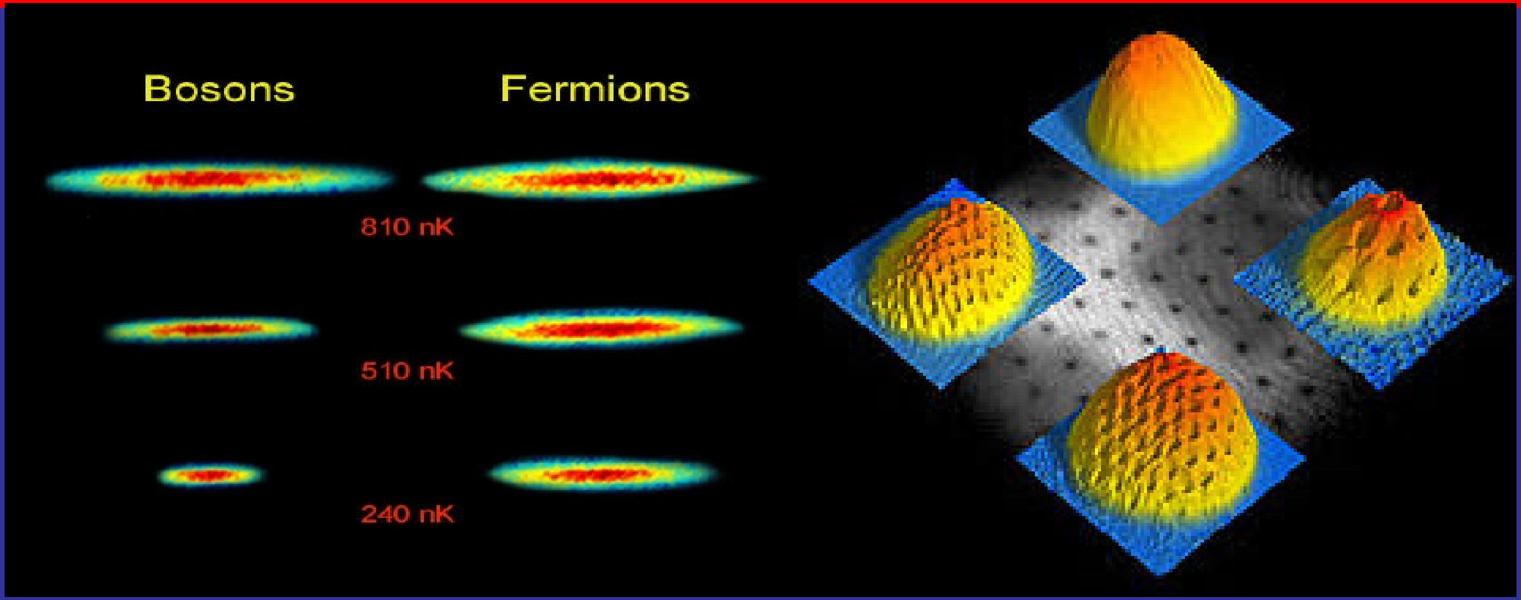
Onset of the pseudogap phase in ultracold atomic gases



Piotr Magierski
Warsaw University of Technology/University of Washington

Collaborators: Aurel Bulgac – University of Washington (Seattle),
Joaquin E. Drut – Ohio State University (Columbus),
Gabriel Wlazłowski – Warsaw University of Technology

Setting the problem:

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$$\boxed{n\, r_0^{\,3} \ll 1} \quad \boxed{n\, |a|^3 \gg 1}$$

n - particle density
a - scattering length
r_0 - effective range

$$\boxed{\text{i.e. } r_0 \to 0, a \to \pm\infty}$$

NONPERTURBATIVE REGIME

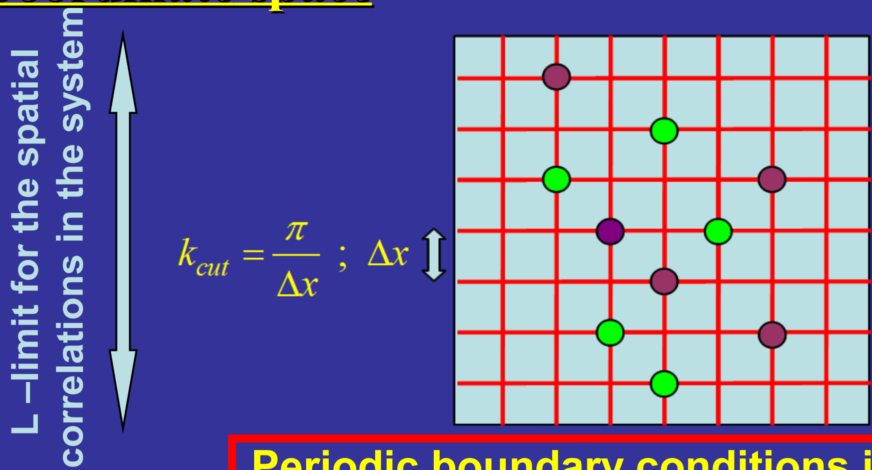
System is dilute but strongly interacting!

UNIVERSALITY: $E = \xi_0 E_{FG}$

AT FINITE TEMPERATURE: $E(T) = \xi\left(\frac{T}{\varepsilon_F}\right) E_{FG}, \quad \xi(0)=\xi_0$

