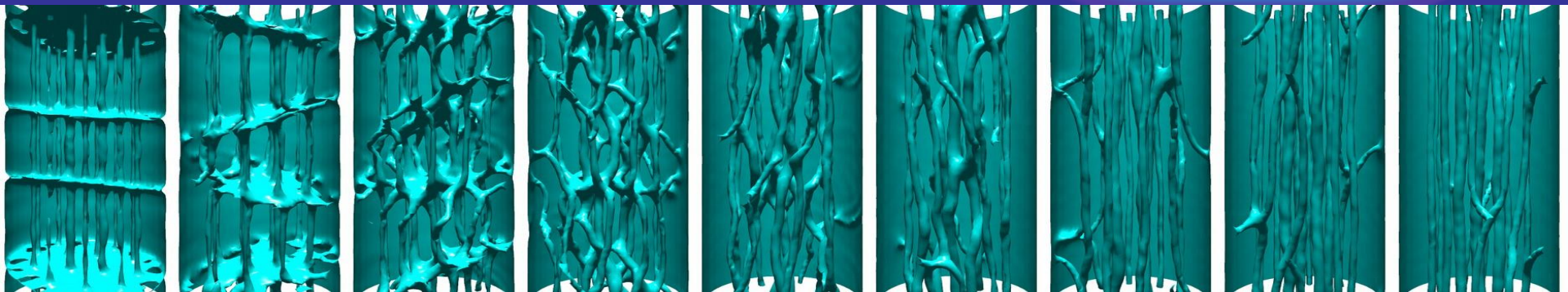
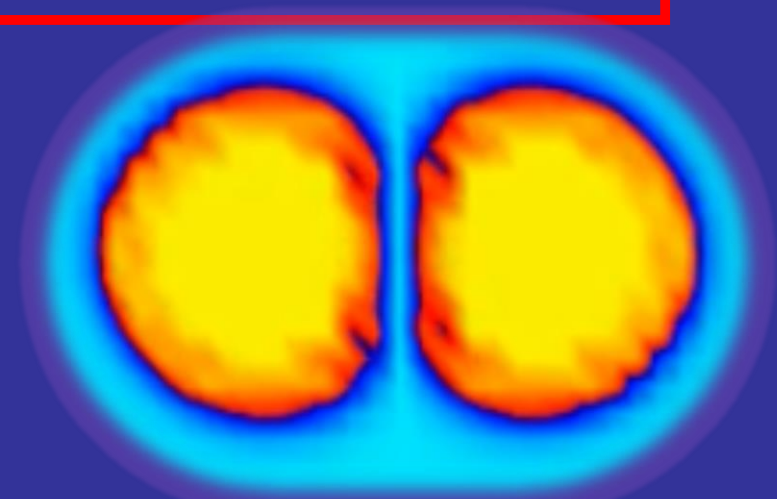
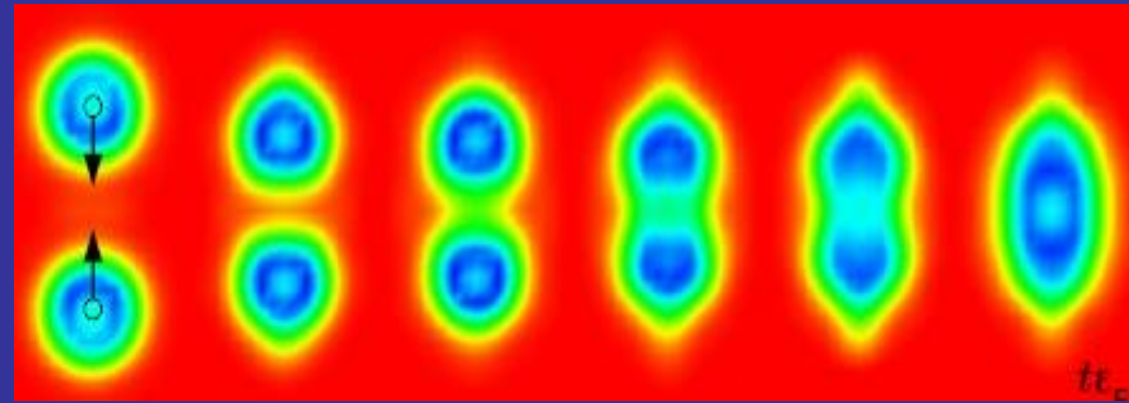


Exotic features of superfluidity far from equilibrium



Piotr Magierski
Warsaw University of Technology (WUT)

Robert B. Laughlin, Nobel Lecture, December 8, 1998:

One of my favorite times in the academic year occurs [...] when I give my class of extremely bright graduate students [...] a take home exam in which they are asked TO DEDUCE SUPERFLUIDITY FROM FIRST PRINCIPLES.

There is no doubt a special place in hell being reserved for me at this very moment for this mean trick, for the task is IMPOSSIBLE. Superfluidity [...] is an **EMERGENT** phenomenon – a low energy collective effect of huge number of particles that CANNOT be deduced from the microscopic equations of motion in a RIGOROUS WAY and that DISAPPEARS completely when the system is taken apart.

[...]students who stay in physics long enough [...] eventually come to understand that the REDUCTIONIST IDEA IS WRONG a great deal of the time and perhaps ALWAYS.

GOAL:

Unified description of superfluid dynamics of fermionic systems far from equilibrium based on microscopic theoretical framework.

Microscopic framework = explicit treatment of fermionic degrees of freedom.

Why Time Dependent Density Functional Theory (TDDFT)?

We need to describe the time evolution of (externally perturbed) spatially inhomogeneous, superfluid Fermi system.

Within current computational capabilities TDDFT allows to describe real time dynamics of strongly interacting, superfluid systems of hundreds of thousands fermions.

Pairing correlations in DFT

One may extend DFT to superfluid systems by defining the pairing field:

$$\Delta(\mathbf{r}\sigma, \mathbf{r}'\sigma') = -\frac{\delta E(\rho, \chi)}{\delta \chi^*(\mathbf{r}\sigma, \mathbf{r}'\sigma')}.$$

L. N. Oliveira, E. K. U. Gross, and W. Kohn, Phys. Rev. Lett. 60 2430 (1988).

O.-J. Wacker, R. Kümmel, E.K.U. Gross, Phys. Rev. Lett. 73, 2915 (1994).

Triggered by discovery of high-Tc superconductors

and introducing anomalous density $\chi(\mathbf{r}\sigma, \mathbf{r}'\sigma') = \langle \hat{\psi}_{\sigma'}(\mathbf{r}') \hat{\psi}_{\sigma}(\mathbf{r}) \rangle$

However in the limit of the local field these quantities diverge unless one renormalizes the coupling constant:

$$\begin{aligned} \Delta(\mathbf{r}) &= g_{eff}(\mathbf{r}) \chi_c(\mathbf{r}) \\ \frac{1}{g_{eff}(\mathbf{r})} &= \frac{1}{g(\mathbf{r})} - \frac{mk_c(\mathbf{r})}{2\pi^2 \hbar^2} \left(1 - \frac{k_F(\mathbf{r})}{2k_c(\mathbf{r})} \ln \frac{k_c(\mathbf{r}) + k_F(\mathbf{r})}{k_c(\mathbf{r}) - k_F(\mathbf{r})} \right) \end{aligned}$$

which ensures that the term involving the kinetic and the pairing energy density is finite:

$$\frac{\tau_c(r)}{2m} - \Delta(r) \chi_c(r), \quad \tau_c(r) = \nabla \cdot \nabla' \rho_c(r, r') \Big|_{r=r'}$$

A. Bulgac, Y. Yu, Phys. Rev. Lett. 88 (2002) 042504

A. Bulgac, Phys. Rev. C65 (2002) 051305

It allows to reduce the size of the problem for static calculations by introducing the energy cutoff

Pairing correlations in time-dependent superfluid local density approximation (TDSLDA)

$$S = \int_{t_0}^{t_1} \left(\left\langle 0(t) \left| i \frac{d}{dt} \right| 0(t) \right\rangle - E[\rho(t), \chi(t)] \right) dt$$

Stationarity requirement produces the set of equations:

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} U_\mu(\mathbf{r}, t) \\ V_\mu(\mathbf{r}, t) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r}, t) & \Delta(\mathbf{r}, t) \\ \Delta^*(\mathbf{r}, t) & -h^*(\mathbf{r}, t) \end{pmatrix} \begin{pmatrix} U_\mu(\mathbf{r}, t) \\ V_\mu(\mathbf{r}, t) \end{pmatrix} :$$

$$B(t) = \begin{pmatrix} U(t) & V^*(t) \\ V(t) & U^*(t) \end{pmatrix} = \exp[iG(t)] \quad G(t) = \begin{pmatrix} h(t) & \Delta(t) \\ \Delta^\dagger(t) & -h^*(t) \end{pmatrix}$$

Orthogonality and completeness has to be fulfilled: $B^\dagger(t)B(t) = B(t)B^\dagger(t) = I$,

In order to fulfill the completeness relation of Bogoliubov transform all states need to be evolved!

Otherwise Pauli principle is violated, i.e. the evolved densities do not describe a fermionic system (spurious bosonic effects are introduced).

Consequence: the computational cost increases considerably.

Solving time-dependent problem for superfluids...

The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations

$$h \sim f_1(n, \nu, \dots) \nabla^2 + \mathbf{f}_2(n, \nu, \dots) \cdot \nabla + f_3(n, \nu, \dots)$$

$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_{n,a}(\mathbf{r}, t) \\ u_{n,b}(\mathbf{r}, t) \\ v_{n,a}(\mathbf{r}, t) \\ v_{n,b}(\mathbf{r}, t) \end{pmatrix} = \begin{pmatrix} h_a(\mathbf{r}, t) & 0 & 0 & \Delta(\mathbf{r}, t) \\ 0 & h_b(\mathbf{r}, t) & -\Delta(\mathbf{r}, t) & 0 \\ 0 & -\Delta^*(\mathbf{r}, t) & -h_a^*(\mathbf{r}, t) & 0 \\ \Delta^*(\mathbf{r}, t) & 0 & 0 & -h_b^*(\mathbf{r}, t) \end{pmatrix} \begin{pmatrix} u_{n,a}(\mathbf{r}, t) \\ u_{n,b}(\mathbf{r}, t) \\ v_{n,a}(\mathbf{r}, t) \\ v_{n,b}(\mathbf{r}, t) \end{pmatrix}$$

We explicitly track fermionic degrees of freedom!

where h and Δ depends on “densities”:

$$n_\sigma(\mathbf{r}, t) = \sum_{E_n < E_c} |v_{n,\sigma}(\mathbf{r}, t)|^2, \quad \tau_\sigma(\mathbf{r}, t) = \sum_{E_n < E_c} |\nabla v_{n,\sigma}(\mathbf{r}, t)|^2,$$

$$v(\mathbf{r}, t) = \sum_{E_n < E_c} u_{n,\uparrow}(\mathbf{r}, t) v_{n,\downarrow}^*(\mathbf{r}, t), \quad \mathbf{j}_\sigma(\mathbf{r}, t) = \sum_{E_n < E_c} \text{Im}[v_{n,\sigma}^*(\mathbf{r}, t) \nabla v_{n,\sigma}(\mathbf{r}, t)],$$

huge number of nonlinear coupled 3D Partial Differential Equations
(in practice $n=1, 2, \dots, 10^5 - 10^6$)

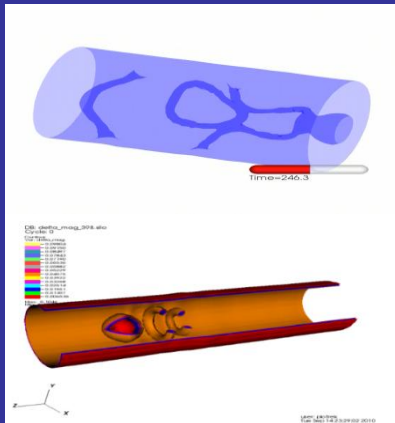
Present computing capabilities:

