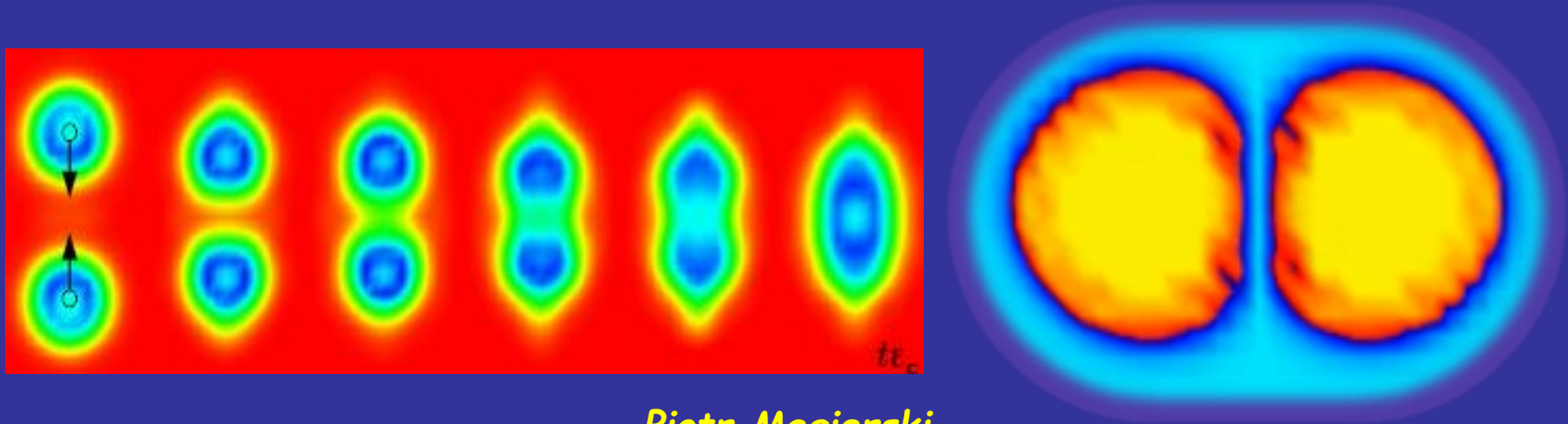


# *Exotic features of superfluid dynamics and collisions: stable and unstable pairing nodal structures*

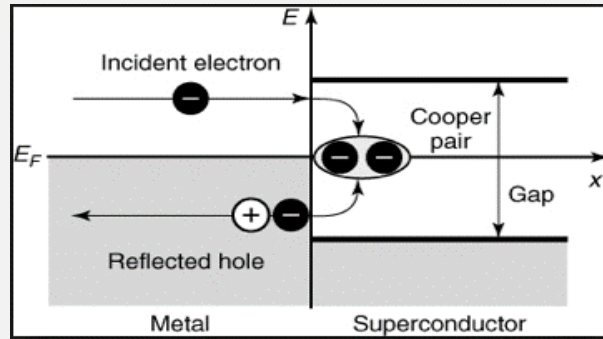


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Buğra Tüzemen (WUT - Ph.D. student)  
Marek Tylutki (WUT)  
Gabriel Wlazłowski (WUT)

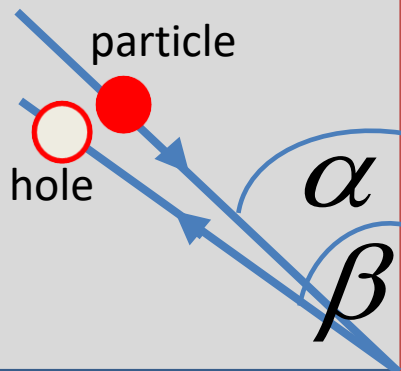
# Pairing induced quantized states

Scattering of particles/holes on the pairing potential:



Schematic idea of Andreev reflection

## Normal metal



## Superconductor

Andreev reflection law:

$$\sqrt{\varepsilon_F + E} \sin \alpha = \sqrt{\varepsilon_F - E} \sin \beta$$

### Consequences:

Andreev reflection gives rise to the appearance of localized quantized states - *Andreev states*.

Due to the properties of Andreev reflection “shell effects” induced by Andreev states are quite strong (Andreev reflection effectively reduces 3D problem to 1D problem)

Andreev reflection provides an effective mechanism to *localize states* at the Fermi surface in inhomogenous systems.

(see: P. Magierski, Phys. Rev. C75, 012803(R), 2007)

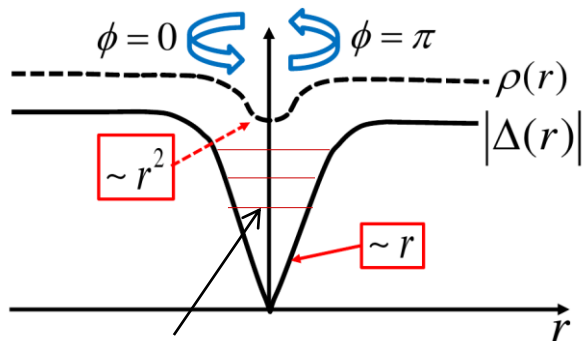
It generates strong Casimir-like force between superfluid grains.

(see: A. Bulgac, P. Magierski, A. Wirzba, Eur. Lett.72,327,2005)

# Andreev states and anatomy of the vortex core

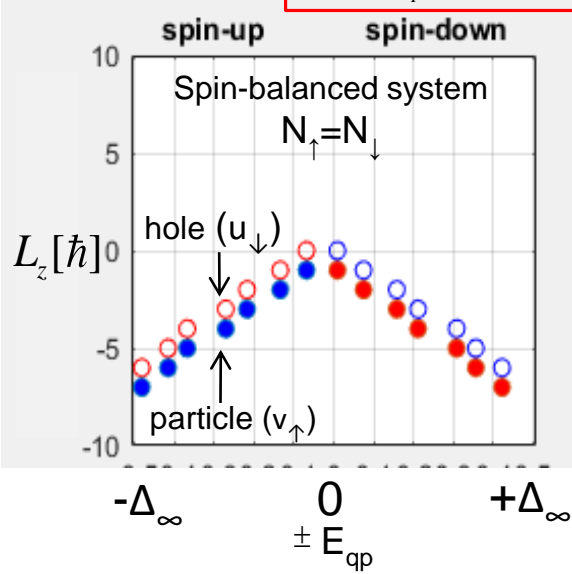
## FERMIONS:

Vortex structure: Fermi gas → BdG eq.