Auxiliary field Monte Carlo for fermions on the lattice

Coordinate space



$Volume = L^3$

$lattice\ spacing = \Delta x$

- Spin up fermion

- Spin down fermion

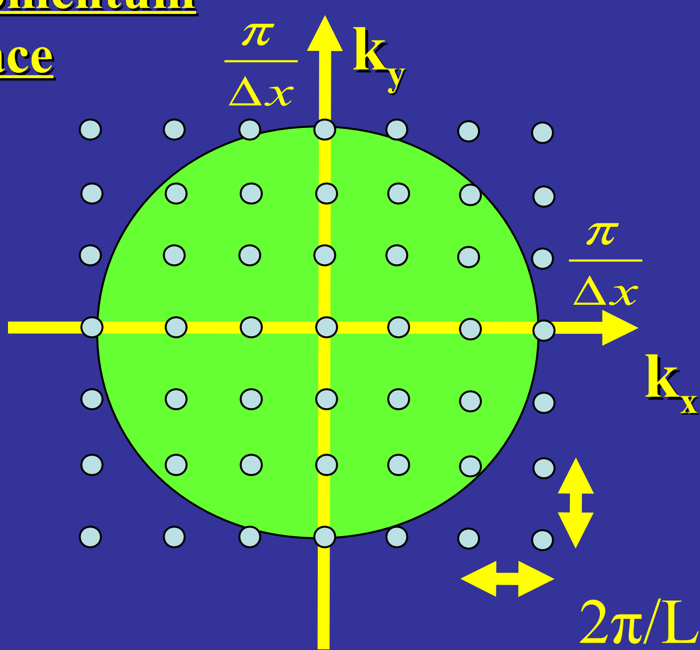
Periodic boundary conditions imposed

External conditions:

T - temperature

μ - chemical potential

Momentum space



UV momentum cutoff $\Lambda_{UV} = \frac{\pi}{\Delta x}$

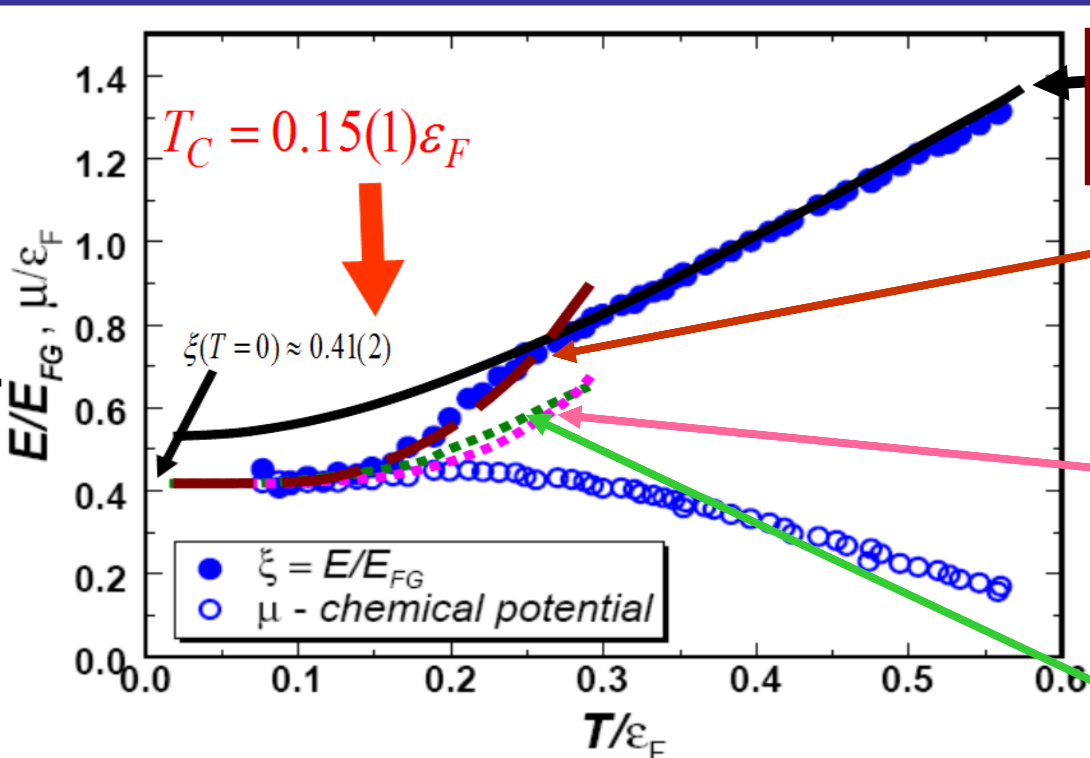
IR momentum cutoff $\Lambda_{IR} = \frac{2\pi}{L}$

$$\frac{\hbar^2 \Lambda_{IR}^2}{2m} << \varepsilon_F, \ \Delta << \frac{\hbar^2 \Lambda_{UV}^2}{2m}$$

-Equation of state of the unitary Fermi gas,

-Critical temperature for the superfluid-normal transition

$a = \pm\infty$



Normal Fermi Gas
(with vertical offset, solid line)

Bogoliubov-Anderson phonons and quasiparticle contribution (dashed line)

Bogoliubov-Anderson phonons contribution only (dotted line)

Quasi-particle contribution only (dotted line)

$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}} \exp\left(-\frac{\Delta}{T}\right)$$

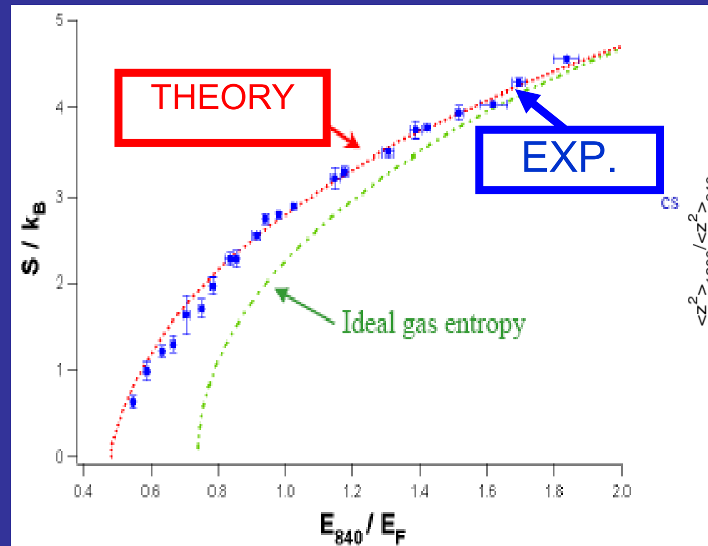
$$\Delta = \left(\frac{2}{e}\right)^{7/3} \varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.41$$