- ▶ full 3D (unconstrained) superfluid dynamics
 - ▶ spatial mesh up to 100^3
 - ▶ max. number of particles of the order of 10^4
 - ▶ up to 10^6 time steps
- (for cold atomic systems - time scale: a few ms
for nuclei - time scale: 100 zs)

Superconducting systems of interest

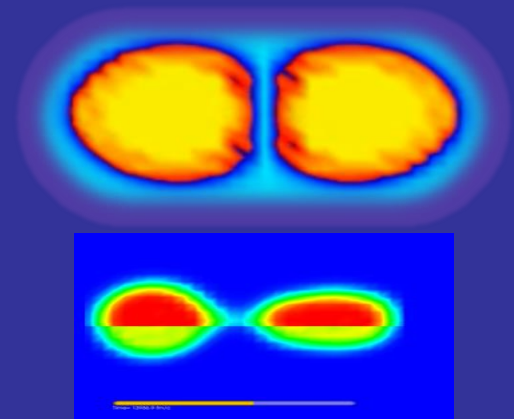
$$\frac{\Delta}{\mathcal{E}_F} \leq 0.5$$

Ultracold atomic (fermionic) gases.
Unitary regime.
 Dynamics of quantum vortices, solitonic excitations, quantum turbulence



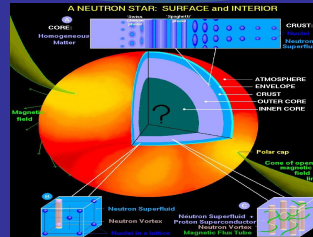
$$\frac{\Delta}{\mathcal{E}_F} \leq 0.03$$

Nuclear physics.
 Induced nuclear fission, fusion, collisions.



$$\frac{\Delta}{\mathcal{E}_F} \leq 0.1-0.2$$

Astrophysical applications.
 Modelling of neutron star interior (glitches): vortex dynamics, dynamics of inhomogeneous nuclear matter.

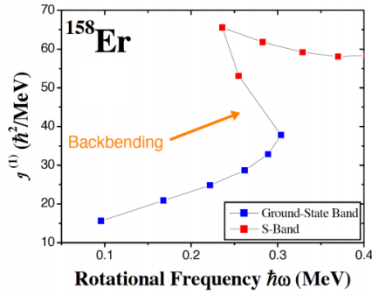


$$\frac{\Delta}{\mathcal{E}_F} \text{ - Pairing gap to Fermi energy ratio}$$

What do we know about pairing correlations in atomic nuclei?

Odd-even mass staggering gives us estimate of the pairing strength (unfortunately obscured by polarization effects)

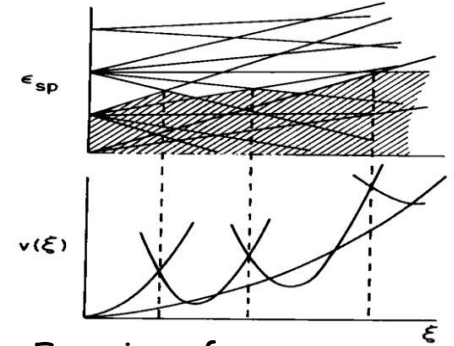
$$|\Delta| \approx \frac{12}{\sqrt{A}} \text{ MeV}$$



A. Johnson, H. Ryde, S.A. Hjorth, Nucl. Phys. A179, 753 (1972)

High spin experimental data: backbending of moments of inertia produced by the alignment of the correlated nucleon pair is a sensitive function of pairing correlations.

Theoretical description of large amplitude nuclear motion require to include pairing correlations.



While a nucleus elongates its Fermi surface becomes oblate and its sphericity must be restored Hill and Wheeler, PRC, 89, 1102 (1953); Bertsch, PLB, 95, 157 (1980)

Can we probe the pairing field phase in nuclei?

Nuclear Josephson junction: enhancement of neutron pair transfer in nuclear collision

V.I. Gol'danskii, A.I. Larkin JETP 53, 1032 (1967)

K. Dietrich, Phys.Lett. B32, 428 (1970)

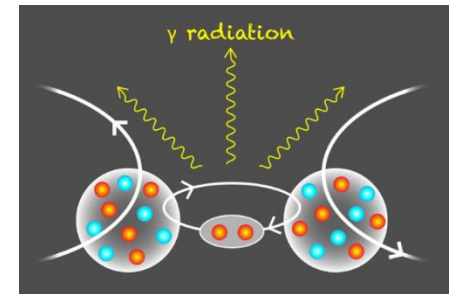
(Unfortunately experimental data are not conclusive)

Recent attempt: oscillatory pair transfer (AC Josephson junction)

C.Potel, F.Barranco, E.Vigezzi, R.A. Broglia, Phys.Rev. C103, L021601(2021)

surprising agreement of gamma spectra with experiment!

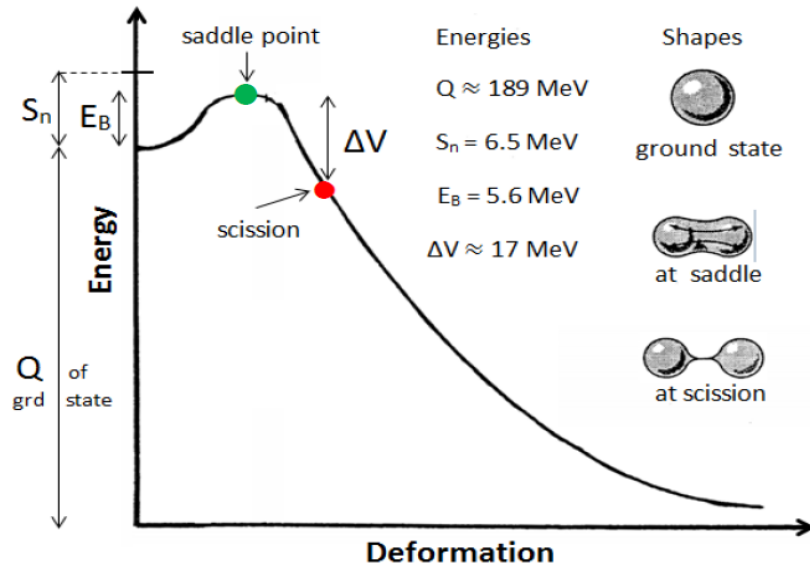
(Although just one reaction: $^{116}\text{Sn} + ^{60}\text{Ni}$ has been studied)



From P.M., Physics 14,27(2021)

Nuclear fission dynamics

Potential energy versus deformation



From F. Gonnemann FIESTA2014

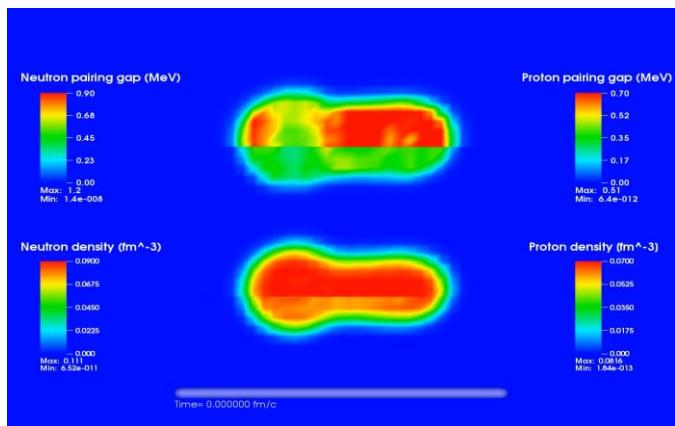
Estimation of characteristic time scales for low energy fission (<10 MeV):

- Ground state to saddle - 1 000 000 zs
- Saddle to scission - 10-100 zs
- Acceleration of fission fragments to 90% of their final velocity - 10 zs
- Neutron evaporation - 1 000 zs

$1 \text{ zs} = 10^{-21} \text{ s}$

Total kinetic energy of the fragments

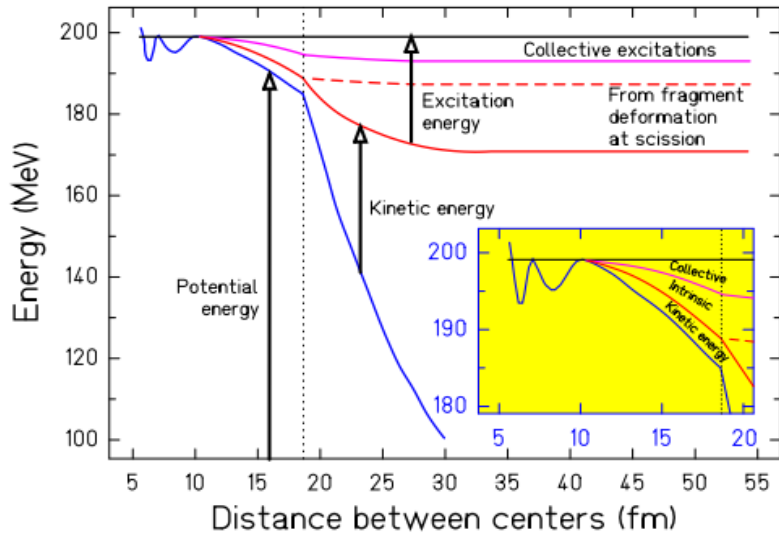
Fission dynamics of ^{240}Pu within TDSLDA



E^* (MeV)	E_n (MeV)	TKE_{TDSLDA} (MeV)	TKE_{syst} (MeV)	err (%)	Z_L	N_L
8.08	1.542	173	177.26	1.95	40.825	62.246
9.60	3.063	174	176.73	1.13	40.500	61.536
10.10	3.560	179	176.56	1.43	41.625	62.783
10.57	4.032	173	176.39	1.55	40.092	61.256
10.58	4.043	173	176.39	1.70	40.146	61.388
10.58	4.047	175	176.39	0.72	40.313	61.475
10.60	4.065	174	176.38	0.92	40.904	62.611
11.07	4.534	176	176.22	0.14	41.495	63.134
11.56	5.024	175	176.05	0.51	40.565	61.894
12.05	5.515	176	175.88	0.49	40.412	61.809
12.15	5.610	176	175.84	0.29	40.355	61.695
12.16	5.626	176	175.84	0.15	41.386	62.764

Calculated TKEs reproduce experimental data with accuracy $< 2\%$

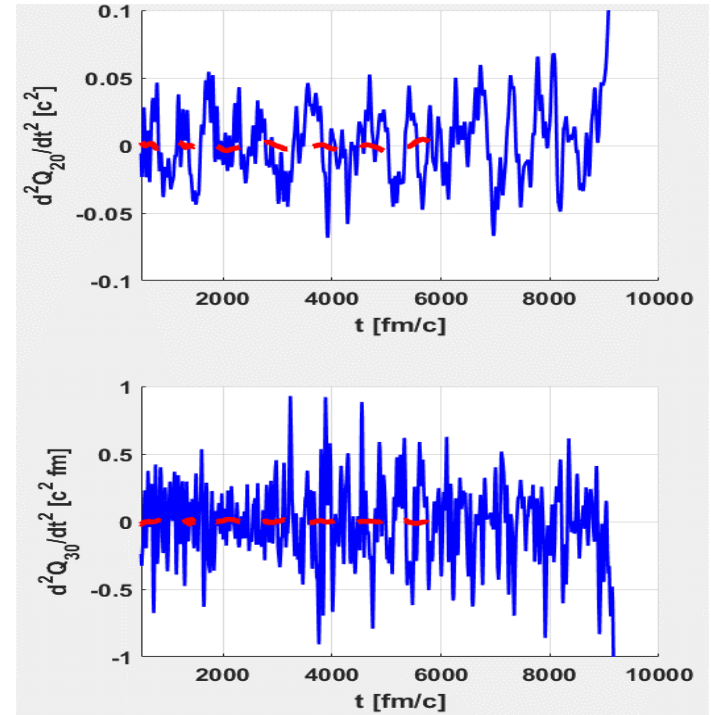
Q: Excitation energy sharing of the fragments



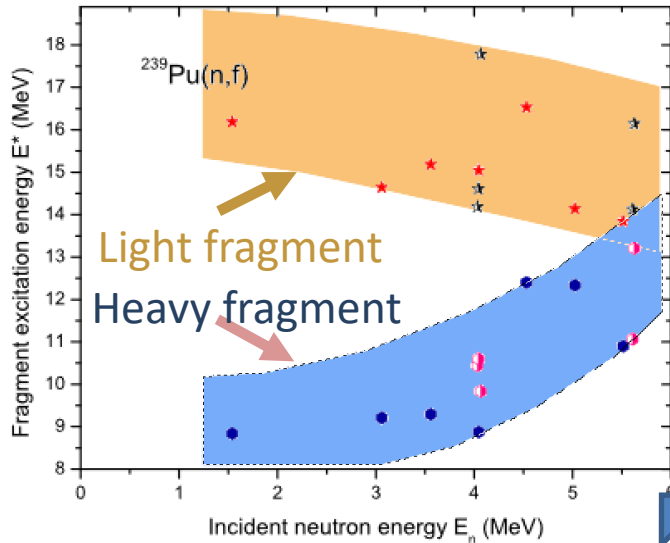
Schmidt&Jurado:Phys.Rev.C83:061601,2011

Character of nuclear motion along the fission path – from TDSLDA

Accelerations in quadrupole and octupole moments



TDSLDA energy sharing between fragments



It is important to realize that these results indicate that the motion is not adiabatic, although it is slow.

