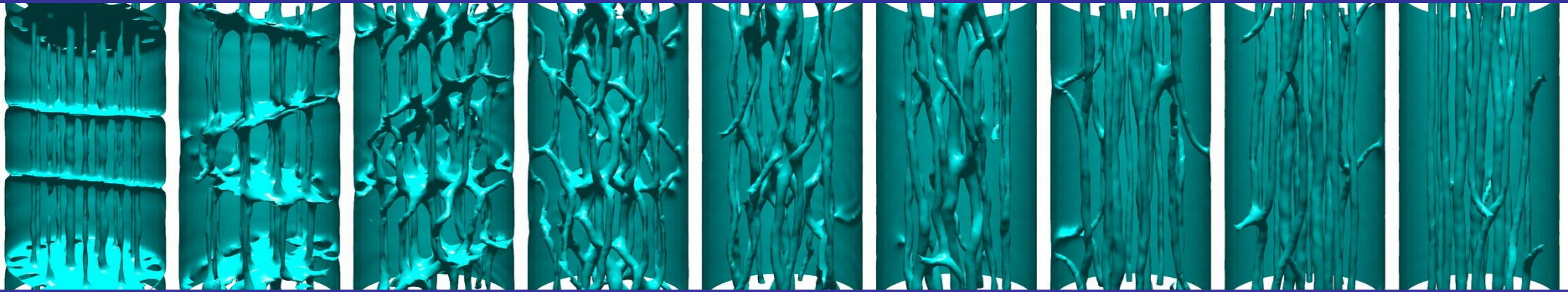
Quantum vortices in fermionic superfluids: from ultracold atoms to neutron stars

Piotr Magierski
Warsaw University of Technology (WUT)



Generation and decay of fermionic turbulence

Collaborators:

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Nicolas Chamel (ULB)
Konrad Kobuszewski (WUT - PhD student)
Andrzej Makowski (WUT - PhD student)
Daniel Pęcak (WUT)

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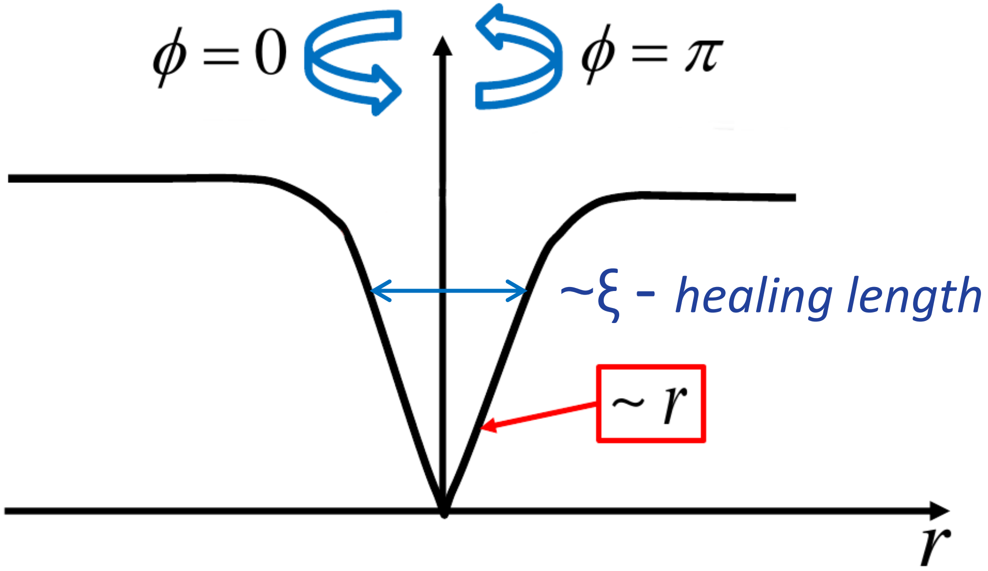
Tomasz Zawiślak (WUT -> Univ. Trento)
and LENS exp. group - Giacomo Roati et al.

MASOVIAN SYMPOSIUM ON QUANTUM, OPTICAL AND ATOMIC PHYSICS 10-11.06.2024

Anatomy of the vortex core

Bosonic vortex structure:
weakly interacting Bose gas at T=0 → Gross-Pitaevskii eq. (GPE)

$$\left[-\frac{1}{2m}\nabla^2 + g|\psi(\vec{r})|^2 + V_{ext}(\vec{r})\right]\psi(\vec{r}) = \mu\psi(\vec{r})$$



Order parameter:

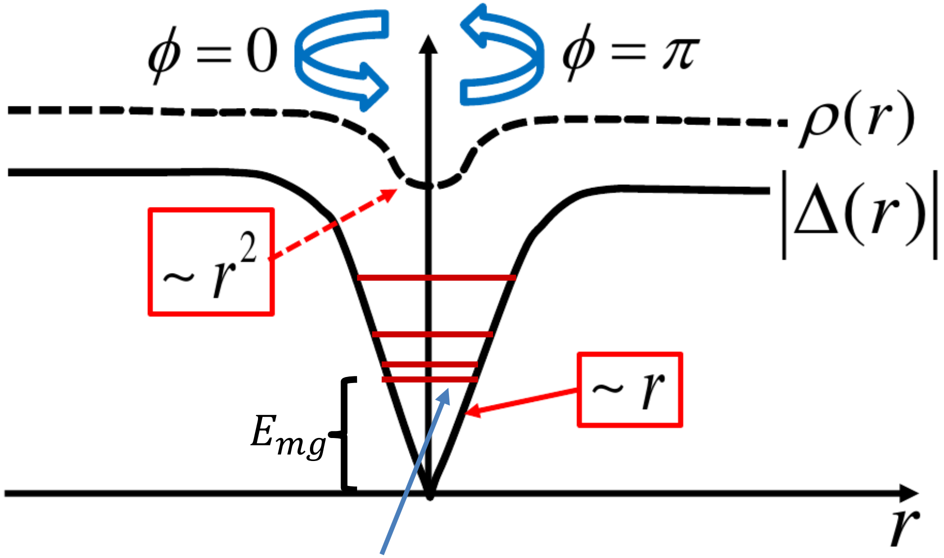
$$\psi(\vec{r}) = \sqrt{n(\vec{r})}e^{i\phi}$$

$$v_S = \frac{\hbar}{m}\nabla\phi$$

$$\kappa = \oint d\vec{l}\cdot\vec{v}_S = \frac{\hbar}{m}$$

Fermionic vortex structure:
Weakly interacting Fermi gas → Bogoliubov de Gennes (BdG) eqs.

$$\begin{pmatrix} h_\uparrow & \Delta \\ \Delta^* & -h_\downarrow^* \end{pmatrix} \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix} = \varepsilon_n \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix}$$



Form of the vortex-like solutions:

$$u_\eta(\mathbf{r}) = u_{nmk_z}(\rho) e^{im\varphi} e^{ik_z z}$$
$$v_\eta(\mathbf{r}) = v_{nmk_z}(\rho) e^{i(m+1)\varphi} e^{ik_z z},$$

CdGM (Andreev) states

C. Caroli, P. de Gennes, J. Matricon, Phys. Lett. 9, 307 (1964):

Minigap: $E_{mg} \sim \dfrac{|\Delta_\infty|^2}{\varepsilon_F}$ - energy scale for vortex core excitations.

Density of states: $g(\varepsilon) \sim \dfrac{\varepsilon_F}{|\Delta_\infty|^2}$; $\varepsilon \ll |\Delta_\infty|$

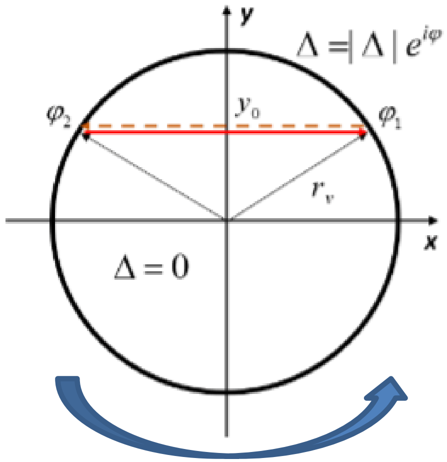
Vortex core structure in Andreev approximation:

$$\frac{E(0,L_z)}{\varepsilon_F}k_F r_V\sqrt{1-\left(\frac{L_z}{k_F r_V}\right)^2}+\arccos\left(\frac{-L_z}{k_F r_V}\right)-\arccos\left(\frac{E(0,L_z)}{|\Delta_\infty|}\right)=0$$

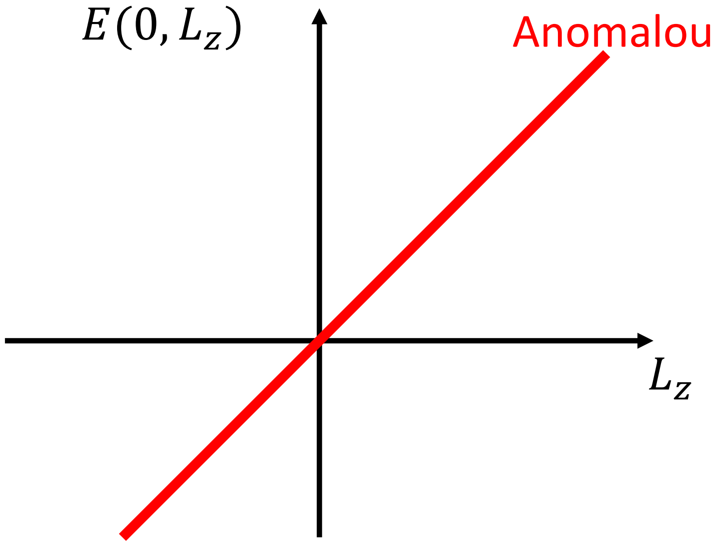
$$E(0,L_z)=E(0)L_z,\ E\ll|\Delta_\infty|$$