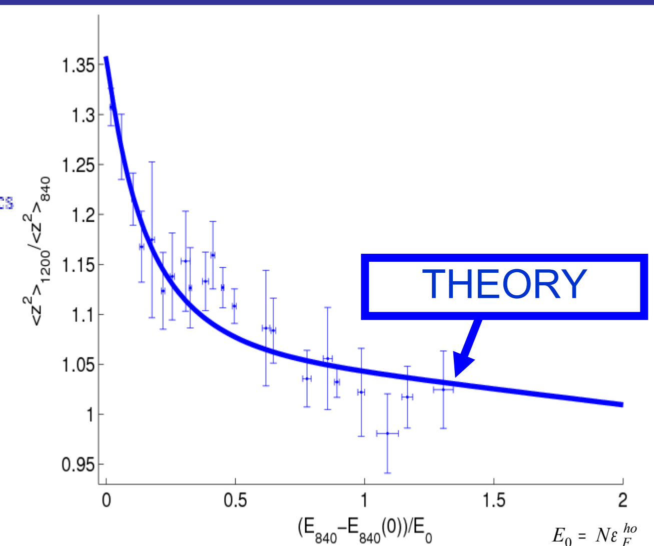
A. Bulgac, J.E. Drut, P. Magierski, PRL96, 090404(2006)

Local density approximation with Monte Carlo input.

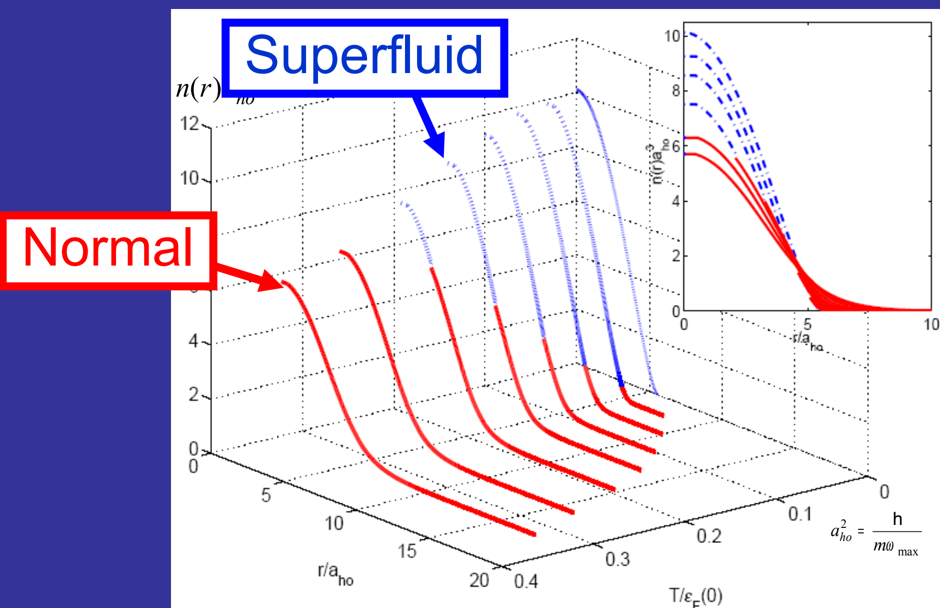
Unitary Fermi gas in a harmonic trap



Entropy as a function of energy (relative to the ground state) for the unitary Fermi gas in the harmonic trap.



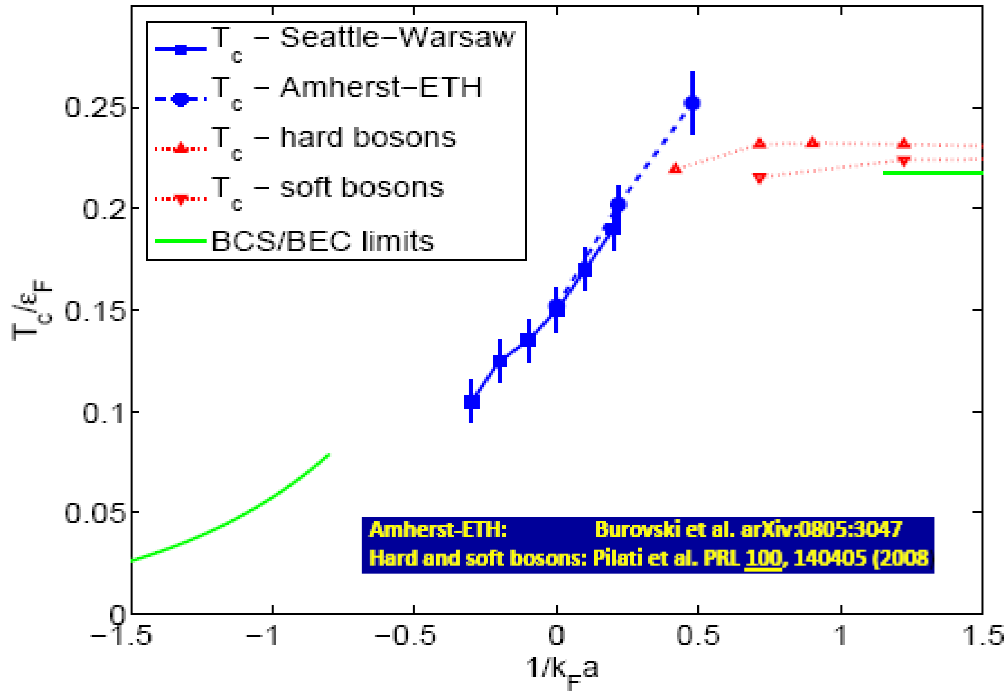
Ratio of the mean square cloud size at B=1200G to its value at unitarity (B=840G) as a function of the energy. Experimental data are denoted by point with error bars.

