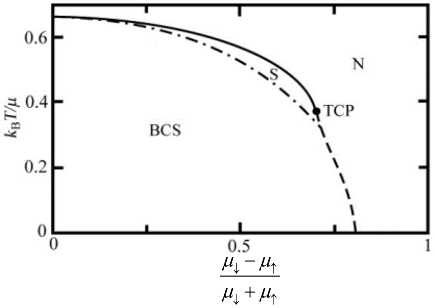
$$\left|\mu_\downarrow - \mu_\uparrow\right| \propto \Delta$$





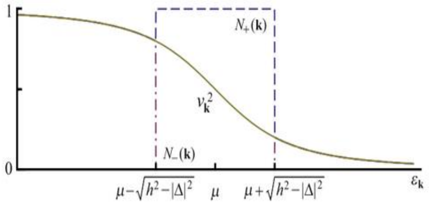
splitting of quasiparticle exc. energy branches for spin-up and spin-down fermions.

$$h = \frac{1}{2}\left|\mu_\downarrow - \mu_\uparrow\right|$$



Sarma (interior gap) phase

G. Sarma, J. Phys. Chem. Solids 24 (1963) 1029.
W.V. Liu, F. Wilczek, Phys. Rev. Lett. 90 (2003) 047002.



Unstable for balanced masses at T=0

K.B. Gubbels, H.T.C. Stoof / Physics Reports 525 (2013) 255–313

Phase separation in momentum space

Inhomogeneous systems: Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase

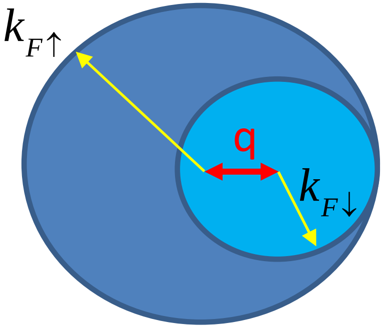
Larkin-Ovchinnikov (LO): $\Delta(r) \sim \cos(\vec{q}\cdot\vec{r})$

Fulde-Ferrell (FF): $\Delta(r) \sim \exp(i\vec{q}\cdot\vec{r})$

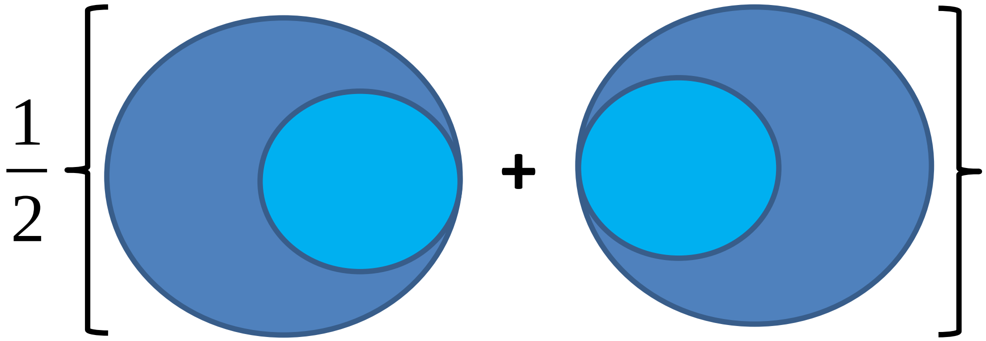
A.I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP 20, 762 (1965)
P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964)