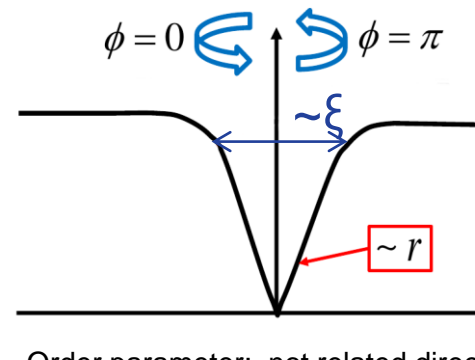
Andreev states inside the core give rise to **anomalous branch of excitations** (of chiral fermions):

$$E_m \approx \frac{4}{3} \frac{|\Delta|^2}{\varepsilon_F} m ; |m|=1,2,\dots$$



## BOSONS:

Vortex structure: Bose gas → Gross-Pitaevskii eq. (GPE)



Order parameter:

$$\Psi = \sqrt{\rho(r)} e^{i\phi}$$

$$\mathbf{v}_s = \frac{\hbar}{M} \nabla \phi$$

$$\kappa = \oint dl \cdot \mathbf{v}_s = \frac{h}{M}$$

Order parameter: not related directly to density

## Consequences:

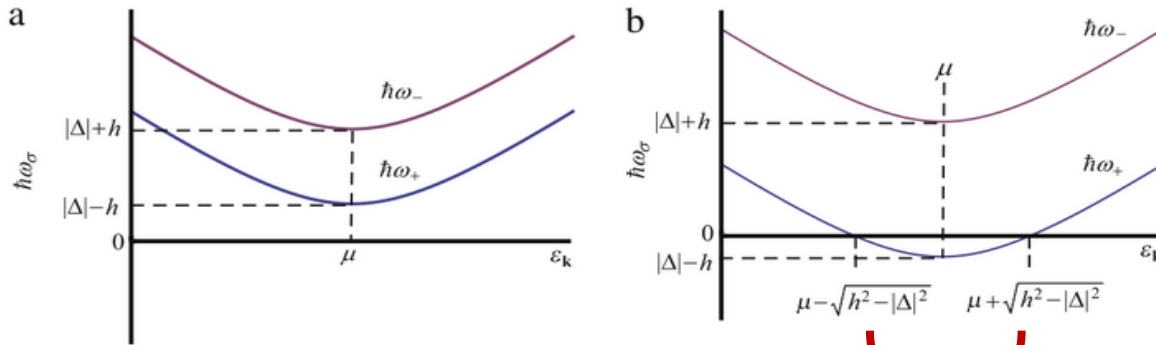
- Long range interaction in the system of vortices is the same for bosons and fermions, as it is governed by the superfluid flow  $v_s$
- **Short range physics** (e.g. reconnection rate) is **different** due to the population of Andreev states in the core. It significantly modifies **the decay of the turbulent state** (Wlazłowski, Kobuszewski, Sekizawa, Magierski, in preparation)
- Note that Andreev states define the energy scale:

$$\delta E \approx \frac{4}{3} \frac{|\Delta|^2}{\varepsilon_F} < |\Delta| - \text{minigap, which affects thermal and dissipative properties of the system of vortices.}$$

# Pairing in spin imbalanced superfluids

Clogston-Chandrasekhar condition sets the limit for the chemical potential difference at which superfluidity is lost:

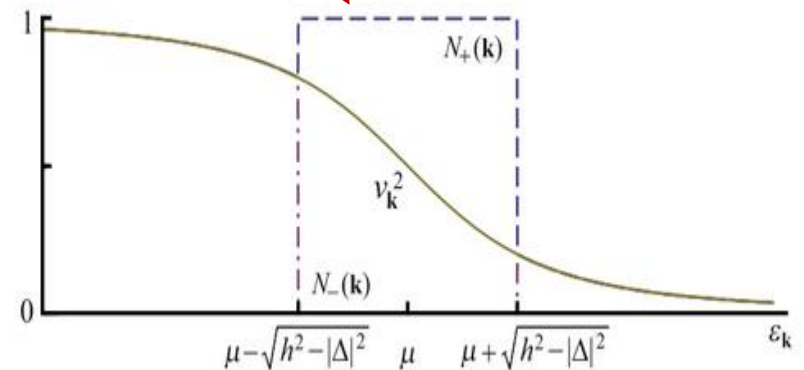
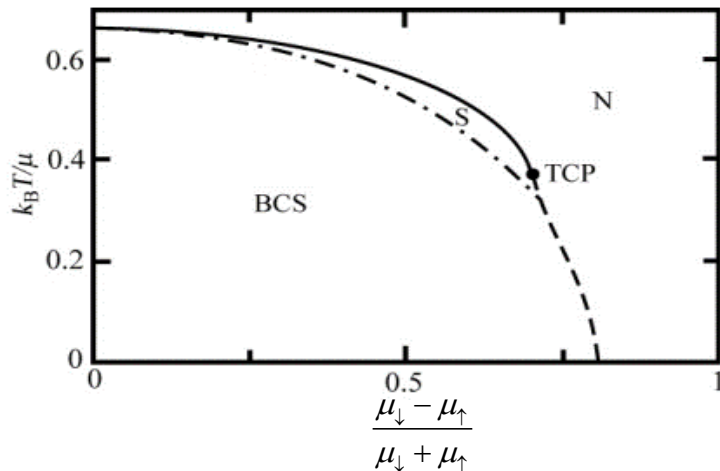
$$|\mu_{\downarrow} - \mu_{\uparrow}| \propto \Delta$$



## Sarma phase (interior gap) phase

G. Sarma, J. Phys. Chem. Solids 24 (1963) 1029.

W.V. Liu, F. Wilczek, Phys. Rev. Lett. 90 (2003) 047002.