Although the average collective velocity is constant till the very last moment before scission, the system heats up as the energy flows irreversibly from collective to intrinsic degrees of freedom.

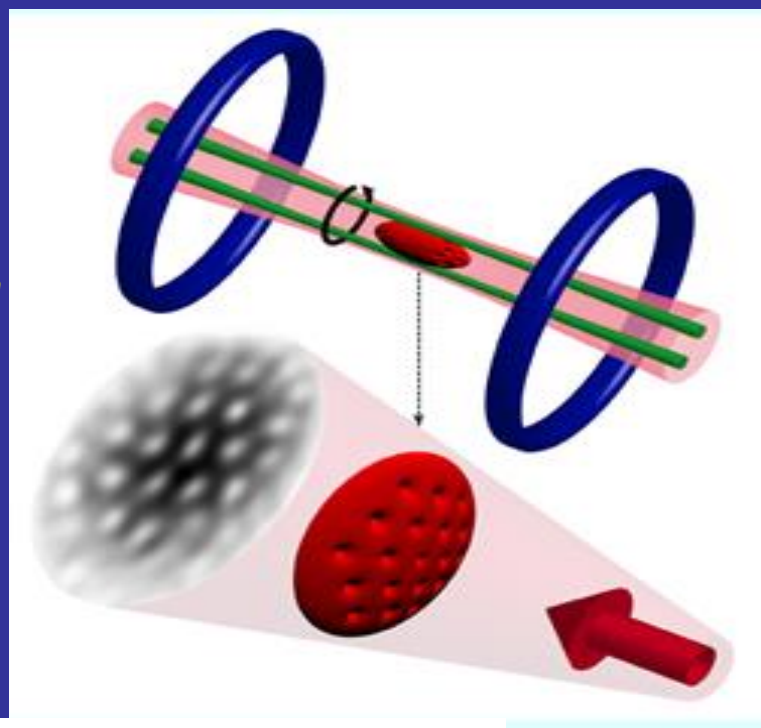
Severe test for the theory – unfortunately no exp. data are available yet.

Ultracold Fermi atoms:

- ✓ In 1999 DeMarco and Jin created a degenerate atomic Fermi gas.
- ✓ In 2005 Zwierlein/Ketterle group observed quantum vortices which survived when passing from BEC to unitarity - evidence for superfluidity!

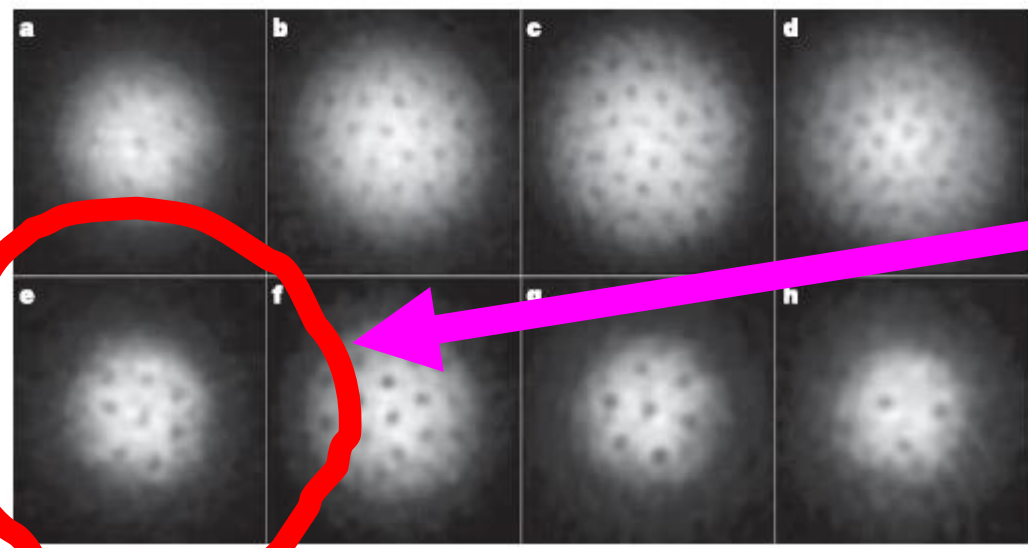
system of fermionic ${}^6\text{Li}$ atoms

Feshbach resonance: $B=834\text{G}$



BEC side:
 $a > 0$

BCS side:
 $a < 0$

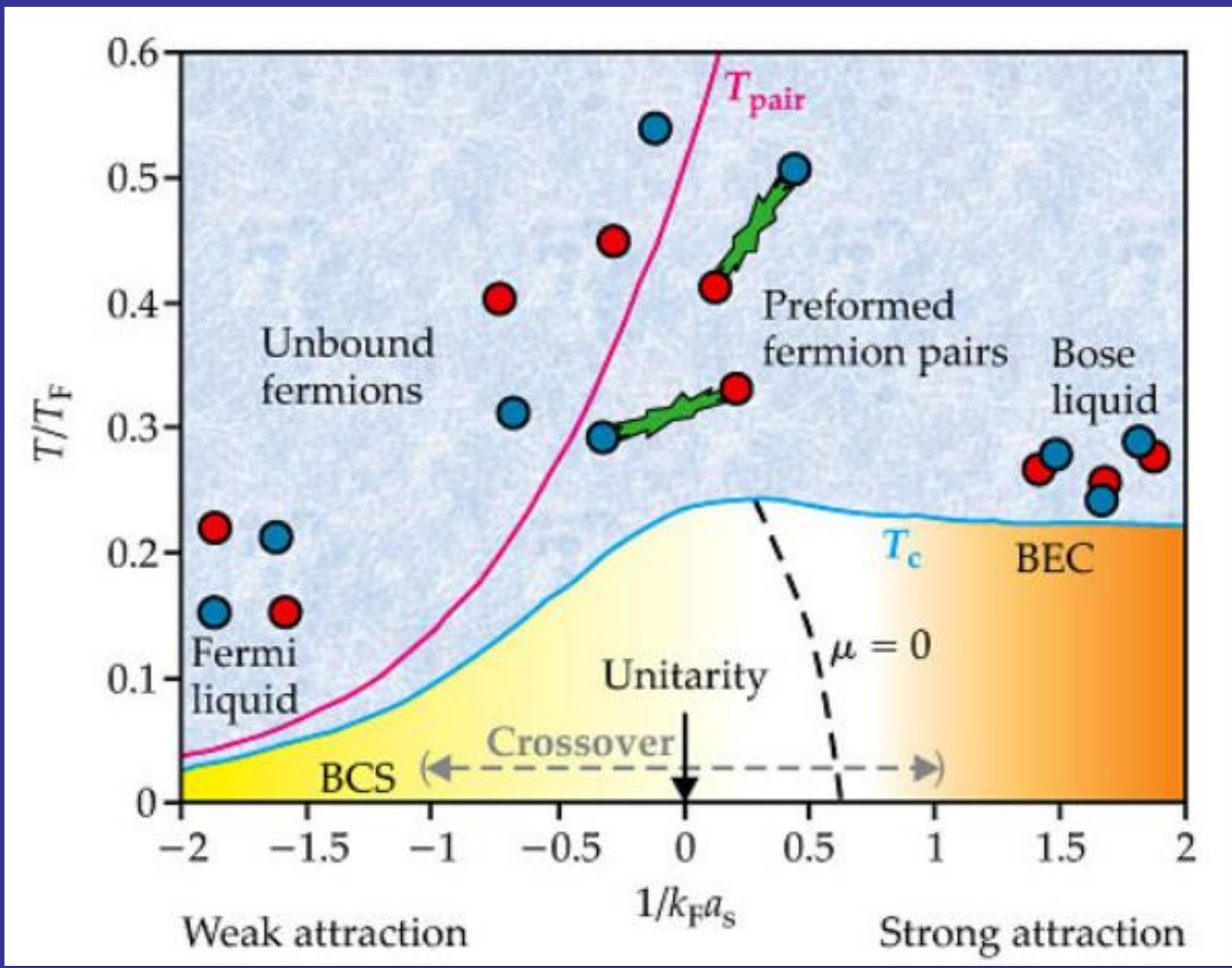


UNITARY REGIME

Figure 2 | Vortices in a strongly interacting Fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) or 500 ms (b–h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the

magnetic field was ramped to 735 G for imaging (see Fig. 1). The magnetic fields were 740 G (a), 766 G (b), 792 G (c), 843 G (f), 853 G (g) and 863 G (h). The field of view is $880 \mu\text{m} \times 880 \mu\text{m}$.