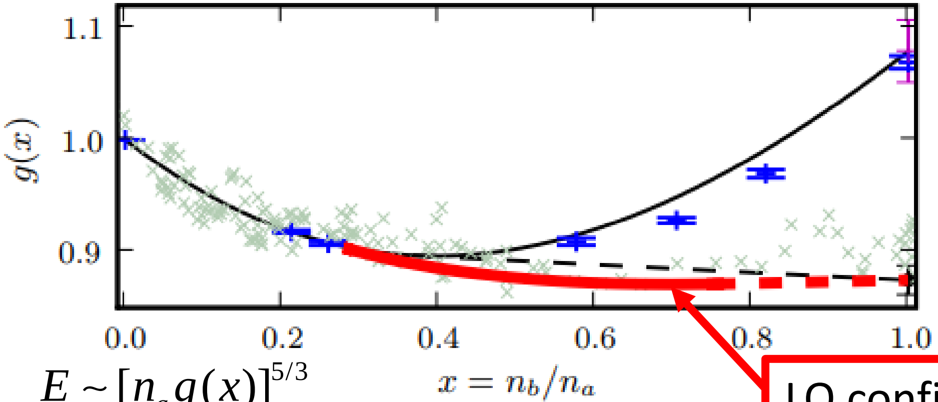
Spatial modulation of the pairing field cost energy proportional to q^2 but may be compensated by an increased pairing energy due to the mutual shift of Fermi spheres:

FF:



LO:





$E \sim [n_a g(x)]^{5/3}$

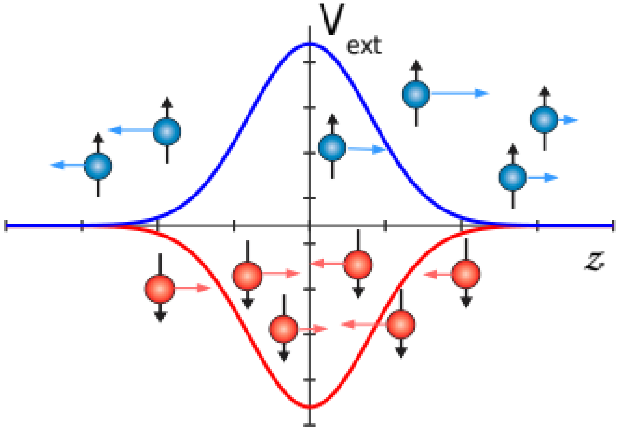
Bulgac & Forbes have shown, within DFT, that Larkin-Ovchinnikov (LO) phase may exist in the unitary Fermi gas (UFG) (realized experimentally in ultracold atomic clouds)

LO configuration – supersolid state

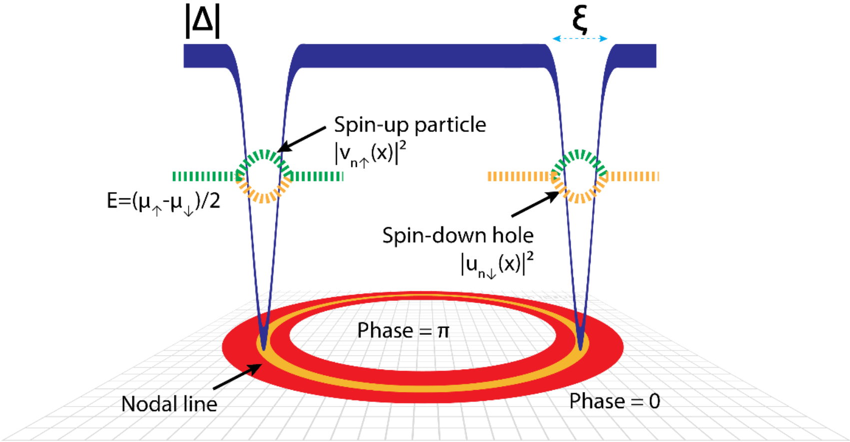
A. Bulgac, M.M.Forbes, Phys. Rev. Lett. 101,215301 (2008)
See also review of mean-field theories : Radzihovsky,Sheehy, Rep.Prog. Phys.73,076501(2010)

Engineering the structure of nodal surfaces

Apply the spin-selective potential of a certain shape:

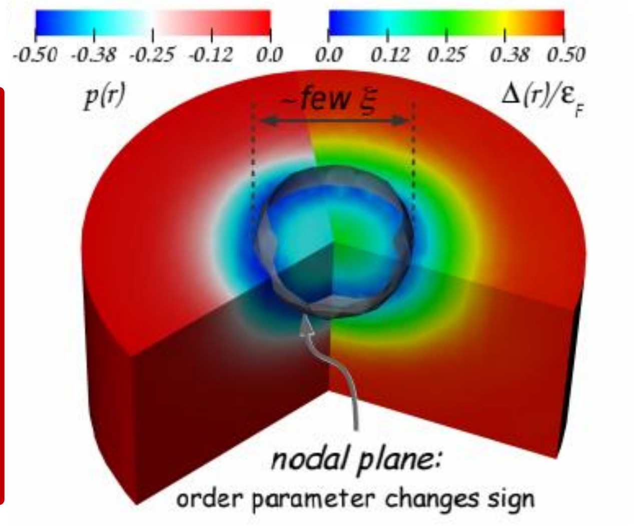


Pairing field nodal structure



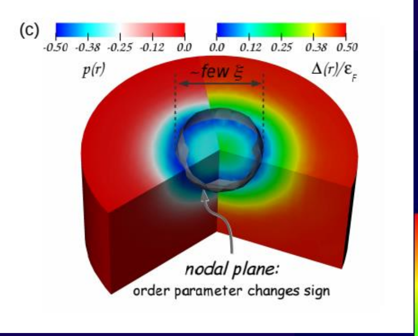
Wait until the proximity effects of the pairing field generate the nodal structure and remove the potential.

For example the spherical nodal structure:

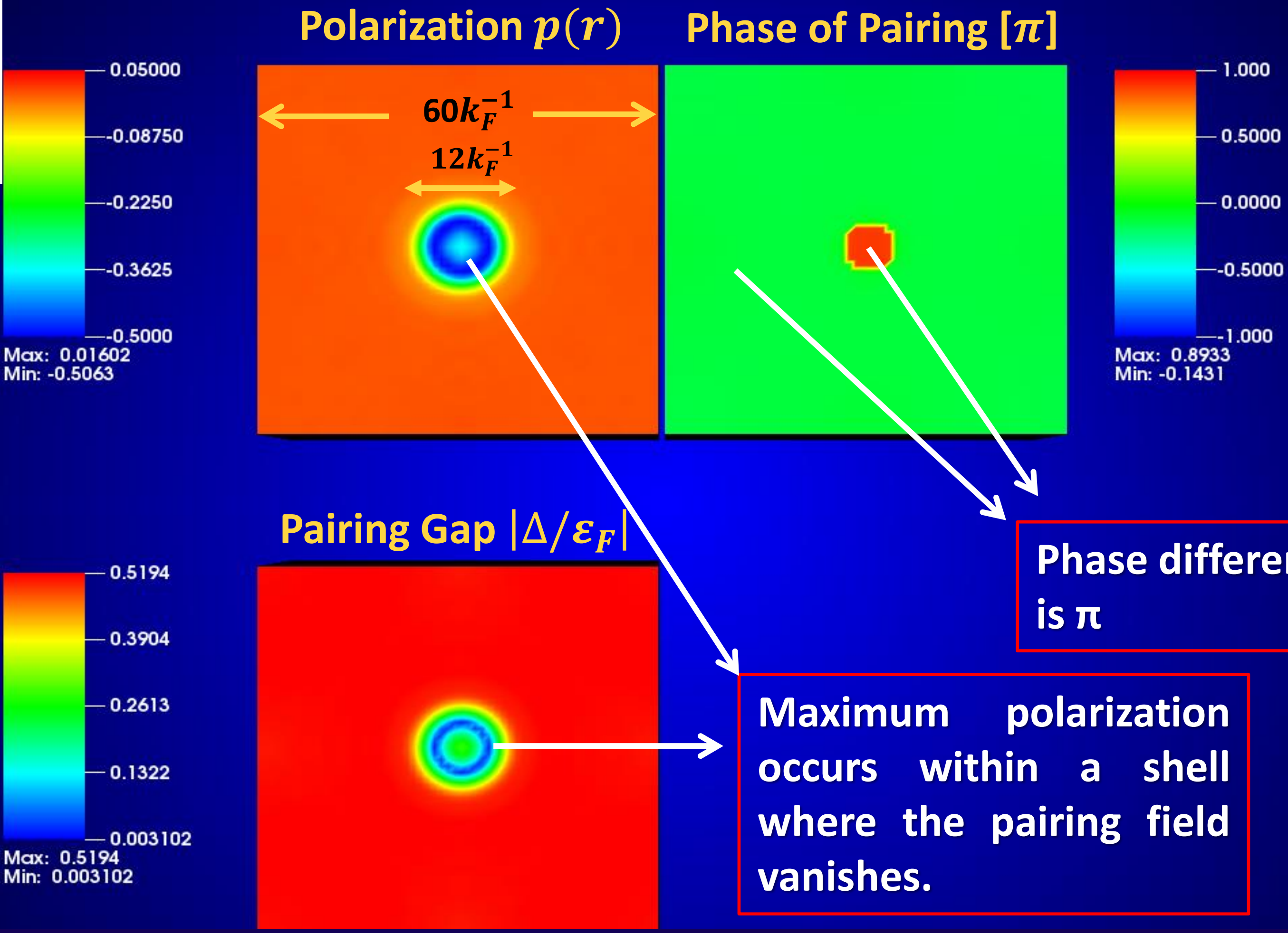


Important!

Nodal structure is **unstable** without spin-polarization.
And vice versa: **spin-polarization** (ie. excess of the majority spin particles) is expelled from superfluid unless pairing nodal structure is created.



Forming a stable spherical nodal surface in Unitary Fermi Gas (UFG) - TDDFT calcs.