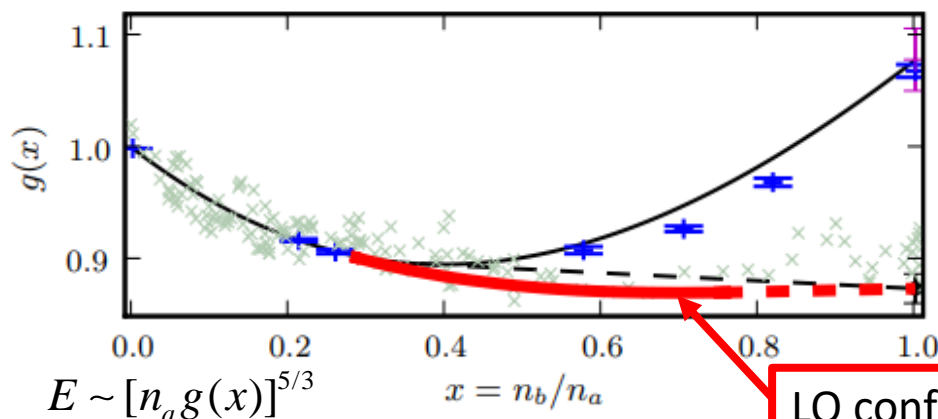
Phase separation in momentum space

Unstable for balanced masses at  $T=0$

# Inhomogeneous systems: Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase

Larkin-Ovchinnikov:  $\Delta(r) \sim \cos(qr)$   
 Fulde-Ferrell:  $\Delta(r) \sim \exp(iqr)$

A.I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP 20, 762 (1965)  
 P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964)



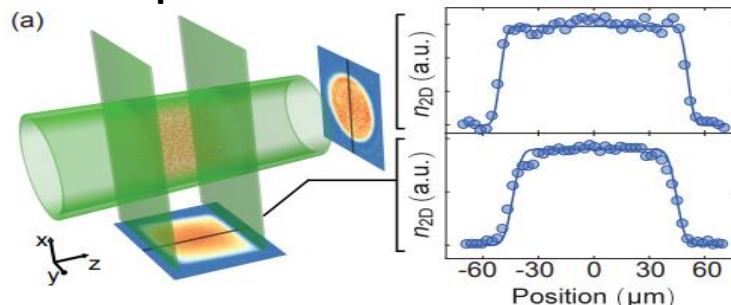
Bulgac & Forbes have shown, within DFT, that Larkin-Ovchinnikov (LO) phase may exist in the unitary Fermi gas (UFG)

LO configuration – supersolid state

A. Bulgac, M.M.Forbes, Phys. Rev. Lett. 101,215301 (2008)

See also review of mean-field theories : Radzihovsky,Sheehy, Rep.Prog. Phys.73,076501(2010)

Trapping ultracold atoms in a uniform potential recently become possible:



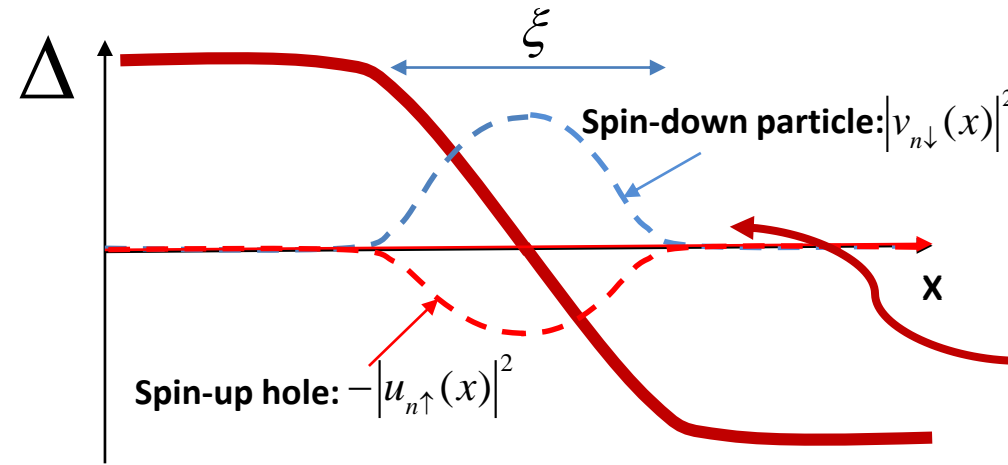
B. Mukherjee et al. Phys. Rev. Lett. 118, 123401 (2017)

**The problem:**

In the trap the volume where LOFF phase may be created is relatively small .

unless

# Andreev states and pairing nodal points

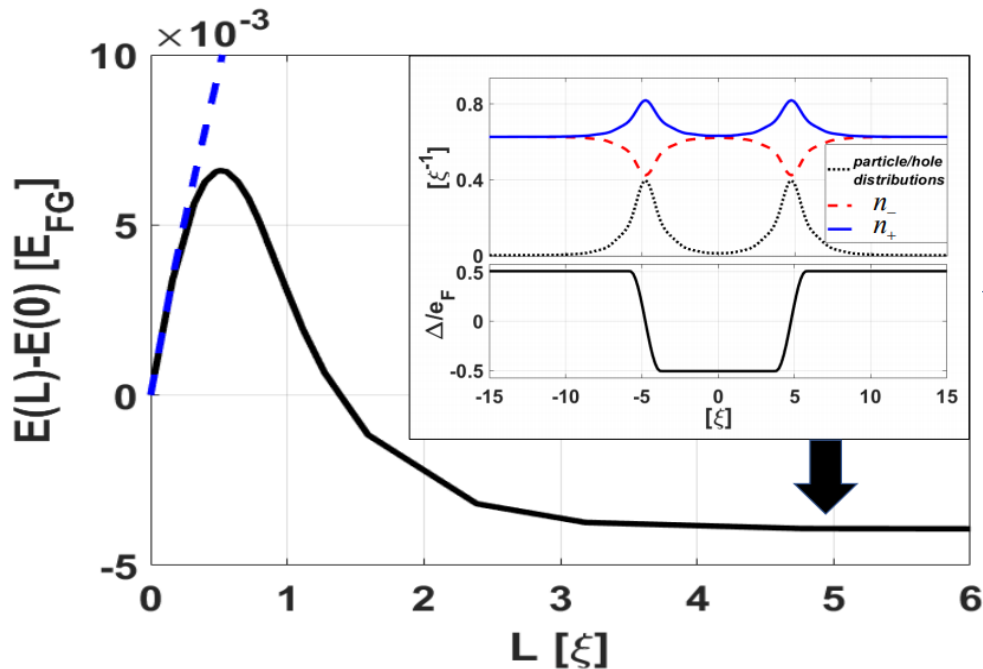


Due to quasiparticle scattering the localized Andreev states appear at the nodal point. These states induce local spin-polarization

BdG in the Andreev approx. ( $\Delta \ll k_F^2$ )