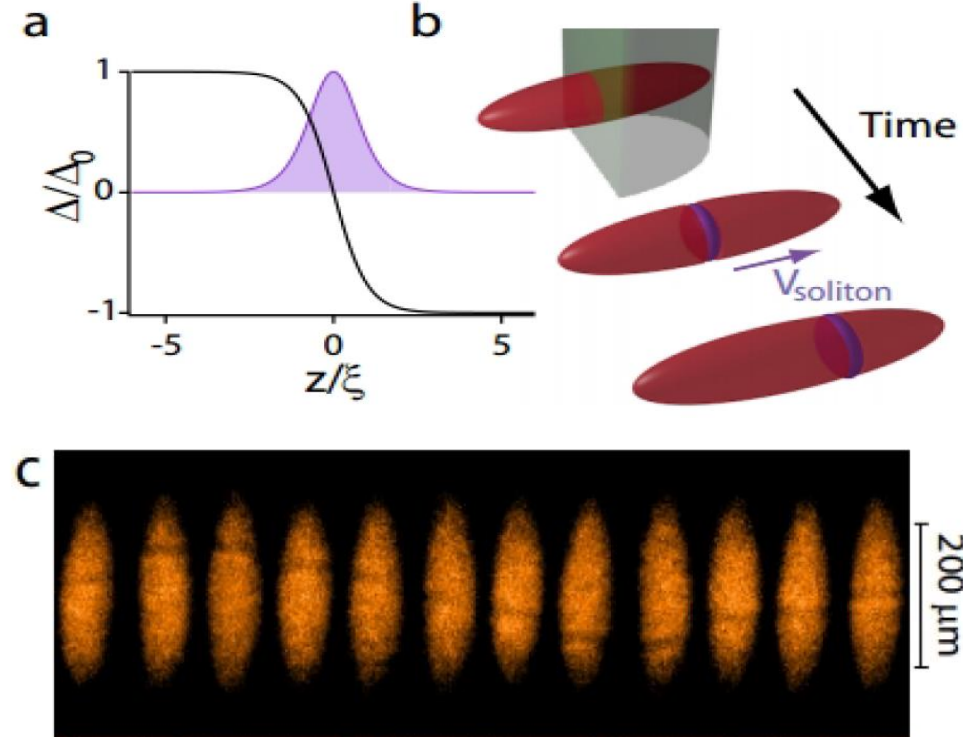
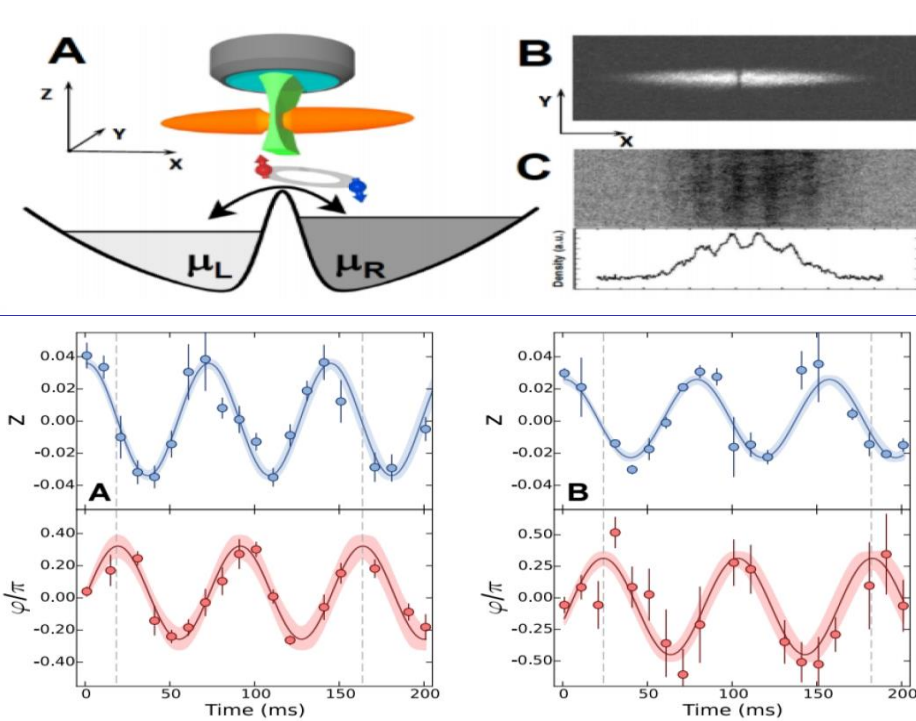
M.W. Zwierlein et al.,
Nature, 435, 1047 (2005)



Two regimes for phase-induced effects in fermionic superfluids

Weak coupling (weak link)

Strong coupling



Observation of **AC Josephson effect** between two 6Li atomic clouds.

G. Valtolina et al., *Science* 350, 1505 (2015).

Superflow is accompanied with creation of topological excitations (vortices) leading to energy dissipation.

G. Wlazłowski, K. Xhani, M. Tylutki, N.P. Proukakis, P. Magierski, *Phys. Rev. Lett.* **130**, 023003 (2023)

Creation of a „heavy soliton“ after merging two superfluid atomic clouds.

T. Yefsah et al., *Nature* 499, 426 (2013);

M.J.H. Ku et al., *Phys. Rev. Lett.* 116, 045304 (2016)

„Heavy soliton“ decays through the unique sequence of topological excitations.

G. Wlazłowski, K. Sekizawa, M. Marchwiany, P. Magierski, *Phys. Rev. Lett.* **120**, 253002 (2018)

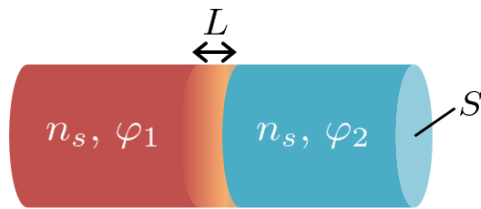
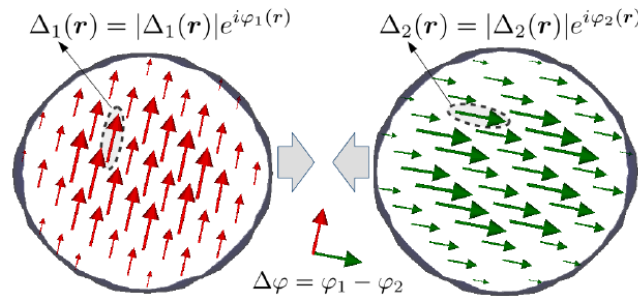
Nuclear collisions

Collisions of superfluid nuclei having different phases of the pairing fields

The main questions are:

- how a possible solitonic structure can be manifested in nuclear system?
- what observable effect it may have on heavy ion reaction:
kinetic energy distribution of fragments, capture cross section, etc.?

Clearly, we cannot control phases of the pairing field in nuclear experiments and the possible signal need to be extracted after averaging over the phase difference.



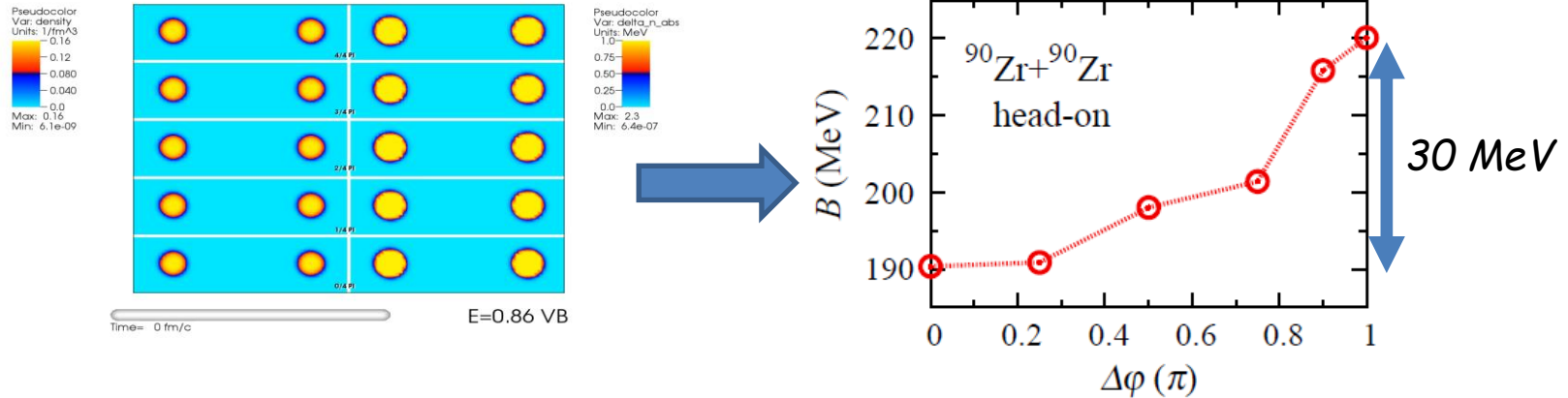
$$\Delta\varphi (\equiv \varphi_1 - \varphi_2)$$

From Ginzburg-Landau (G-L) approach:

$$E_j = \frac{S}{L} \frac{\hbar^2}{2m} n_s \sin^2 \frac{\Delta\varphi}{2}$$

For typical values characteristic for two medium nuclei: $E_j \approx 30\text{MeV}$

Effective barrier height for fusion as a function of the phase difference



What is an average extra energy needed for the capture?

$$E_{extra} = \frac{1}{\pi} \int_0^{\pi} (B(\Delta\phi) - V_{Bass}) d(\Delta\phi) \approx 10 \text{ MeV}$$

The effect is found (within TDDFT) to be of the order of 30 MeV for medium nuclei and occur for energies up to 20-30% of the barrier height.

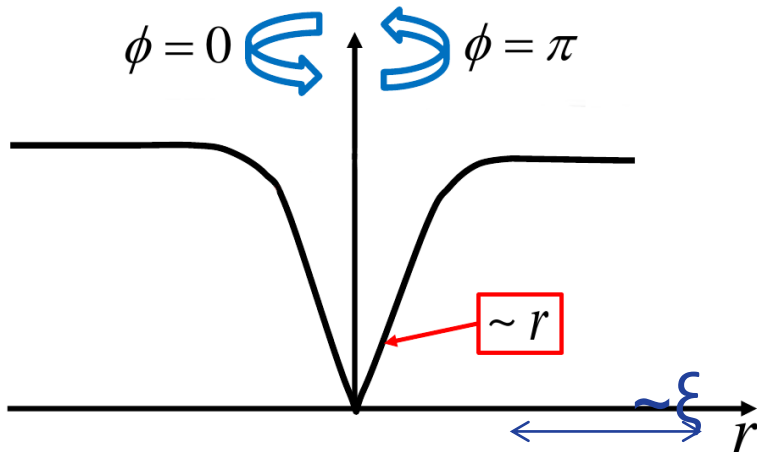
P. Magierski, K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119 042501 (2017)

It raises (again) an interesting (and well known) question:
to what extent systems of hundreds of particles can be described using the concept of pairing field?

G. Scamps, Phys. Rev. C 97, 044611 (2018): barrier fluctuations extracted from experimental data indicate that the effect exists although is weaker than predicted by TDDFT

Anatomy of the vortex core

BOSONS: $\Psi(\vec{r}) = |\Psi(\vec{r})| e^{i\phi(\vec{r})}$



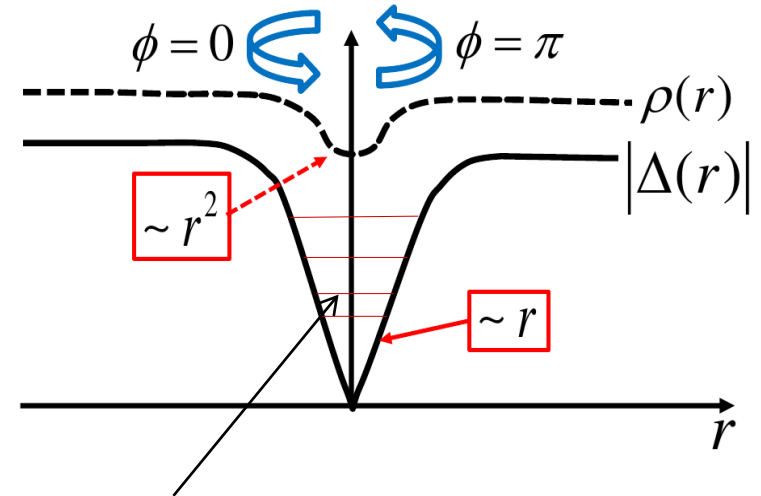
Order parameter:

$$\Psi = \sqrt{\rho(r)} e^{i\phi}$$

$$\mathbf{v}_s = \frac{\hbar}{M} \nabla \phi \quad \kappa = \oint d\mathbf{l} \cdot \mathbf{v}_s = \frac{h}{M}$$

At T=0 the core is empty

FERMIONS: $\Delta(\vec{r}) = |\Delta(\vec{r})| e^{i\phi(\vec{r})}$



Andreev states affect the density distribution inside the core.

Order parameter: $\Delta(\vec{r}, t) = |\Delta(\vec{r}, t)| e^{i\phi(\vec{r}, t)}$
not related directly to density

The core is not empty!