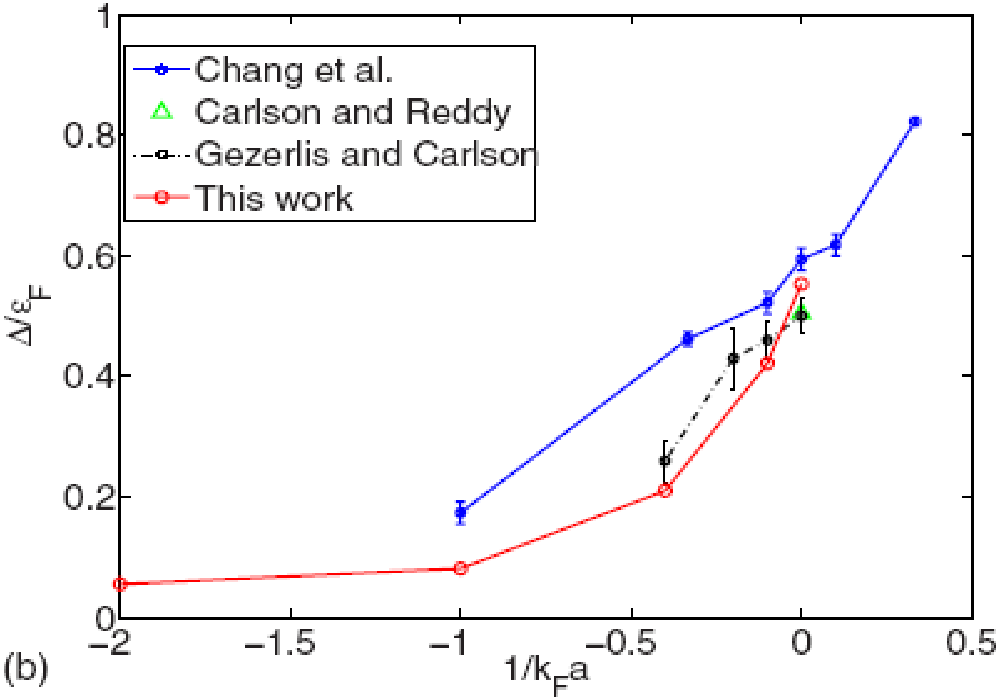


$\varepsilon_F(0)$ - Fermi energy at the center of the trap

Exp: L.Luo, et al. Phys. Rev. Lett. 98, 080402, (2007)
Theory: Bulgac, Drut, Magierski, PRL99, 120401(2007)

The radial (along shortest axis) density profiles of the atomic cloud in the Duke group experiment at various temperatures.





Results in the vicinity of the unitary limit:

-Critical temperature

-Pairing gap at T=0

Note that

- at unitarity: $\Delta/\varepsilon_F \approx 0.5$

- for atomic nucleus: $\Delta/\varepsilon_F \approx 0.03$

BCS theory predicts:

$$\Delta(T=0)/T_C \approx 1.7$$

At unitarity:

$$\Delta(T=0)/T_C \approx 3.3$$

This is NOT a BCS superfluid!

Bulgac, Drut, Magierski, PRA78, 023625(2008)

Single particle gap (density of states)

Spectral weight function: $A(\vec{p},\omega)$

$$G^{ret/adv}(\vec{p},\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega' \frac{A(\vec{p},\omega')}{\omega-\omega' \pm i0^+}$$

$$G(\vec{p},\tau) = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, A(\vec{p},\omega)\frac{e^{-\omega\tau}}{1+e^{-\beta\omega}}$$

From Monte Carlo calcs.

$$G(\vec{p},\tau) = \frac{1}{Z} Tr\{e^{-(\beta-\tau)(\hat{H}-\mu\hat{N})}\hat{\psi}_\uparrow(\vec{p})e^{-\tau(\hat{H}-\mu\hat{N})}\hat{\psi}^+_\uparrow(\vec{p})\}$$

Constraints:

$$\int_{-\infty}^{+\infty} A(\vec{p},\omega)\frac{d\omega}{2\pi} = 1$$

$$\int_{-\infty}^{+\infty} A(\vec{p},\omega)(1+e^{\beta\omega})^{-1}\frac{d\omega}{2\pi} = n(\vec{p})$$

In the limit of independent quasiparticles: $A(\vec{p},\omega) = 2\pi\delta(\omega - E(p))$

Maximum entropy method

From Bayes' theorem: $$P(A|G) \propto P(G|A)P(A)$$

A priori probability: $$P(A) \propto \exp(\alpha S)$$

Relative entropy: $$S(\mathcal{M}) = \sum_{k=1}^{n_A} \Delta\omega \left[A(\omega_k) - \mathcal{M}(\omega_k) - A(\omega_k) \ln\left(\frac{A(\omega_k)}{\mathcal{M}(\omega_k)} \right) \right].$$

Likelihood function: $$P(G|A) \propto \exp\left(-\frac{1}{2}\chi^2 \right) \quad \chi^2 = \sum_{i=1}^{n_\tau} \left(\frac{\tilde{\mathcal{G}}_{\tau_i} - \mathcal{G}(\tau_i)}{\sigma_{\tau_i}} \right)^2 \quad \mathcal{G}(\tau_i) = \sum_{k=1}^{n_A} \frac{e^{-\omega_k \tau_i}}{1 + e^{-\omega_k \beta}} A_k \Delta\omega.$$

Maximum entropy method: $$\min_{A(\omega)} \left(\frac{1}{2}\chi^2 - \alpha S \right)$$