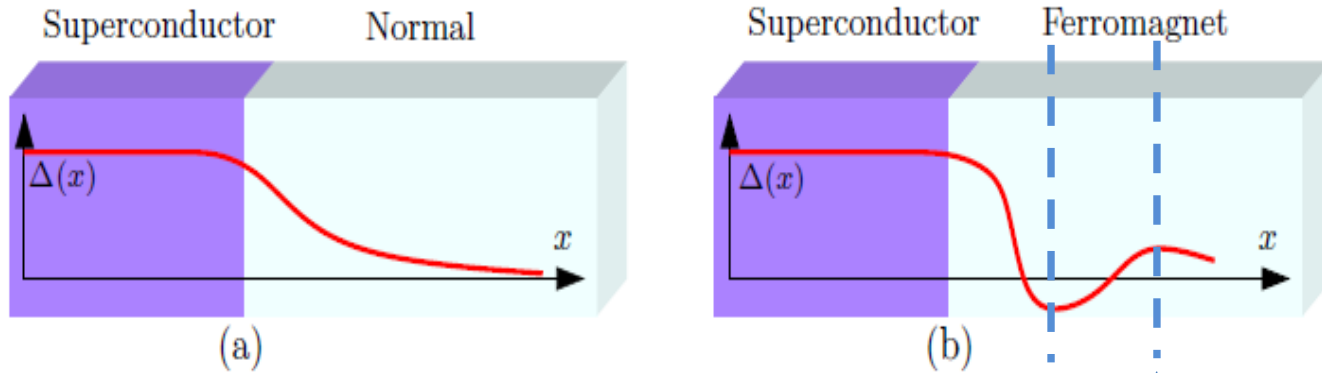
$$\begin{bmatrix} -2ik_F \frac{d}{dx} & \Delta(x) \\ \Delta^*(x) & 2ik_F \frac{d}{dx} \end{bmatrix} \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix} = E_n \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix}$$

## Energetics of two nodal points in 1D system



Two nodal points repel each other.

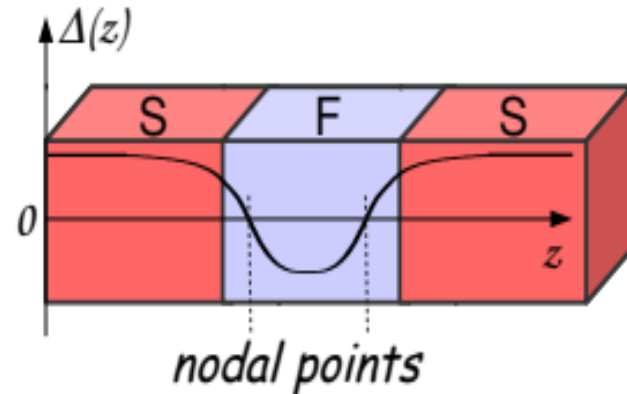
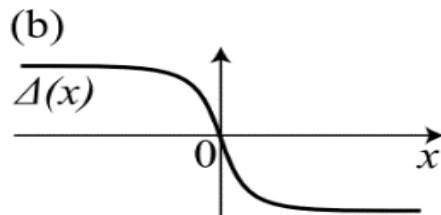
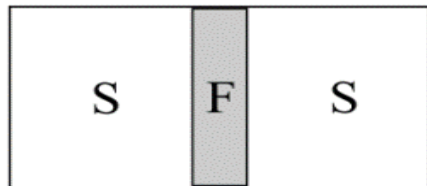
# Another perspective: superconductor-ferromagnet junction



Due to the difference between Fermi momenta of spin-up and spin-down particles:

$$\left. \begin{aligned} k_{F\uparrow} &= k_F + \delta k_F \\ k_{F\downarrow} &= k_F - \delta k_F \end{aligned} \right\}$$

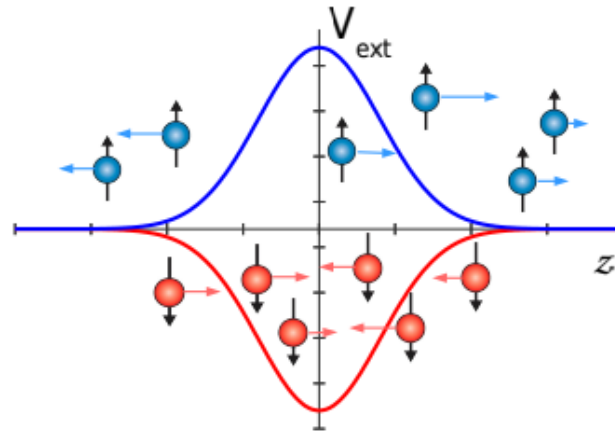
Induces spatial modulation of the order parameter of the period:  $\frac{\pi}{\delta k_F}$



Josephson-Pi junction

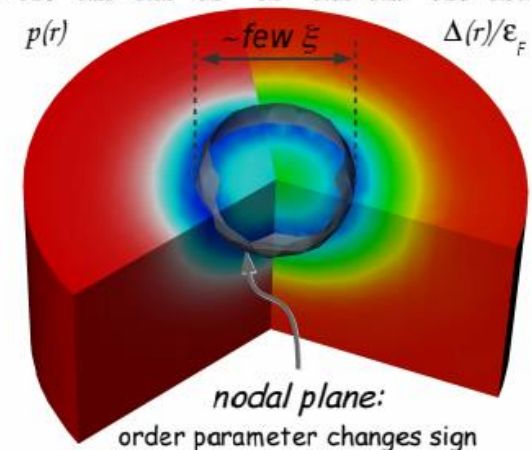
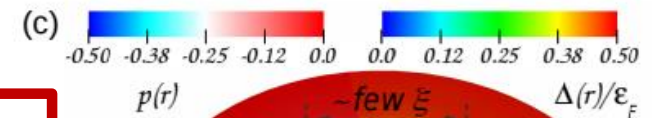
# Engineering the structure of nodal surfaces

Apply the spin-selective potential of a certain shape:



Wait until the proximity effects of the pairing field generate the nodal structure and remove the potential.

For example the spherical nodal structure:

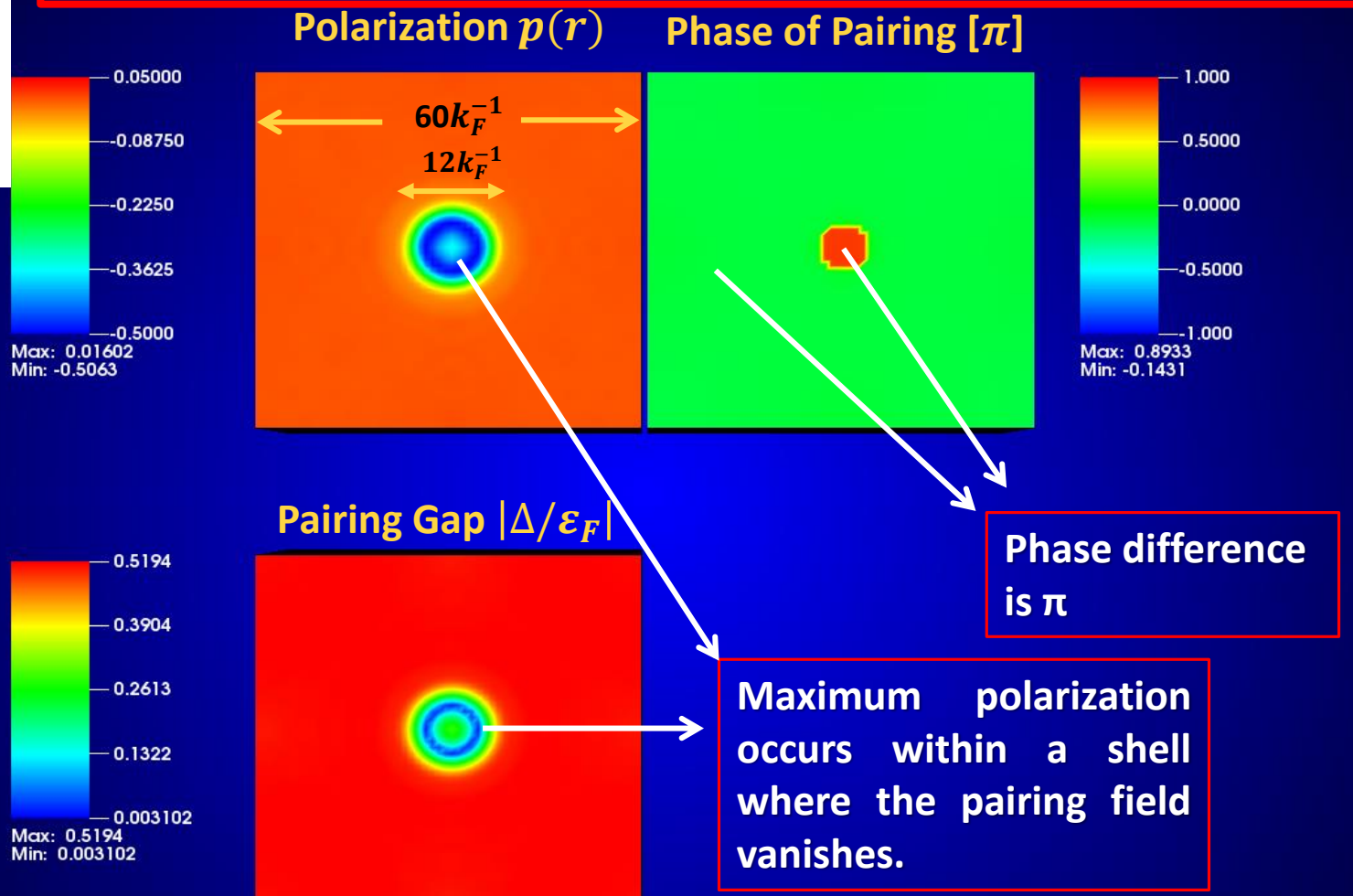
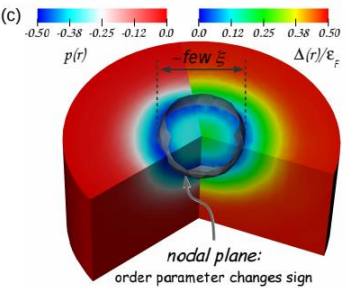


## **Important!**

Nodal structure is **unstable** without spin-polarization. And vice versa: **spin-polarization** (ie. excess of the majority spin particles) is expelled from superfluid unless pairing nodal structure is created.



# Forming a stable spherical nodal surface in Unitary Fermi Gas (UFG) - TDDFT calcs.



Contraction of the nodal sphere is prevented by the pairing potential barrier.  
Expansion of the nodal sphere will cost the energy due to expansion of polarized shell.

**As a result of the interplay between volume and surface energies keeps the impurity stable**

# Non-central collision of two impurities



## Moving impurity:

From Larkin-Ovchinnikov towards Fulde-Ferrell limit:

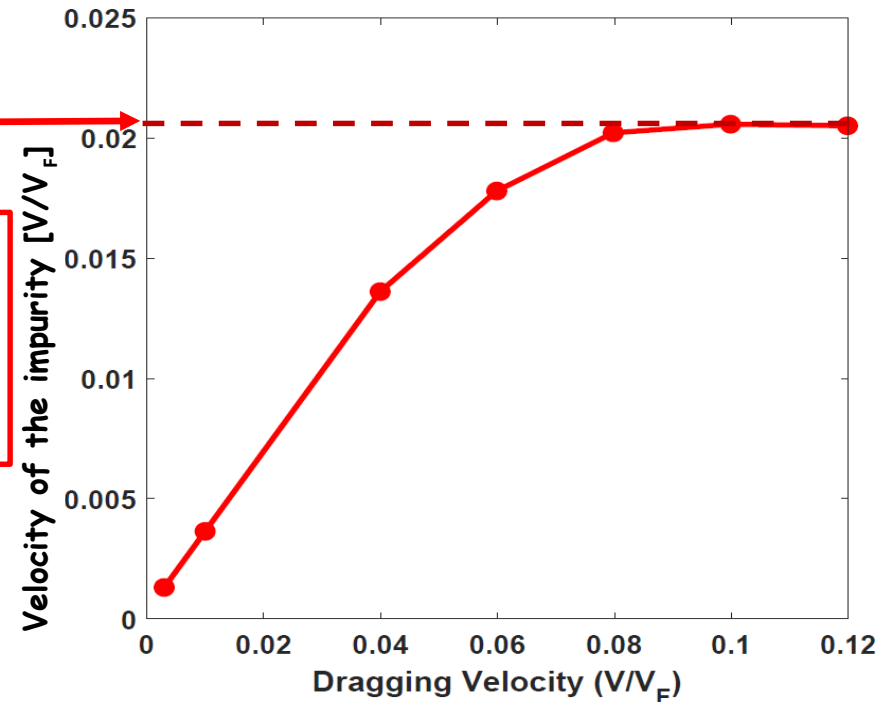
$$\Delta(r) : \cos(qr) \Rightarrow \exp(iqr)$$