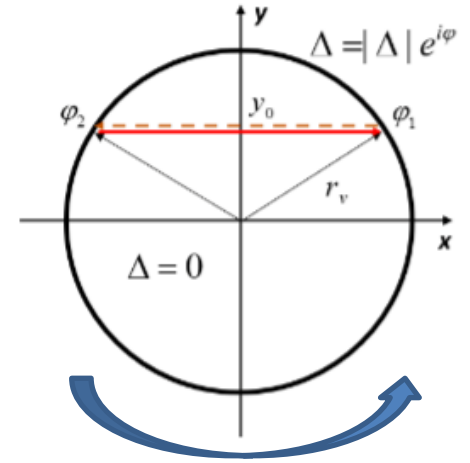
Vortex core structure in Andreev approximation:

$$\frac{E(0, L_z)}{\varepsilon_F} k_F r_V \sqrt{1 - \left(\frac{L_z}{k_F r_V}\right)^2} + \arccos\left(\frac{-L_z}{k_F r_V}\right) - \arccos\left(\frac{E(0, L_z)}{|\Delta_\infty|}\right) = 0$$

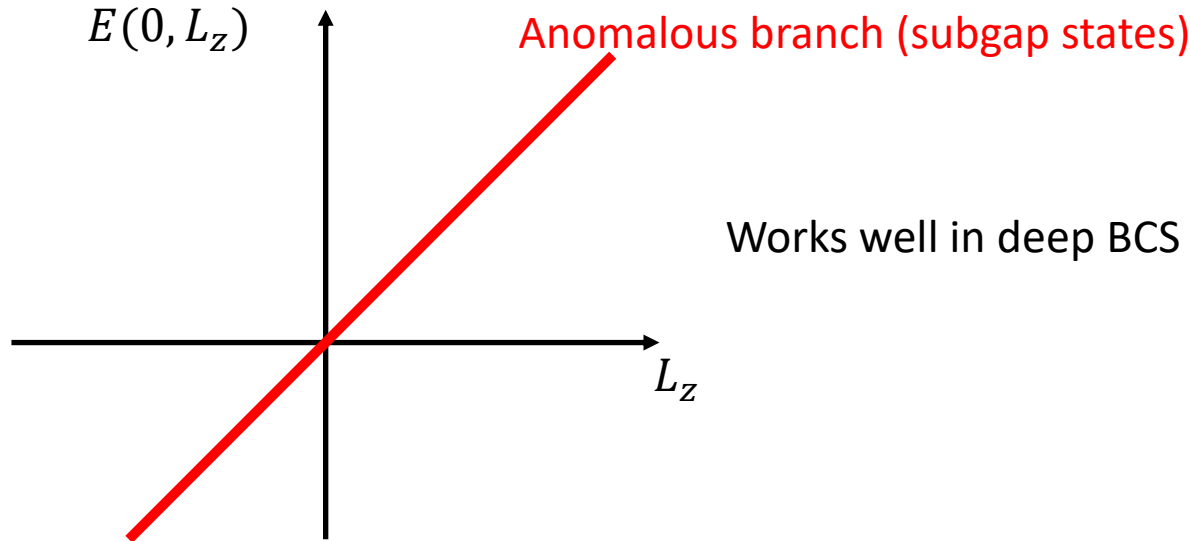
$$E(0, L_z) = E(0)L_z, \quad E \ll |\Delta_\infty|$$

$$E(0, L_z) \approx \frac{|\Delta_\infty|^2}{\varepsilon_F \frac{r_V}{\xi} \left(\frac{r_V}{\xi} + 1\right)} \frac{L_z}{\hbar}, \quad \xi = \frac{\varepsilon_F}{k_F |\Delta_\infty|}$$

Schematic section of the core

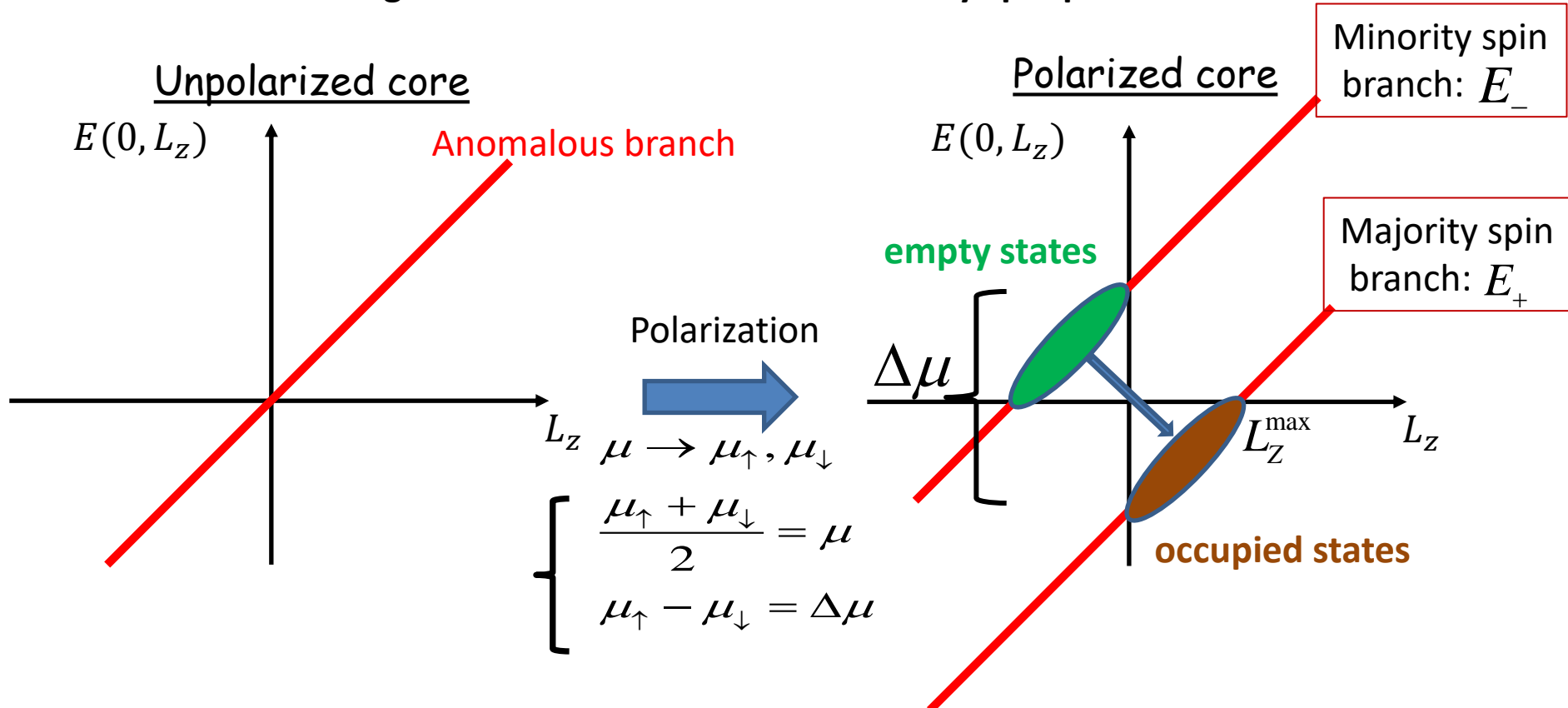


Spectrum of in-gap states



Works well in deep BCS limit: $\frac{1}{k_F a_S} \ll 0$

Changes of the core structure induced by spin polarization



Branches are split proportionally to polarization

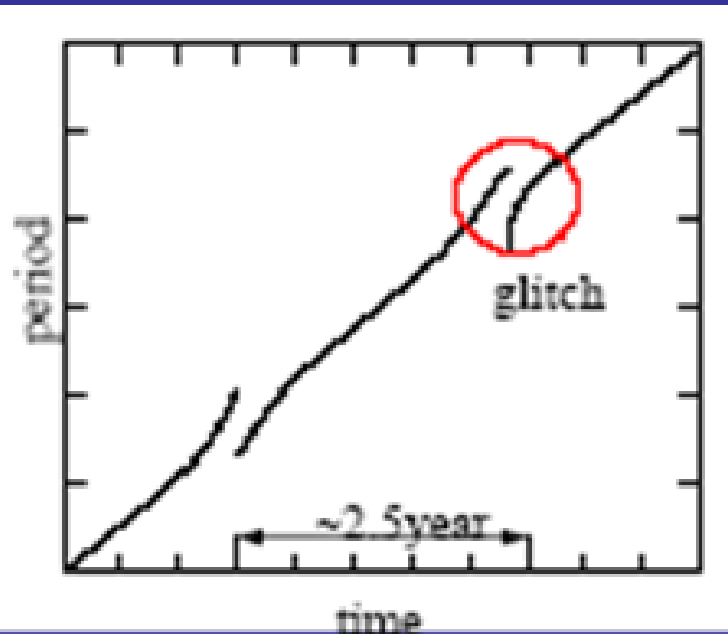
$$E_{\pm}(0, L_z) \approx \frac{|\Delta_{\infty}|^2}{\varepsilon_F \frac{r_V}{\xi} \left(\frac{r_V}{\xi} + 1 \right)} \frac{L_z}{\hbar} \mp \frac{\Delta\mu}{2}$$

Certain fraction of majority spin particles rotate in the opposite direction!

$$L_z^{\max} \approx \frac{1}{2} \frac{\varepsilon_F}{|\Delta_{\infty}|^2} \frac{r_V}{\xi} \left(\frac{r_V}{\xi} + 1 \right) \hbar \Delta\mu$$

Neutron stars and quantum turbulence

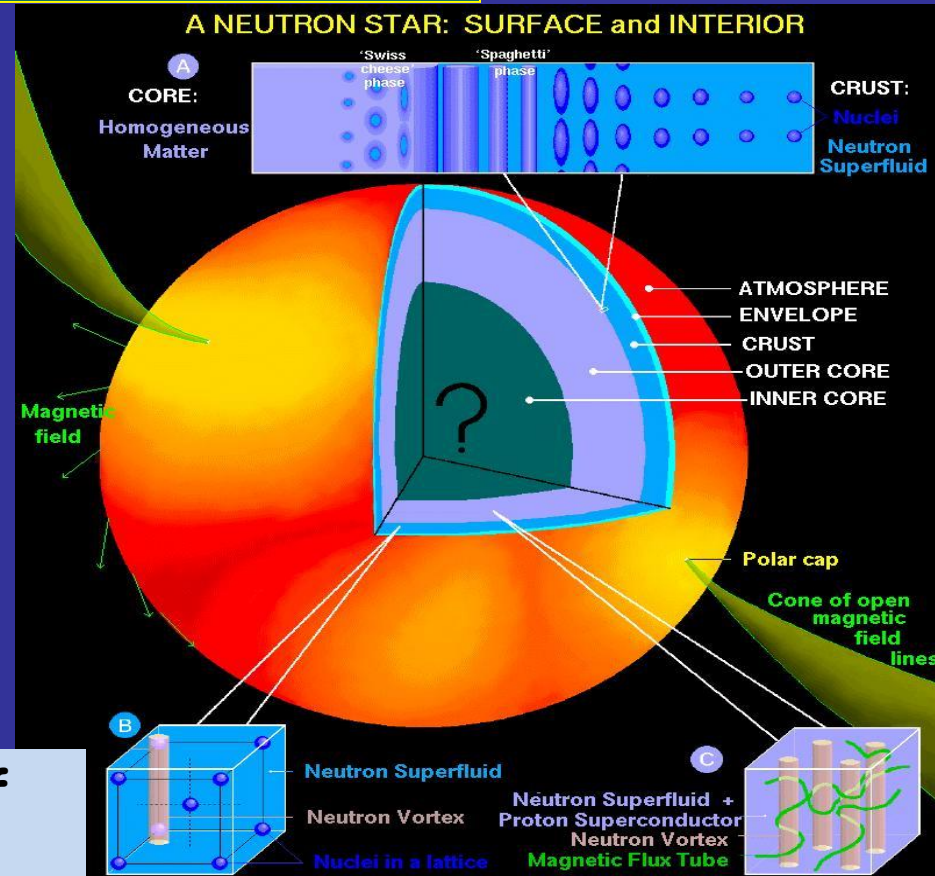
Neutron star is a huge superfluid



glitch phenomenon = a sudden speed up of rotation.