SVD method

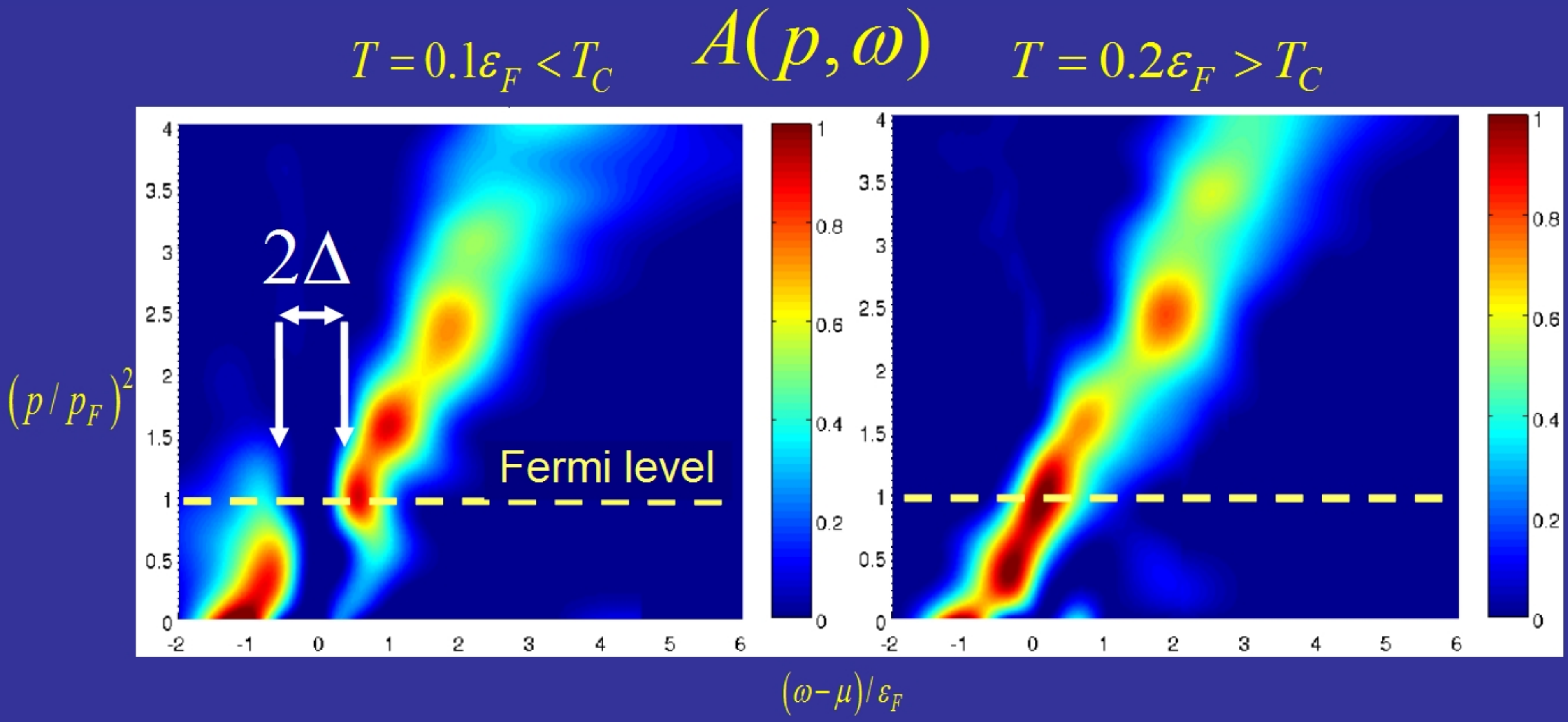
$$\mathcal{G}(\boldsymbol{p}, \tau_i) = (\mathcal{K}A)(\boldsymbol{p}, \tau_i).$$

$$\mathcal{K}u_i = \lambda_i \vec{v}_i, \quad \mathcal{K}^* \vec{v}_i = \lambda_i u_i,$$

$$u_i(\omega) = \frac{1}{\sigma_i} \sum_{k=1}^{n_\tau} (\vec{v}_i)_k \phi_{\tau_k}(\omega) = -\frac{1}{2\pi\sigma_i} \sum_{k=1}^{n_\tau} (\vec{v}_i)_k \frac{e^{-\omega\tau_k}}{1 + e^{-\omega\beta}}.$$

$$A(\boldsymbol{p}, \omega) = \sum_{i=1}^{r} b_i(\boldsymbol{p}) u_i(\omega), \quad b_i(\boldsymbol{p}) = \frac{1}{\lambda_i} (\vec{\mathcal{G}}(\boldsymbol{p}) \cdot \vec{v}_i),$$

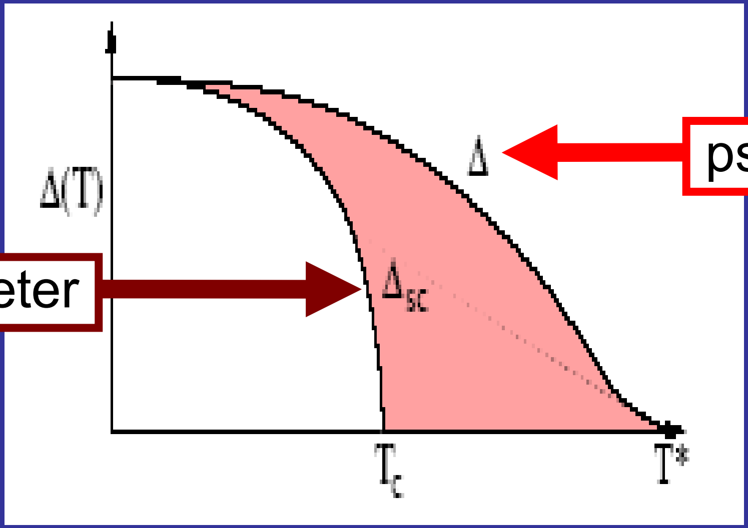
Spectral weight function at unitarity

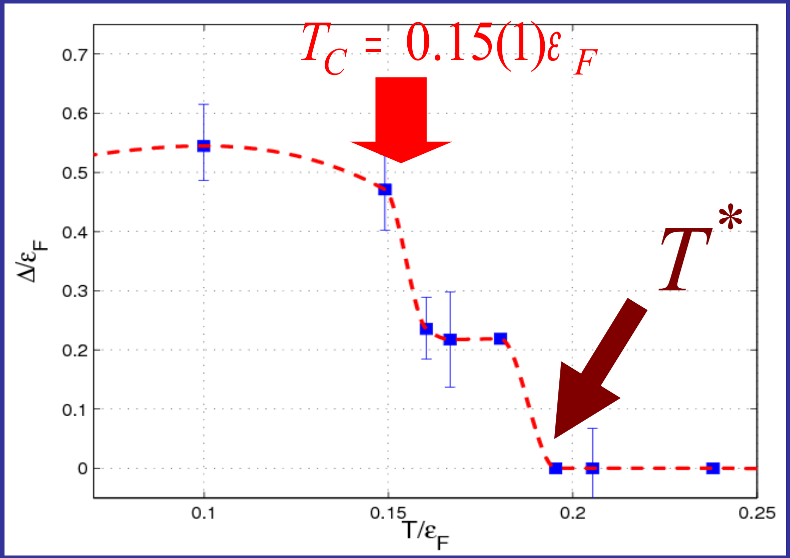


P.Magierski, G. Wlazłowski, A. Bulgac, J.E. Drut, PRL103, 210403 (2009)