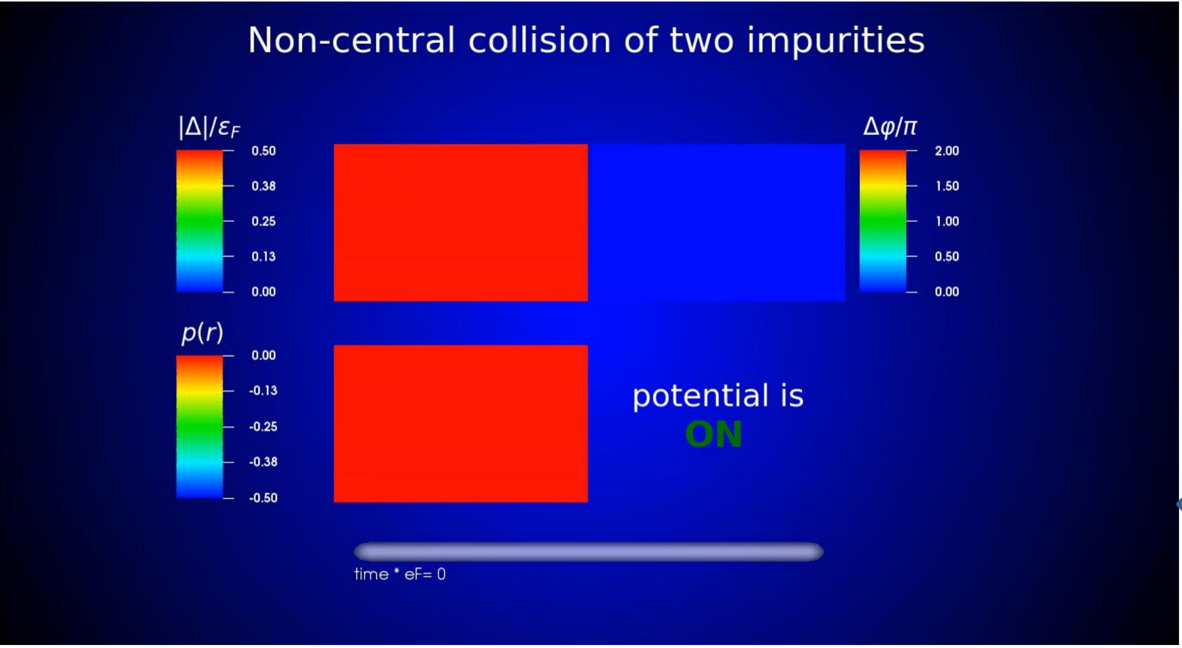
Phase difference is π

Maximum polarization occurs within a shell where the pairing field vanishes.

Contraction of the nodal sphere is prevented by the pairing potential barrier.
Expansion of the nodal sphere will cost the energy due to expansion of polarized shell.

As a result of the interplay between volume and surface energies keeps the impurity stable

P. Magierski, B.Tüzemen, G.Wlazłowski, Phys. Rev. A 100, 033613 (2019); Phys. Rev. A 104, 033304 (2021)



Moving impurity:

From Larkin-Ovchinnikov
towards
Fulde-Ferrell limit:

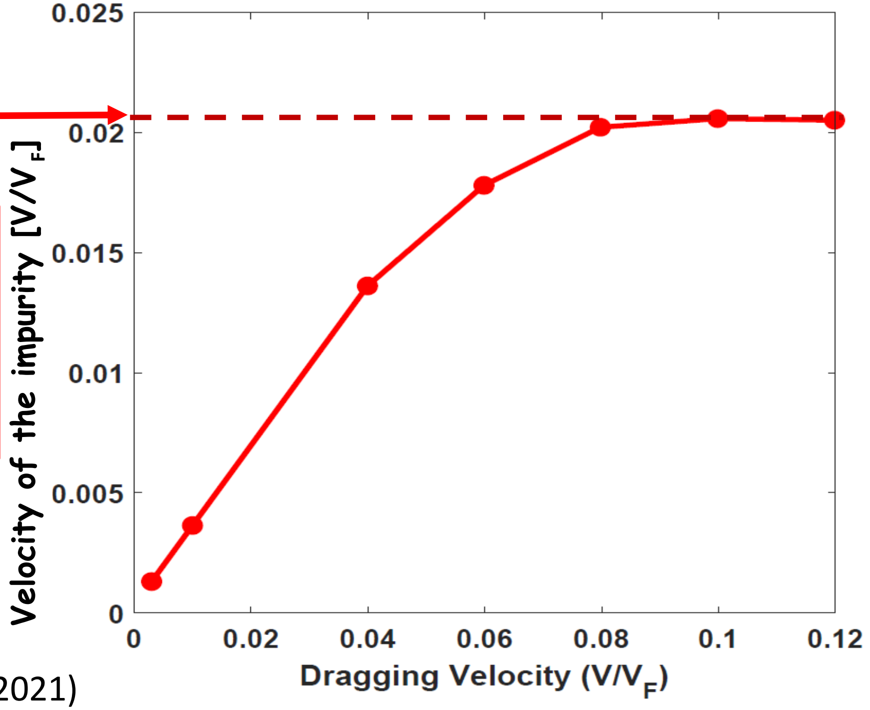
$$\Delta(r): cos(\vec{q}\cdot\vec{r}) \Rightarrow \exp(i\vec{q}\cdot\vec{r})$$

Surprisingly, the nodal structure remains stable even during collisions

The velocities of impurites are about 30% of the velocity of sound.