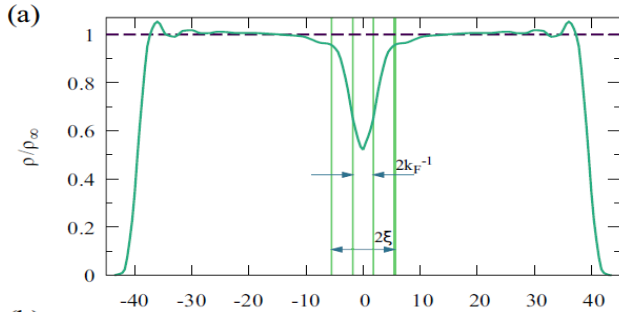
To date more than 300 glitches have been detected in more than 100 pulsars

Glitch phenomenon is commonly believed to be related to rearrangement of vortices in the interior of neutron stars. It would require however a correlated behavior of huge number of quantum vortices and the mechanism of such collective rearrangement is still a mystery.

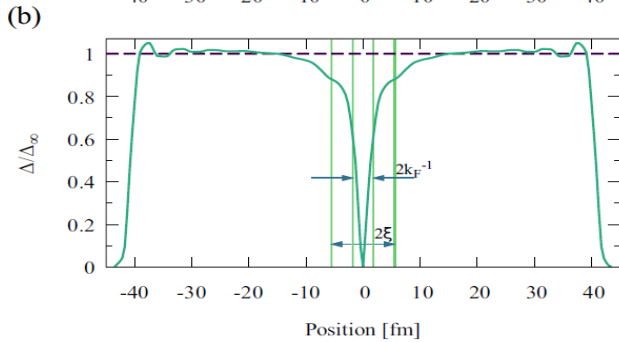


Example: vortices across the neutron star crust

Section through the vortex core



Normal density

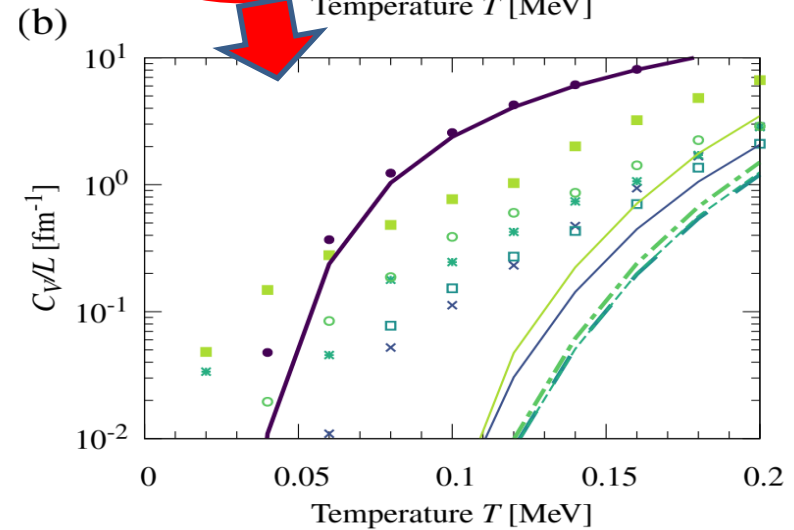
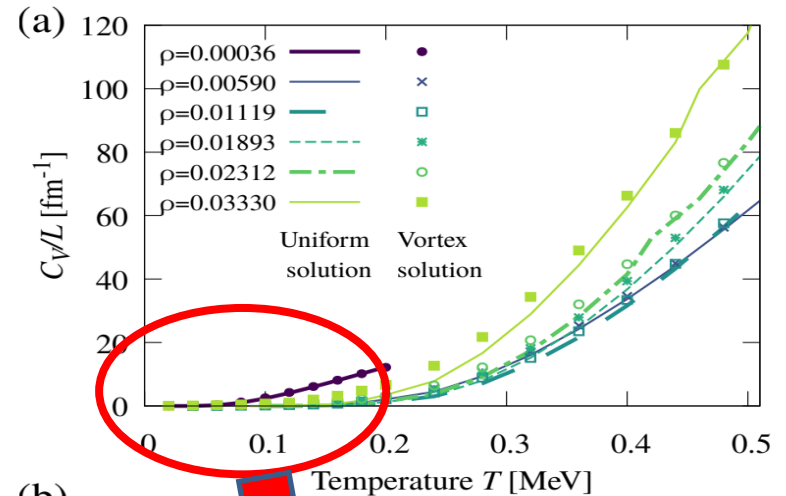


Pairing field

Note two different length scales inside the core as explained by:
Sensarma, Randeria, Ho,
Phys. Rev. Lett. 96, 090403 (2006)

ρ_∞ (fm ⁻³)	0.00036	0.0059	0.0112	0.0189	0.0231	0.0333
k_F^{-1} (fm)	4.52	1.79	1.45	1.21	1.14	1.01
ξ (fm)	8.44	5.53	5.97	7.00	7.78	10.28
R_{VFM} (fm)	15.0	10.5	10.5	12.0	13.5	16.5
Δ_∞ (MeV)	0.35	1.33	1.53	1.55	1.50	1.28
T_{crit} (MeV)	0.20	0.76	0.87	0.88	0.85	0.73
ε_F (MeV)	1.01	6.48	9.93	14.09	16.10	20.53
μ (MeV)	0.80	4.21	5.80	7.30	7.91	9.09
E_{mg} (MeV)	0.090	0.308	0.261	0.199	0.152	0.009
B_{crit} (10 ¹⁵ G)	7.76	26.5	22.5	17.2	13.1	0.82

Specific heat contribution vs uniform matter



Minigap values

Magnetic field needed to polarize the core

How can we measure the influence of core states in ultracold gases?

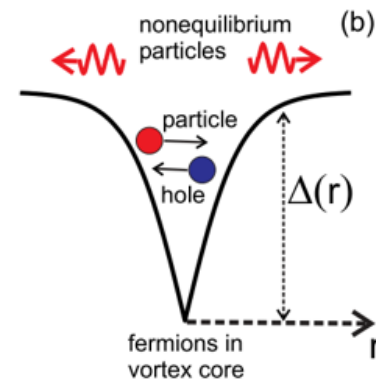
Dissipative processes involving vortex dynamics.

- Silaev, Phys. Rev. Lett. 108, 045303 (2012)
- Kopnin, Rep. Prog. Phys. 65, 1633 (2002)
- Stone, Phys. Rev. B54, 13222 (1996)
- Kopnin, Volovik, Phys. Rev. B57, 8526 (1998)

....

Classical treatment of states in the core (Boltzmann eq.).

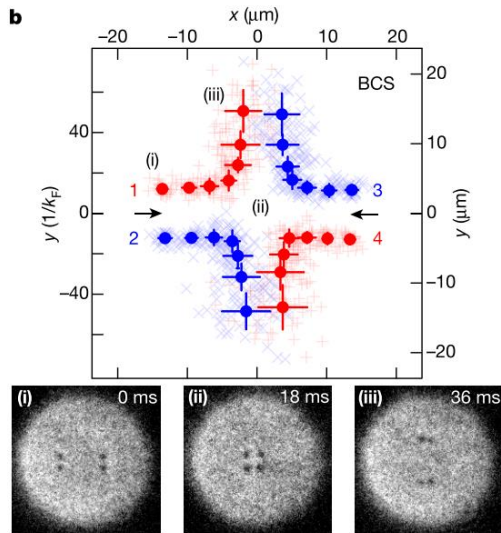
More applicable in deep BCS limit unreachable in ultracold atoms.



Vortex-antivortex scattering in 2D

„Further, our few-vortex experiments extending across different superfluid regimes reveal non-universal dissipative dynamics, suggesting that fermionic quasiparticles localized inside the vortex core contribute significantly to dissipation, thereby opening the route to exploring new pathways for quantum turbulence decay, vortex by vortex.”

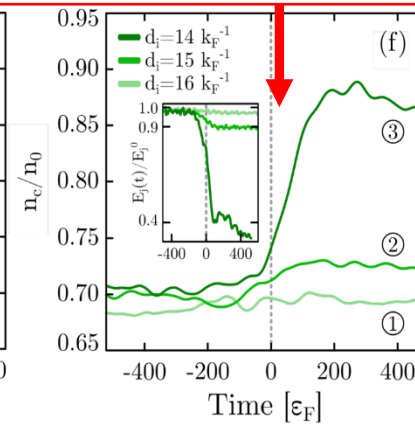
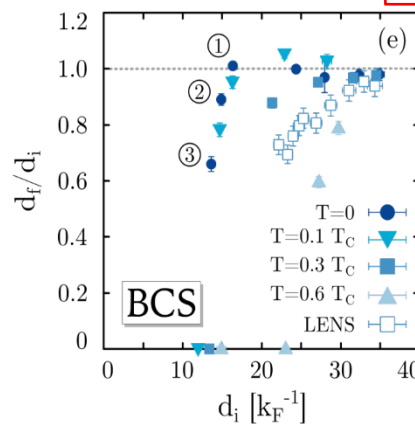
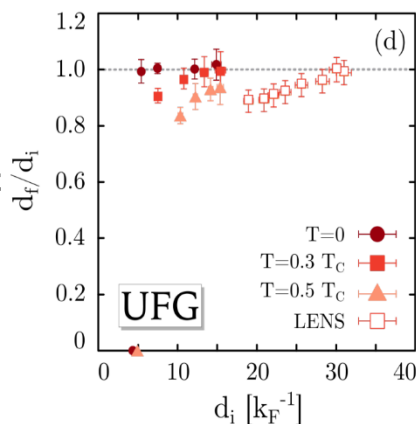
W.J. Kwon et al. Nature **600**, 64 (2021)



Exciting quasiparticles in the vortex core

Indeed quasiparticles in the core are excited due to vortex acceleration but the effect is too weak to account for the total dissipation rate.

A. Barresi, A. Boulet, P.M., G. Wlazłowski, Phys. Rev. Lett. 130, 043001 (2023)



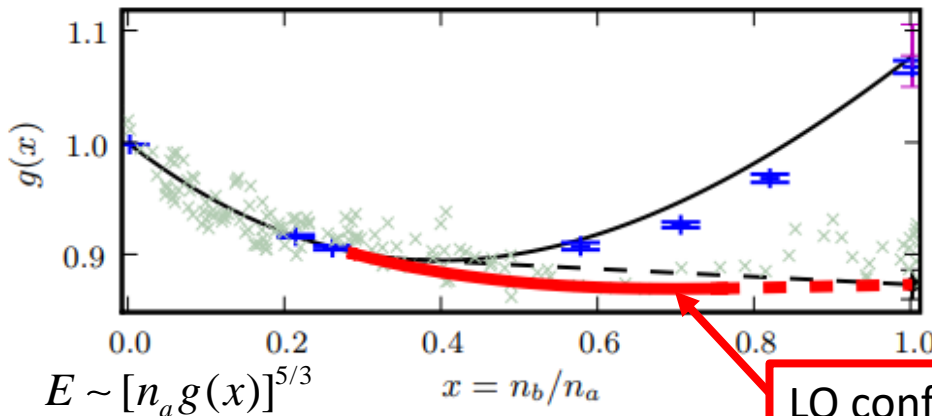
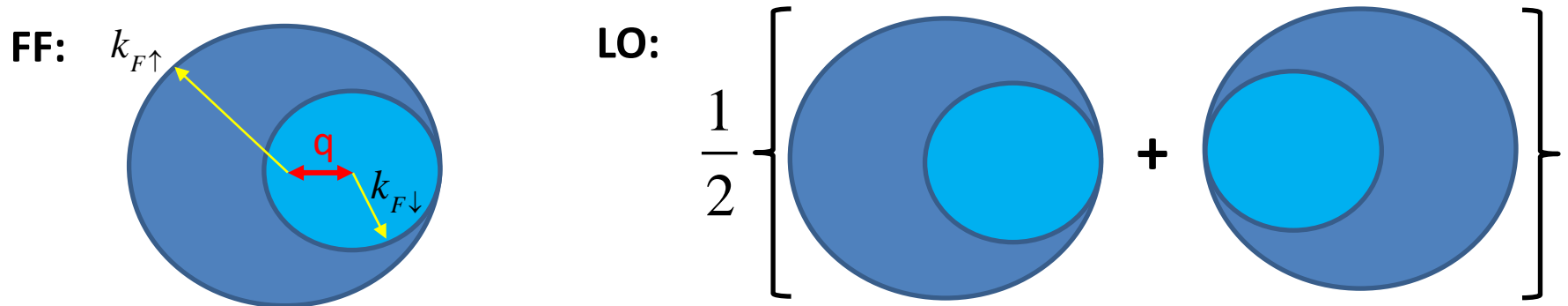
Inhomogeneous systems: Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase

Larkin-Ovchinnikov (LO): $\Delta(r) \sim \cos(\vec{q} \cdot \vec{r})$

Fulde-Ferrell (FF): $\Delta(r) \sim \exp(i\vec{q} \cdot \vec{r})$

A.I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP 20, 762 (1965)
 P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964)

Spatial modulation of the pairing field cost energy proportional to q^2 but may be compensated by an increased pairing energy due to the mutual shift of Fermi spheres:



Bulgac & Forbes have shown, within DFT, that Larkin-Ovchinnikov (LO) phase may exist in the unitary Fermi gas (UFG) (realized experimentally in ultracold atomic clouds)

LO configuration – supersolid state

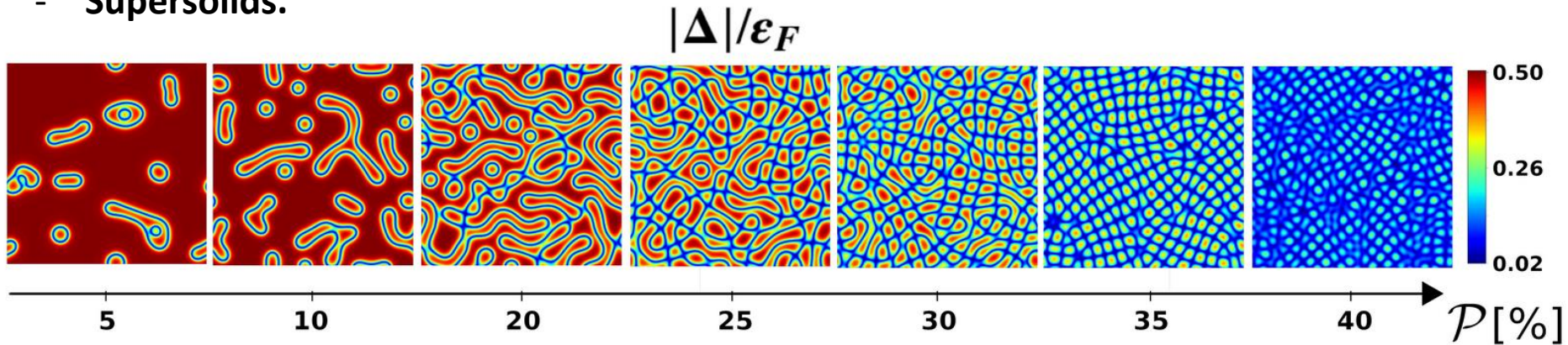
A. Bulgac, M.M.Forbes, Phys. Rev. Lett. 101,215301 (2008)

See also review of mean-field theories : Radzihovsky,Sheehy, Rep.Prog. Phys.73,076501(2010)

What is going to happen if we keep increasing spin imbalance?

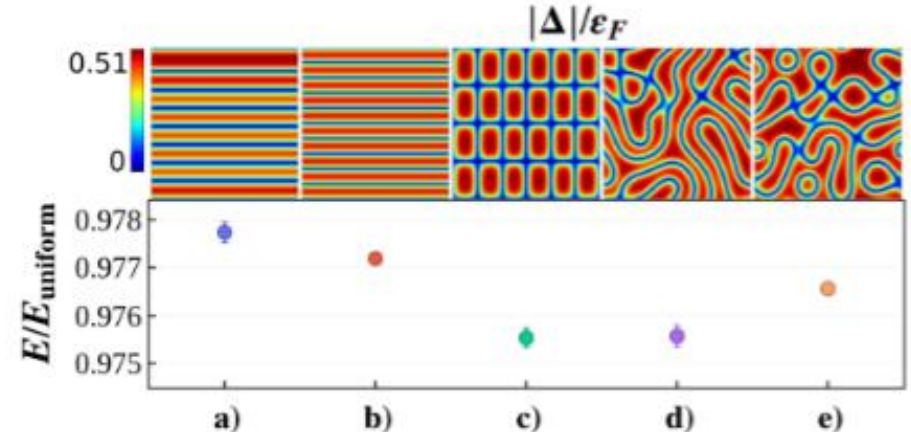
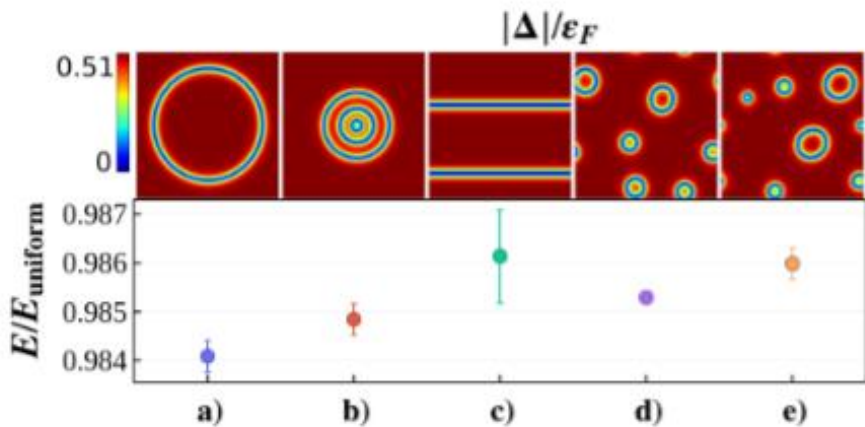
In general it will generate distortions of Fermi spheres locally and triggering the appearance of **pairing field inhomogeneity** leading to various patterns involving:

- **Separate impurities (fermons),**
- **Liquid crystal-like structure,**
- **Supersolids.**

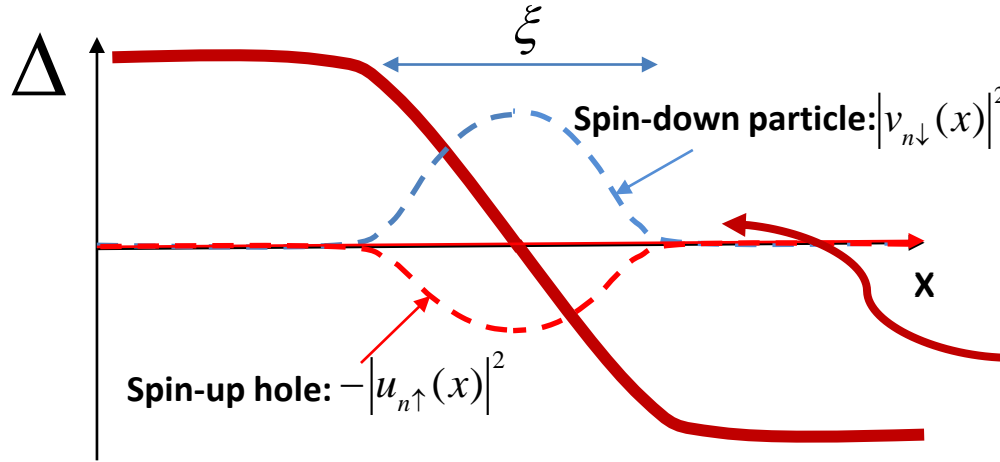


B. Tüzemen, T. Zawiślak, P.M., G. Wlazłowski, New J. Phys. 25, 033013 (2023)

$$P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$



Andreev states and stability of pairing nodal points



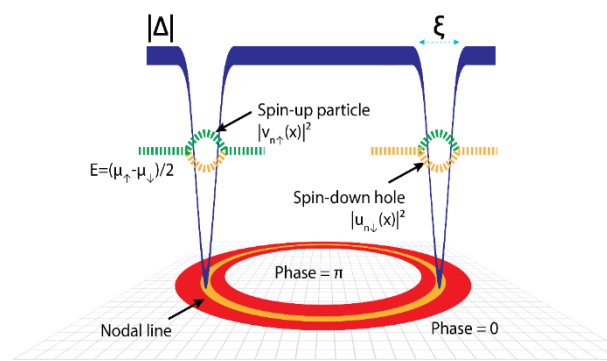
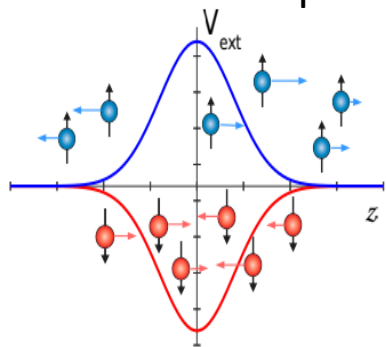
Due to quasiparticle scattering the localized Andreev states appear at the nodal point. These states induce local spin-polarization

BdG in the Andreev approx. ($\Delta \ll k_F^2$)

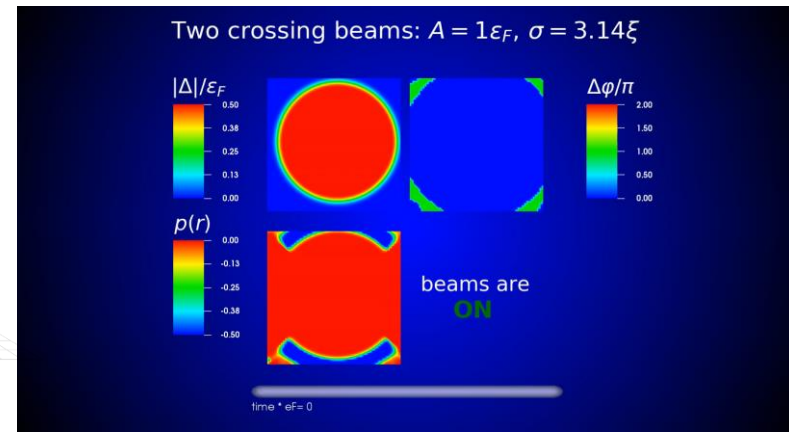
$$\begin{bmatrix} -2ik_F \frac{d}{dx} & \Delta(x) \\ \Delta^*(x) & 2ik_F \frac{d}{dx} \end{bmatrix} \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix} = E_n \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix}$$

Engineering the structure of nodal surfaces

Apply the spin-selective potential of a certain shape:



Wait until the proximity effects of the pairing field generate the nodal structure and remove the potential.