Pairing gap and pseudogap

Outside the BCS regime close to the unitary limit, but still before BEC, superconductivity/superfluidity emerge out of a very exotic, non-Fermi liquid normal state



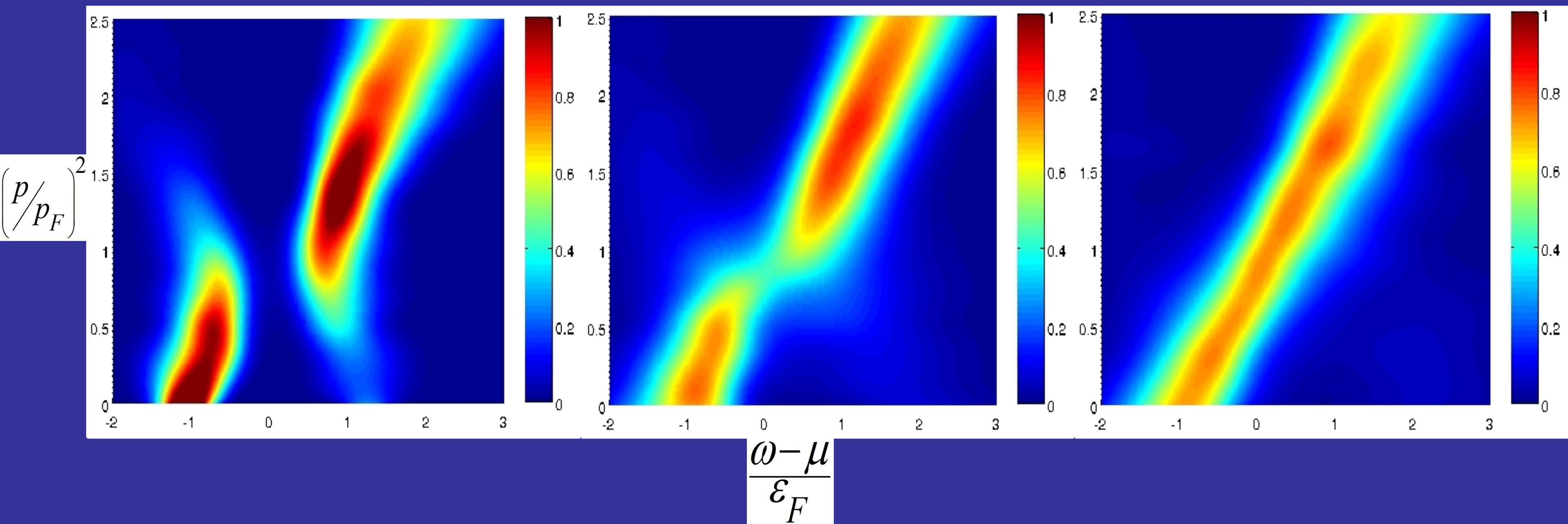


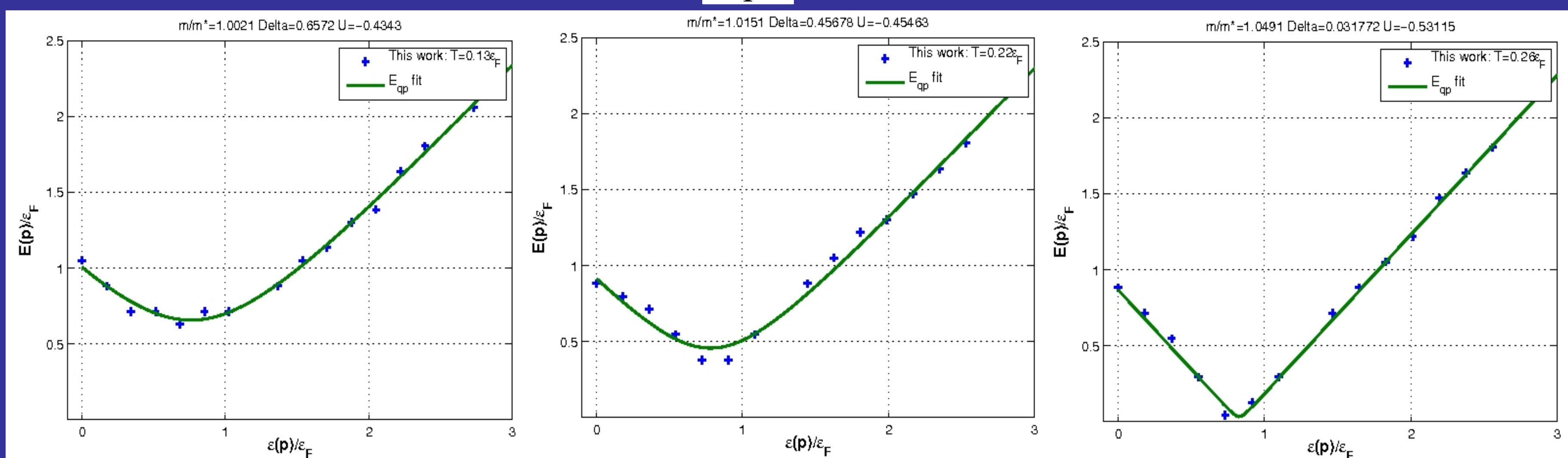