Limiting velocity with respect to superfluid background

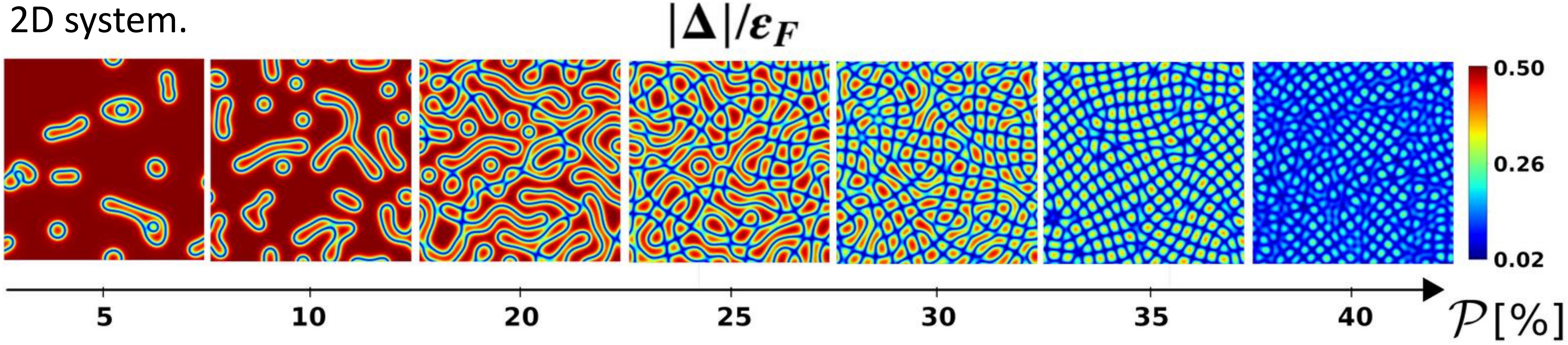
Note that the Fulde-Ferrell limit defines the **critical velocity** which is associated with the maximum spin current that can flow through the impurity $(\sim q = |k_{F\uparrow} - k_{F\downarrow}|)$.



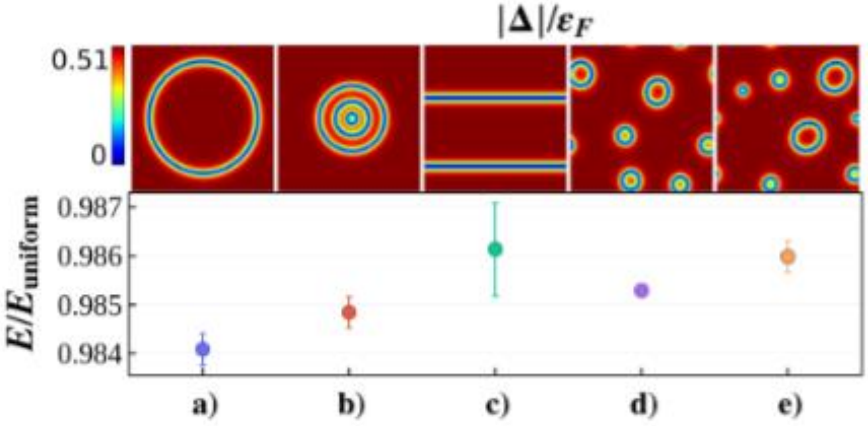
P.Magierski, B.Tüzemen, G.Wlazłowski, Phys. Rev. A 104, 033304 (2021)