$$E(0,L_z)\approx\frac{|\Delta_\infty|^2}{\varepsilon_F\frac{r_V}{\xi}\left(\frac{r_V}{\xi}+1\right)}\frac{L_z}{\hbar},\qquad \xi=\frac{\varepsilon_F}{k_F|\Delta_\infty|}$$

Schematic section of the core



Spectrum of in-gap states



Works well in deep BCS limit: $\frac{1}{k_F a_s}\ll 0$

M. Stone, Phys. Rev. B 54, 13222 (1996)
P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

Quasiparticle mobility along the vortex line

$$E(k_z) = \frac{E(0)}{\sqrt{1-\left(\frac{k_z}{k_F}\right)^2}}\ ;\ k_z < k_F$$

C. Caroli, P. de Gennes, J. Matricon, Phys. Lett. 9, 307 (1964):

In Andreev approximation:

$$\sqrt{\varepsilon_F + E}\sin\alpha = \sqrt{\varepsilon_F - E}\sin\beta$$

$$k_h = \sqrt{2(\varepsilon_F - E)}$$

$$k_p = \sqrt{2(\varepsilon_F + E)}$$

$$v_z = k_z\frac{\sqrt{k_p^2 - k_z^2} - \sqrt{k_h^2 - k_z^2}}{\sqrt{k_p^2 - k_z^2} + \sqrt{k_h^2 - k_z^2}}$$

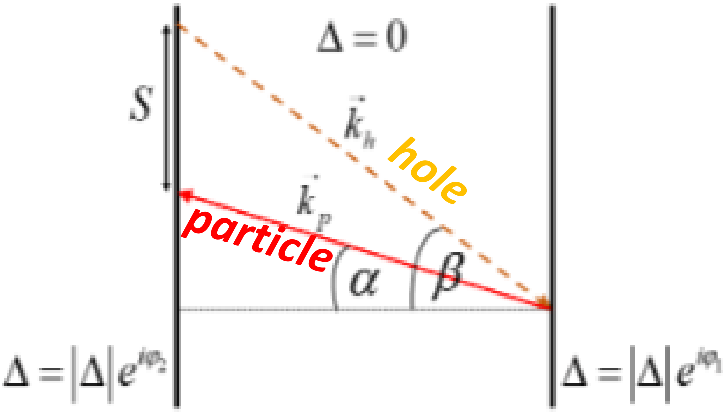
Velocity component along the vortex line

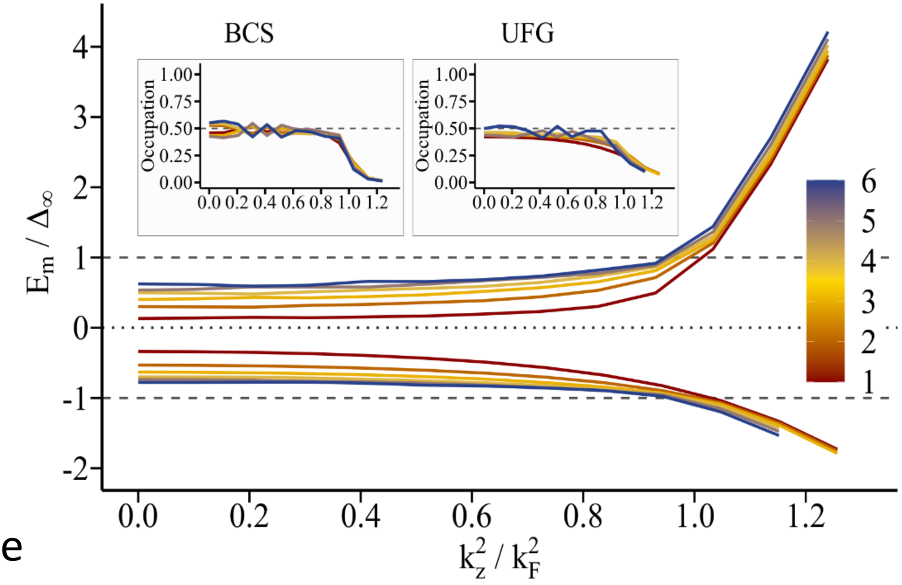
It gives the same dispersion relations as above up to the second order.