Monte Carlo calculations

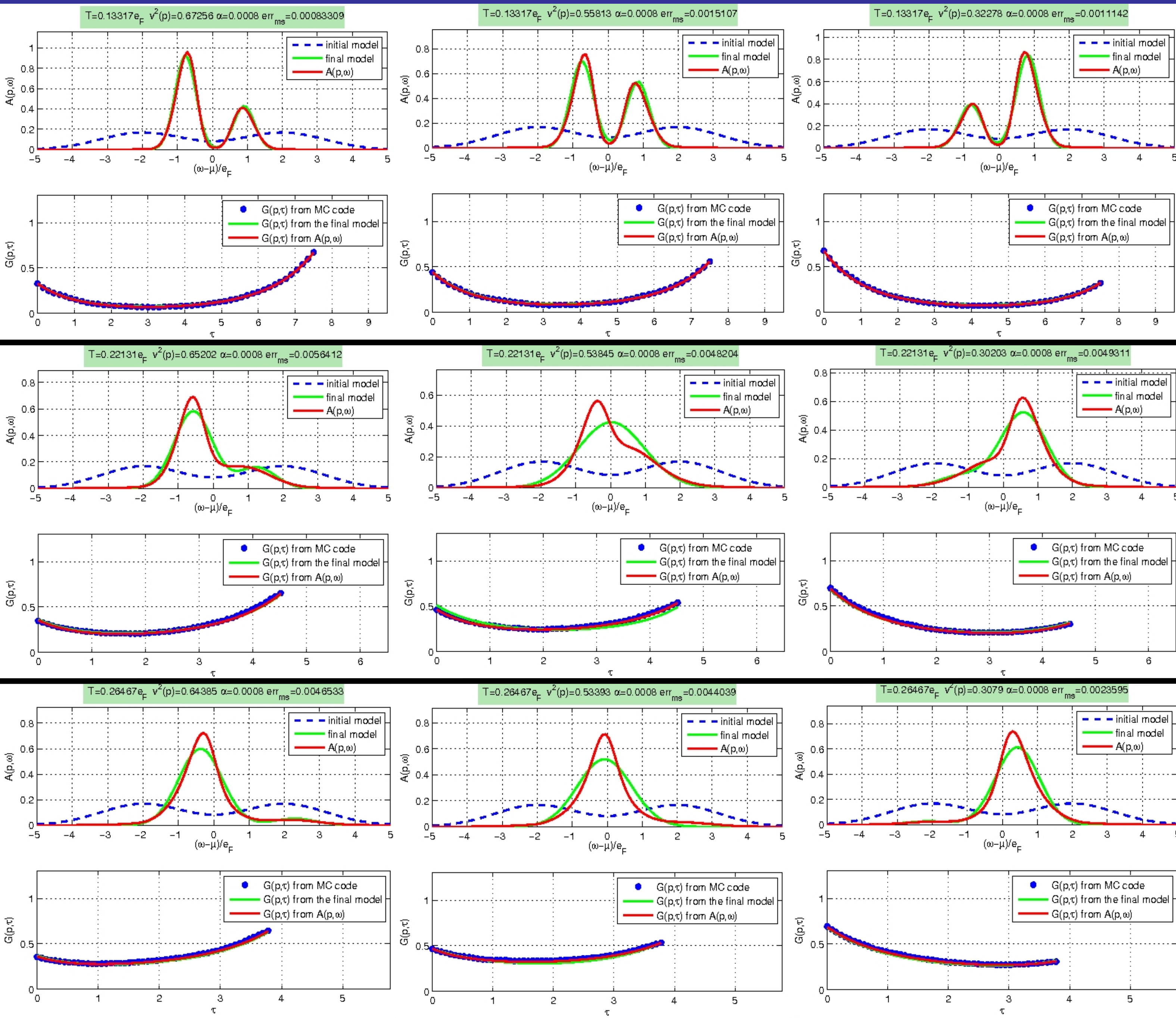
The onset of superconductivity occurs in the presence of fermionic pairs!

Spectral weight function from MC calc. for $1/k_F a = 0.2$

Superfluid region (T=0.13) Pseudogap phase (T=0.22) Normal Fermi gas (T=0.26)