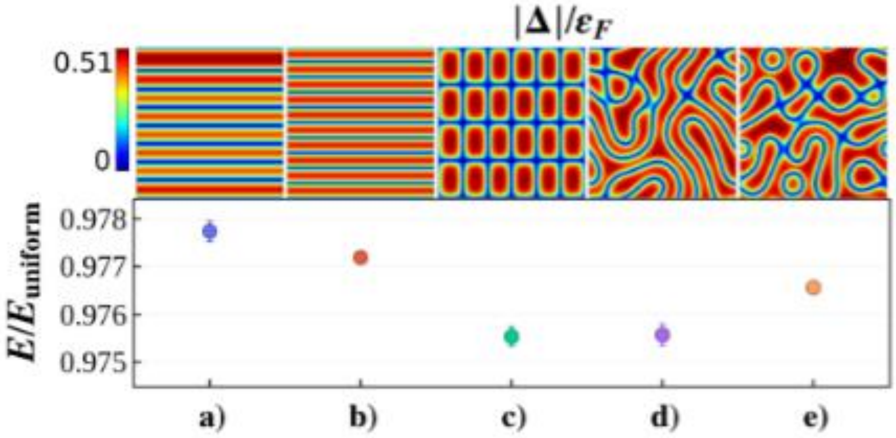
In search of LOFF phase: Supersolid or liquid crystal?

2D system.

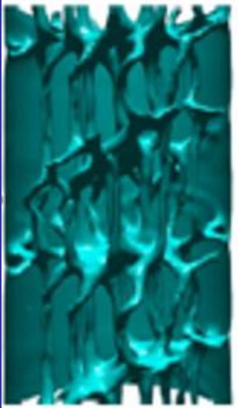


$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$





B. Tüzemen, T. Zawiślak, G. Wlazłowski, P.M. – in preparation

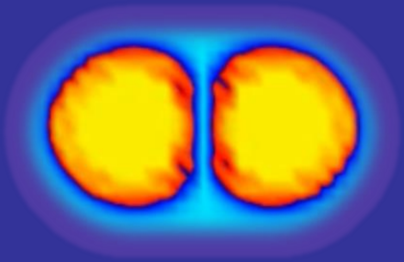


Quantum turbulence

K. Hossain (WSU)
M.M. Forbes (WSU)
K. Kobuszewski (WUT)
S. Sarkar (WSU)
G. Wlazłowski (WUT)

Vortex dynamics in neutron star crust

N. Chamel (ULB)
D. Pęcak (WUT)
G. Wlazłowski (WUT)



Nuclear collisions

M. Barton (WUT)
A. Boulet (WUT)
A. Makowski (WUT)
K. Sekizawa (Tokyo I.)
G. Wlazłowski (WUT)

Josephson junction in atomic Fermi gases - dissipative effects

N. Proukakis (NU)
M. Tylutki (WUT)
G. Wlazłowski (WUT)
K. Xhani (LENS & NU)

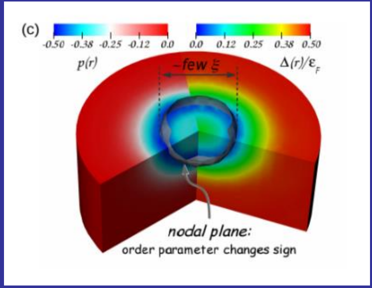
Nonequilibrium superfluidity in Fermi systems

Collisions of vortex-antivortex pairs

A. Barresi (WUT)
A. Boulet (WUT)
G. Wlazłowski (WUT)
and LENS exp. Group

Spin-imbalanced Fermi gases

B. Tuzemen (WUT)
G. Wlazłowski (WUT)
T. Zawiślak (WUT)



Robert B. Laughlin, Nobel Lecture, December 8, 1998:

One of my favorite times in the academic year occurs [..] when I give my class of extremely bright graduate students [..] a take home exam in which they are asked TO DEDUCE SUPERFLUIDITY FROM FIRST PRINCIPLES.
There is no doubt a special place in hell being reserved for me at this very moment for this mean trick, for the task is IMPOSSIBLE. Superfluidity [..] is an EMERGENT phenomenon – a low energy collective effect of huge number of particles that CANNOT be deduced from the microscopic equations of motion in a RIGOROUS WAY and that DISAPPEARS completely when the system is taken apart.

[..]students who stay in physics long enough [..] eventually come to understand that the REDUCTIONIST IDEA IS WRONG a great deal of the time and perhaps ALWAYS.