Surprisingly, the nodal structure remains stable even during collisions

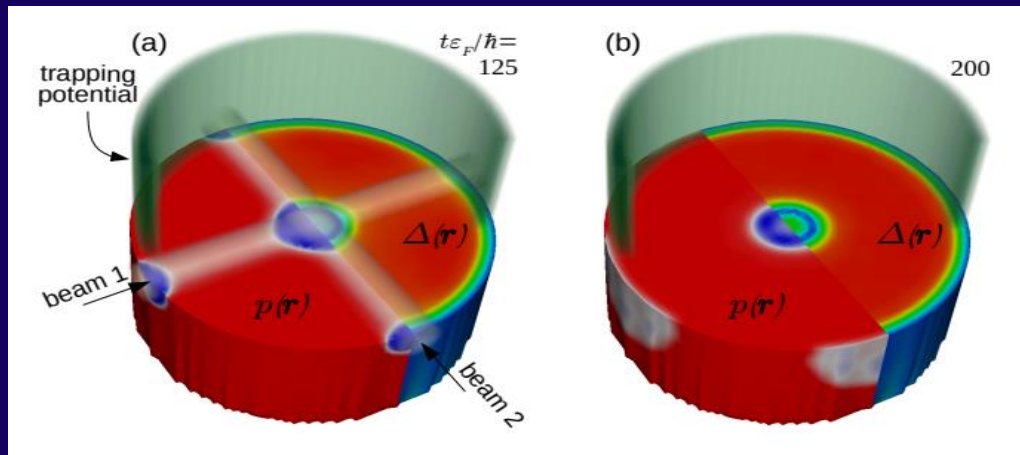
The velocities of impurities are about 30% of the velocity of sound.

## Limiting velocity with respect to superfluid background

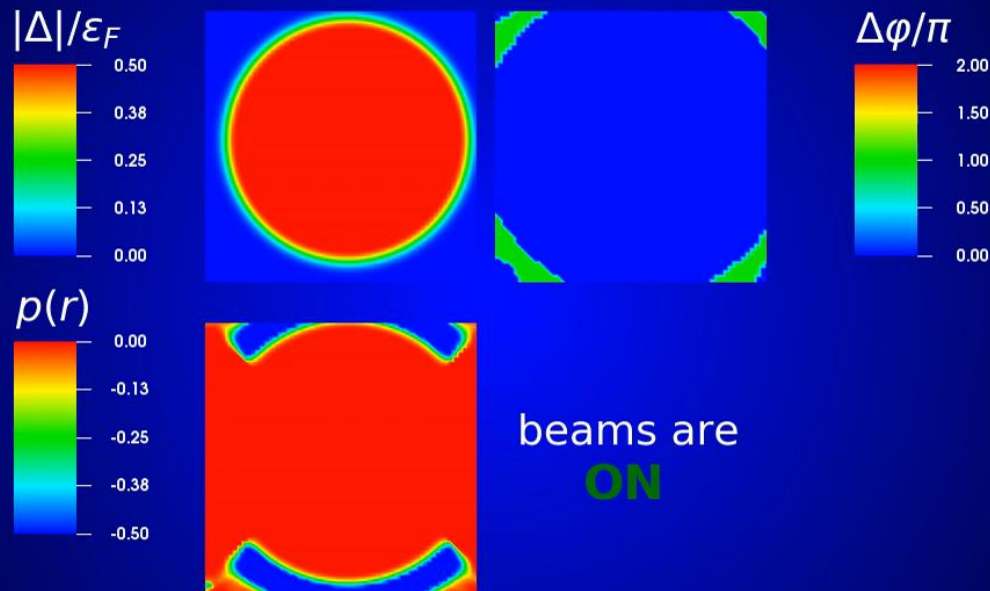
Note that the Fulde-Ferrell limit defines the **critical velocity** which is associated with the maximum spin current that can flow through the impurity ( $\sim q$ ).



# Suggestion for experimental protocol for ultracold atomic gas:



Two crossing beams:  $A = 1\varepsilon_F$ ,  $\sigma = 3.14\xi$



time \* eF = 0

# Summarizing

It is possible to create dynamically stable, locally spin-polarized region in the ultracold Fermi gas.

The stability is due to the peculiar pairing structure characteristic for the FFLO phase.

The conditions of stability:

- do not depend on details of the functional (simple BdG approach predicts qualitatively the same results)
- do not depend whether we are on the BEC-side ( $\alpha > 0$ ) or on the BCS-side ( $\alpha < 0$ ), although UFG may be the best system for experimental realization.

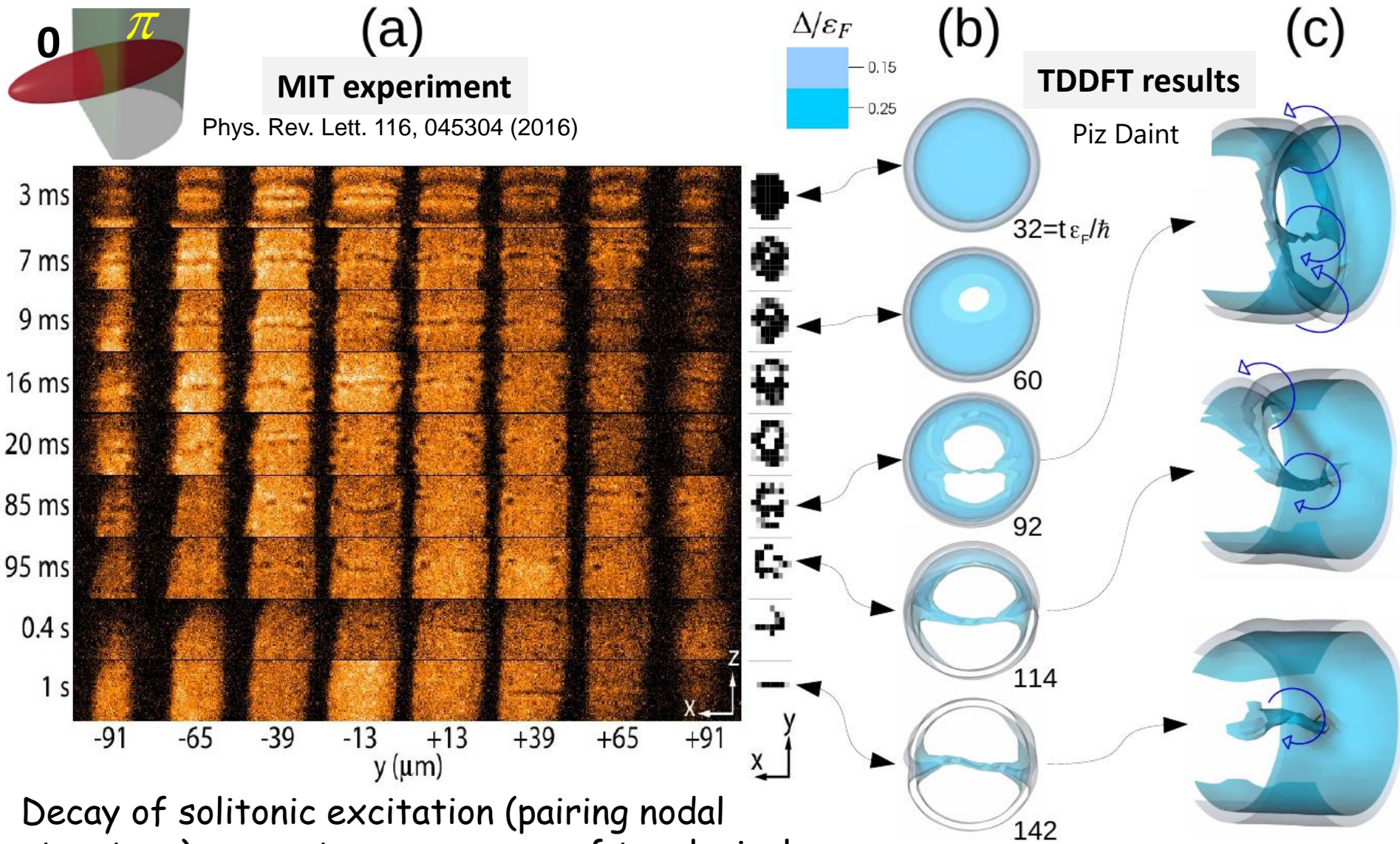
The effect can be viewed as:

- long-lived, spin-polarized excitation mode of UFG
- FFLO droplet (although its size is of few coherence lengths only)

We dubbed it ferron

*How about unstable nodal structures?*

# Unstable pairing nodal structures: atomic cloud collisions



Decay of solitonic excitation (pairing nodal structure) generates a sequence of topological excitations involving: "Phi"-soliton and vortex line.

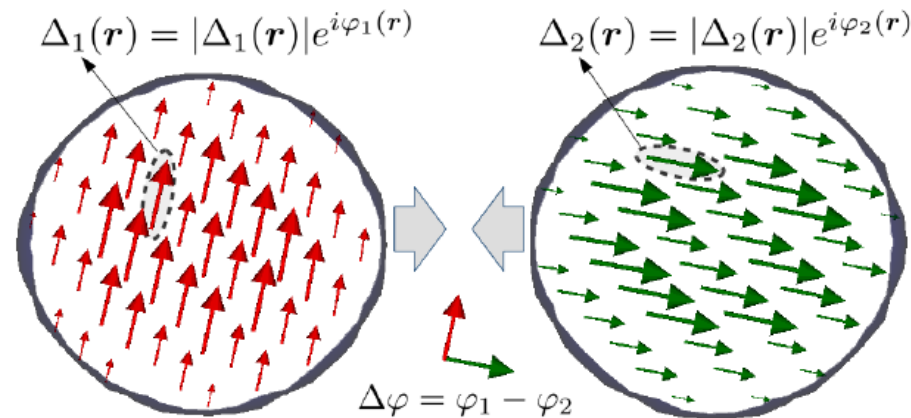
# Unstable pairing nodal structures: nuclear collisions

Collisions of superfluid nuclei having different phases of the pairing fields

The main questions are:

- how a possible solitonic structure can be manifested in nuclear system?
- what observable effect it may have on heavy ion reaction:  
kinetic energy distribution of fragments, capture cross section, etc.?

Clearly, we cannot control phases of the pairing field in nuclear experiments and the possible signal need to be extracted after averaging over the phase difference.

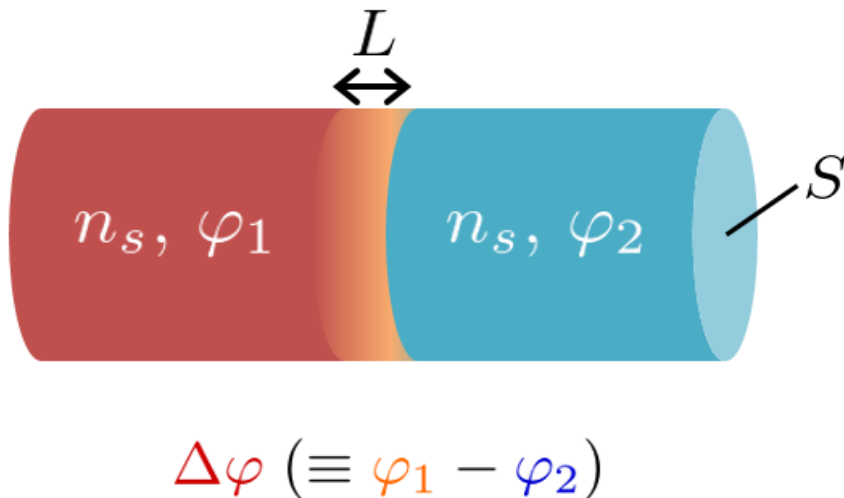


# Estimates for the magnitude of the effect

At first one may think that the magnitude of the effect is determined by the nuclear pairing energy which is of the order of MeV's in atomic nuclei (according to the expression):

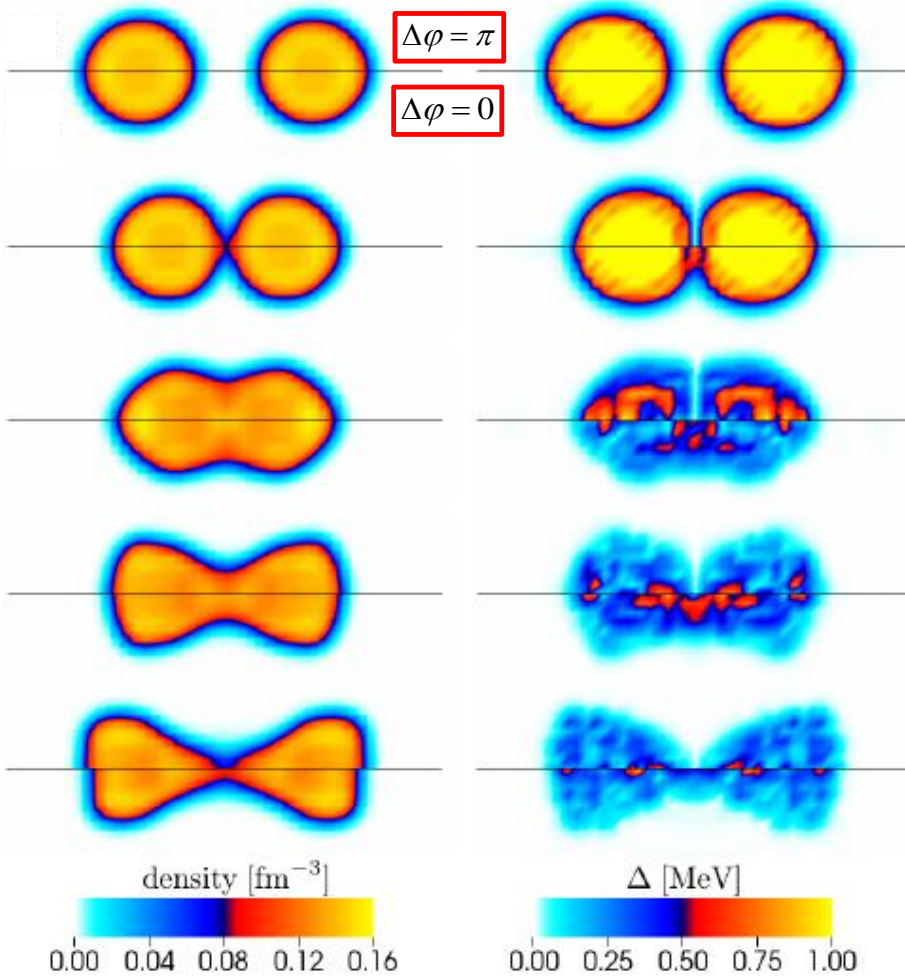
$$\frac{1}{2} g(\varepsilon_F) |\Delta|^2; \quad g(\varepsilon_F) - \text{density of states}$$