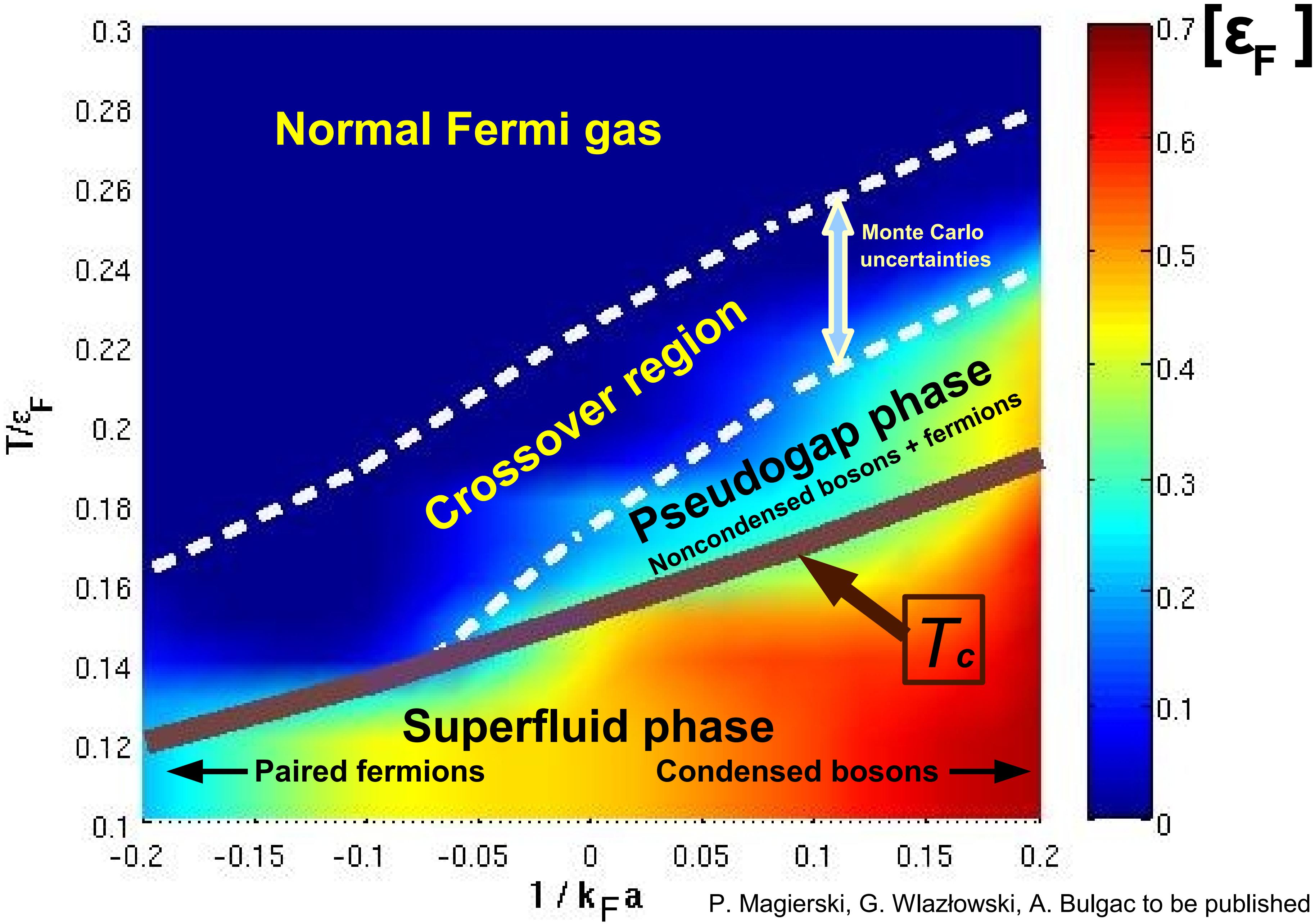


Superfluid phase: T=0.13

Pseudogap phase: T=0.22

Normal Fermi gas: T=0.26

Gap in the single particle fermionic spectrum from MC calcs.



P. Magierski, G. Wlazłowski, A. Bulgac to be published

Preliminary measurements of pseudogap in ultracold atomic gases

^{40}K

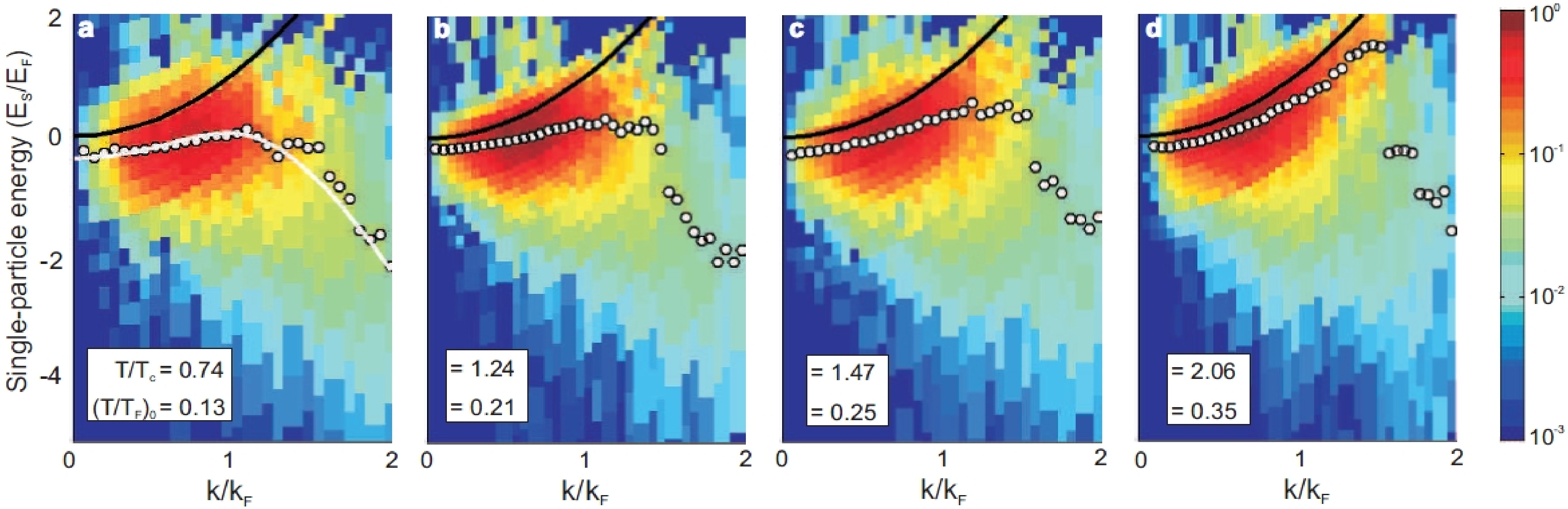


FIG. 1: **Photoemission spectra throughout the pseudogap regime.** Spectra are shown for Fermi gases at four different temperatures, each with an interaction strength characterized by $(k_F a)^{-1} \approx 0.15$. The intensity plots show the fraction of out-coupled atoms as a function of their single-particle energy (normalized to E_F) and momentum (normalized to k_F), where $E = 0$ corresponds to a non-interacting particle at rest. The spectra are normalized so that integrating them over momentum and energy gives unity. White dots indicate the centers extracted from gaussian fits to individual energy distribution curves (traces through the data at fixed momentum). The black curve is the quadratic dispersion expected for a free particle. **a** At $T = 0.74\,T_c$, we observe a BCS-like dispersion with back-bending, consistent with previous measurements [6]. The white curve is a fit to a BCS-like dispersion, Eqn. 1. **b,c** At $T = 1.24\,T_c$ and $T = 1.47\,T_c$, respectively, the dispersion with back-bending persists even though there is no longer any superfluidity. **d** At $T = 2.06\,T_c$, the dispersion does not display back-bending in the range of $0 < k < 1.5\,k_F$. In all the plots there is a negative dispersion for $k/k_F > 1.5$. We attribute this weak feature (note the log scale) to a $1/k^4$ tail and not to the gap.

J. P. Gaebler, J. T. Stewart, T. E. Drake, D. S. Jin, A. Perali, P. Pieri, G. C. Strinati, arXiv:1003.1147

Summary

- Finite temperature MC results provide a strong evidence for the existence of the pseudogap phase which opens up around the unitary limit at about $1/(k_F*a) = -0.1$

Open question:

- How to describe the pseudogap phase within the framework of finite temperature DFT?