Non-central collision of two impurities



Moving impurity:

From Larkin-Ovchinnikov towards Fulde-Ferrell limit:

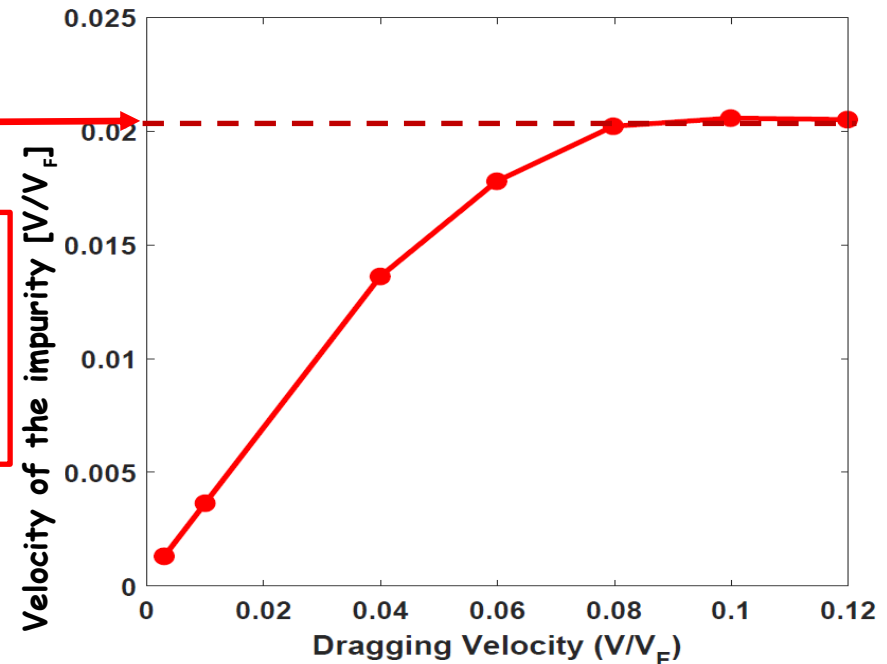
$$\Delta(r) : \cos(qr) \Rightarrow \exp(iqr)$$

Surprisingly, the nodal structure remains stable even during collisions

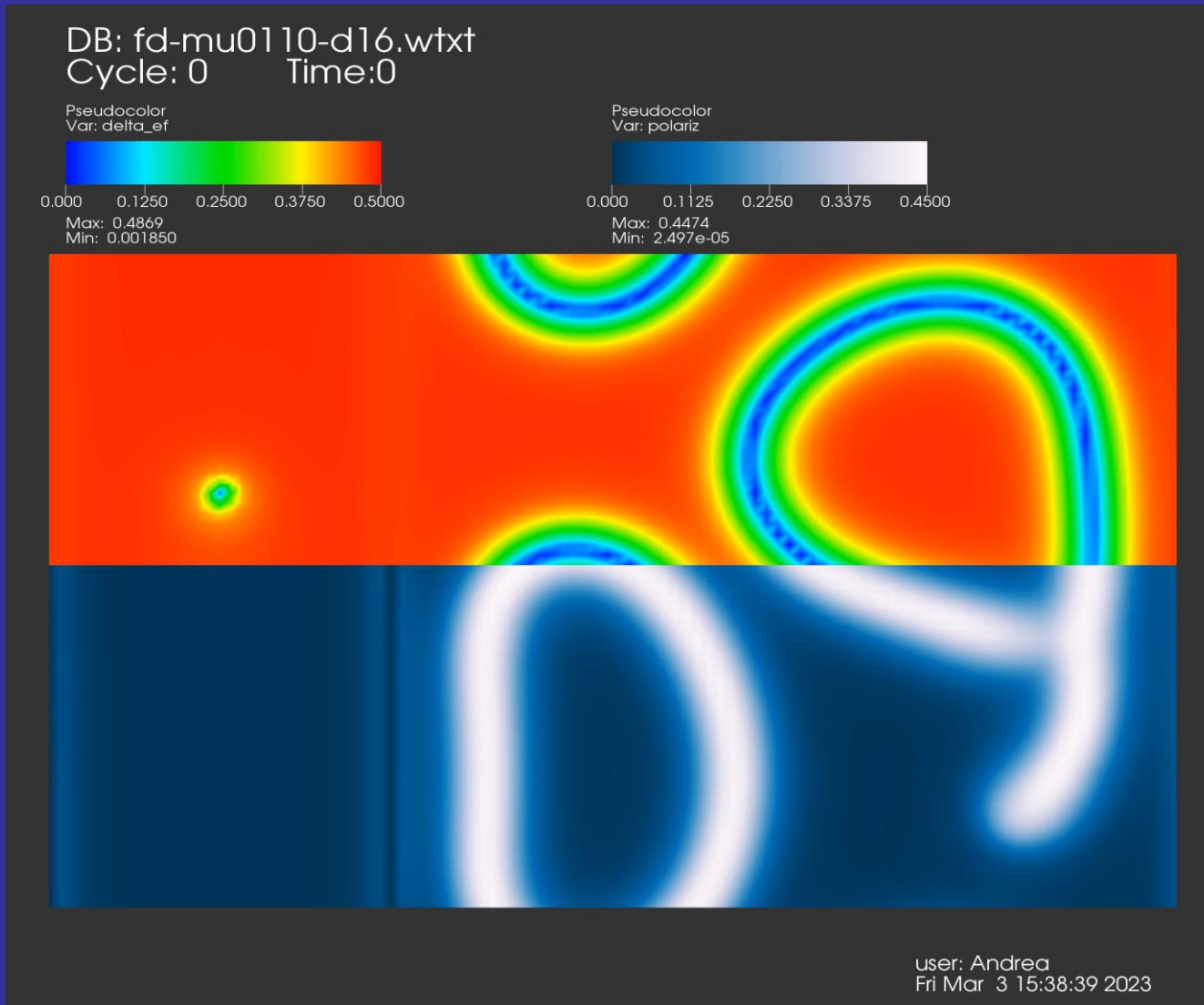
The velocities of impurities are about 30% of the velocity of sound.

Limiting velocity with respect to superfluid background

Note that the Fulde-Ferrell limit defines the **critical velocity** which is associated with the maximum spin current that can flow through the impurity ($\sim q \sim |k_{F\uparrow} - k_{F\downarrow}|$).



Complex dynamics (strongly damped) of vortices in the spin imbalanced environment



Thanks to A. Barresi *et al.*

THANK YOU

Summary

TDSLDA extended to superfluid systems and based on the local densities offers a flexible tool to study quantum superfluids far from equilibrium.

Open problems

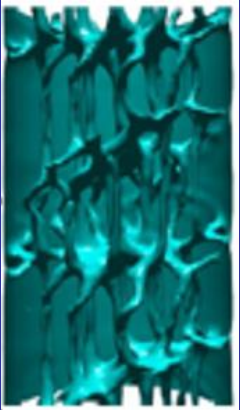
- 1) There are easy and difficult observables in DFT.
In general: easy observables are one-body observables. They are easily extracted and reliable.
- 2) But there are also important observables which are difficult to extract.
For example:
 - S matrix
 - momentum distributions
 - transitional densities (defined in linear response regime)
 - various conditional probabilities
 - mass distributions

Stochastic extensions of TDDFT are under investigation:

D. Lacroix, A. Ayik, Ph. Chomaz, Prog.Part.Nucl.Phys.52(2004)497

S. Ayik, Phys.Lett. B658 (2008) 174

- 3) Dissipation: transition between one-body dissipation regime and two-body dissipation regime.

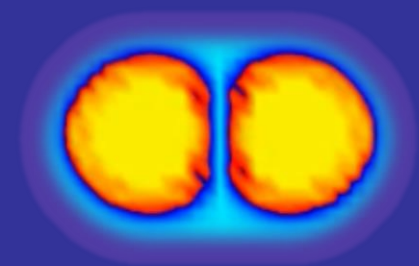


Quantum turbulence

K. Hossain (WSU)
M.M. Forbes (WSU)
K. Kobuszewski (WUT)
S. Sarkar (WSU)
G. Wlazłowski (WUT)

Vortex dynamics in neutron star crust

N. Chamel (ULB)
D. Pęcak (WUT)
G. Wlazłowski (WUT)



Nuclear collisions

M. Barton (WUT)
A. Boulet (WUT)
A. Makowski (WUT)
K. Sekizawa (Tokyo I.)
G. Wlazłowski (WUT)



Nonequilibrium superfluidity in Fermi systems

Josephson junction in atomic Fermi gases - dissipative effects

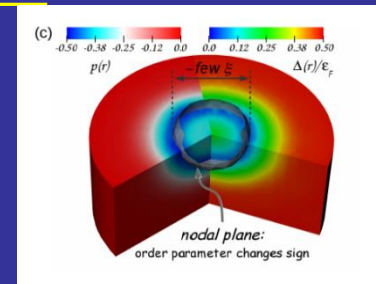
N. Proukakis (NU)
M. Tylutki (WUT)
G. Wlazłowski (WUT)
K. Xhani (LENS & NU)
and LENS exp. Group

Collisions of vortex-antivortex pairs

A. Barresi (WUT)
A. Boulet (WUT)
G. Wlazłowski (WUT)
and LENS exp. Group

Spin-imbalanced Fermi gases

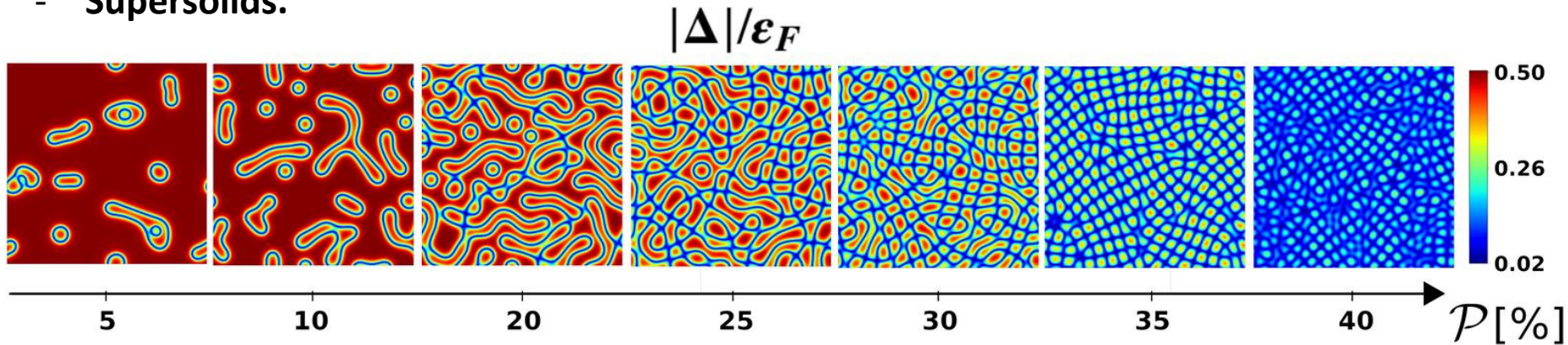
B. Tuzemen (WUT)
G. Wlazłowski (WUT)
T. Zawiślak (WUT)



What is going to happen if we introduce spin imbalance?

In general it will generate distortions of Fermi spheres locally and triggering the appearance of **pairing field inhomogeneity** leading to various patterns involving:

- **Separate impurities (ferrons),**
- **Liquid crystal-like structure,**
- **Supersolids.**



B. Tüzemen, T. Zawiślak, P.M., G. Wlazłowski, New J. Phys. 25, 033013 (2023)

$$P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$

Suppose however that polarization is weak enough, so it does not affect the bulk and only the core of the vortex.

Note that due to the fact that: $E_{mg} < \Delta_{\infty}$

the core will always be affected by polarization before the bulk will respond. It implies also that the vortex core will „suck in” the majority spin particles from the bulk whenever such possibility occurs.

Two consequences of vortex core polarization:

- 1) Minigap vanishes.
- 2) Direction of the current in the core reverses.

- 1) Since the polarization correspond to relative shift of anomalous branches therefore the quasiparticle spectrum of spin-up and spin-down components is asymmetric for $k_z = 0$.

However the symmetry of the spectrum has to be restored in the limit of $k_z \rightarrow \infty$. Since for a straight vortex one can decouple the degree of freedom along the vortex line:

$$H = \begin{pmatrix} h_{2D}(\mathbf{r}) + \frac{1}{2}k_z^2 - \mu_{\uparrow} & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h_{2D}^*(\mathbf{r}) - \frac{1}{2}k_z^2 + \mu_{\downarrow} \end{pmatrix}$$

therefore $E(k_z) \propto \pm k_z^2$ when $k_z \rightarrow \infty$

As a result there must exist a sequence of values: $k_z = \pm k_{z1}, \pm k_{z2}, \dots$ for which:

$$E(\pm k_{z_i}) = 0$$