On the other hand the energy stored in the junction can be estimated from Ginzburg-Landau (G-L) approach:

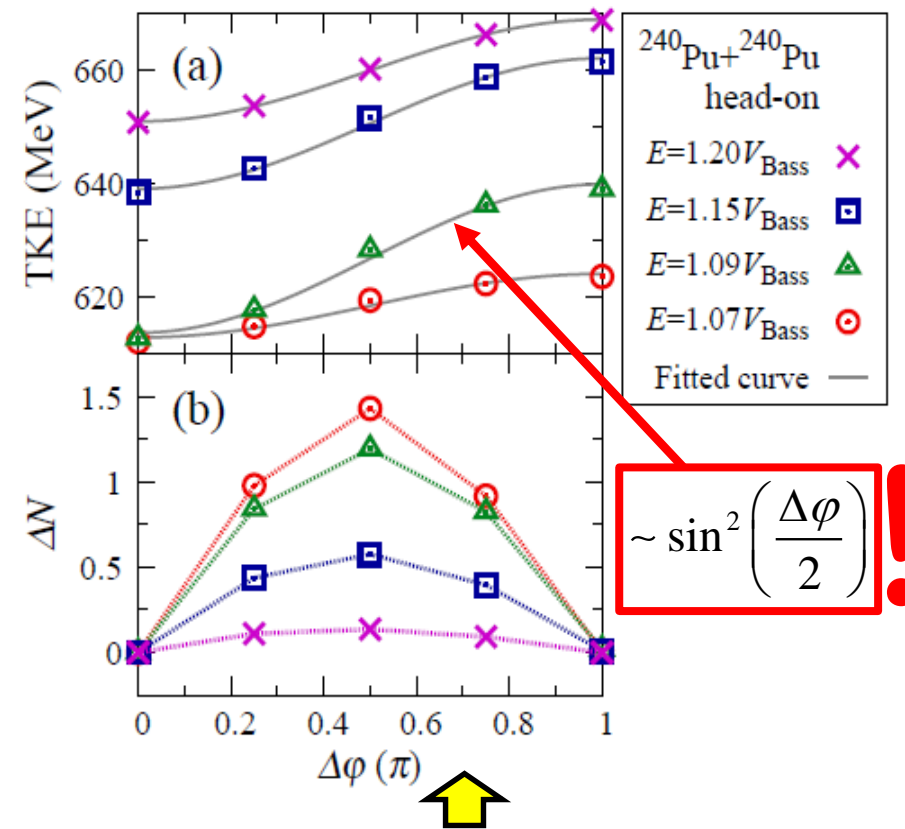


$$E_j = \frac{S}{L} \frac{\hbar^2}{2m} n_s \sin^2 \frac{\Delta\varphi}{2}$$

For typical values characteristic for two medium nuclei:  $E_j \approx 30 \text{ MeV}$



Total kinetic energy of the fragments (TKE)

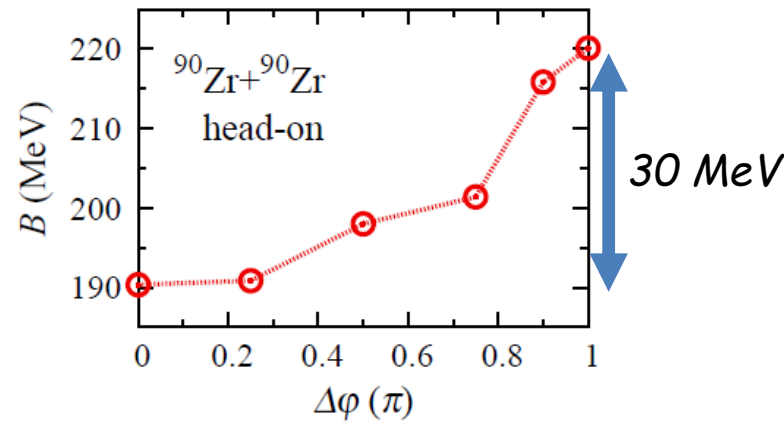


Average particle transfer between fragments.

Creation of the solitonic structure between colliding nuclei prevents energy transfer to internal degrees of freedom and consequently enhances the kinetic energy of outgoing fragments.  
 Surprisingly, the gauge angle dependence from the G-L approach is perfectly well reproduced in the kinetic energies of outgoing fragments!



# Effective barrier height for fusion as a function of the phase difference



What is an average extra energy needed for the capture?

$$E_{extra} = \frac{1}{\pi} \int_0^{\pi} (B(\Delta\varphi) - V_{Bass}) d(\Delta\varphi) \approx 10 \text{ MeV}$$

The effect is found (within TDDFT) to be of the order of 30 MeV for medium nuclei and occur for energies up to 20-30% of the barrier height.

P. Magierski, K. Sekizawa, G. Wlazłowski, Phys. Rev. Lett. 119 042501 (2017)

It raises (again) an interesting (and well known) question:  
**to what extent systems of hundreds of particles can be described using the concept of pairing field?**

G. Scamps, Phys. Rev. C 97, 044611 (2018): barrier fluctuations extracted from experimental data indicate that the effect exists although is weaker than predicted by TDDFT