$$\boxed{M_{eff}^{-1}(L_z) \approx \frac{2}{3}\left(\frac{|\Delta_\infty|}{\varepsilon_F}\right)^2\frac{L_z}{\hbar}}$$

Effective mass of quasiparticle in the core carrying ang. mom. Lz

Schematic picture of Andreev reflection of particle-hole moving along the vortex line

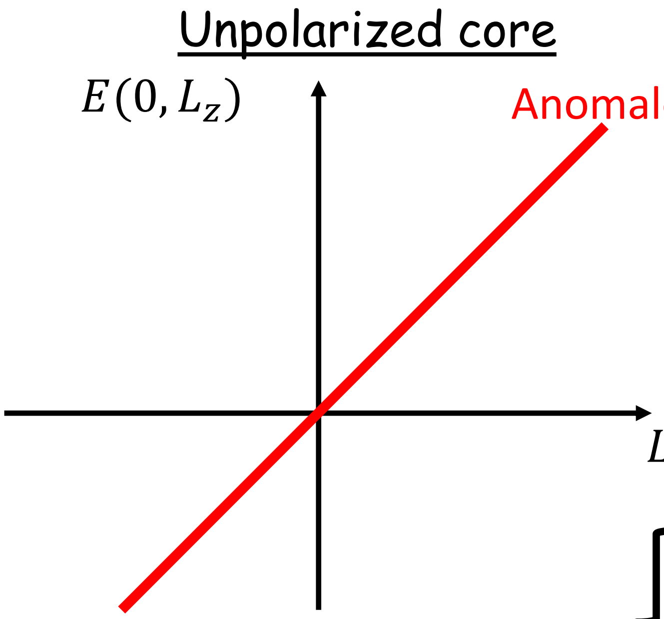




P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

Note that large value of effective mass along the vortex line originate from the fact that the occupations of hole and particle states below the gap are approximately equal.

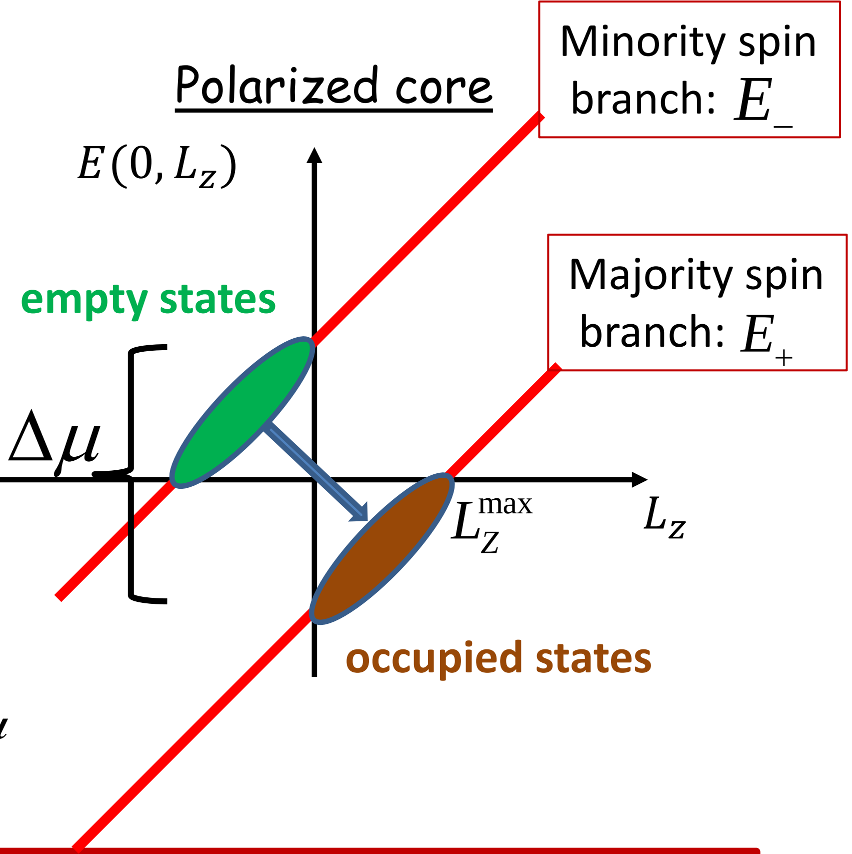
Changes of the core structure induced by spin polarization



Polarization

$$\mu \rightarrow \mu_\uparrow, \mu_\downarrow$$

$$\begin{cases} \dfrac{\mu_\uparrow + \mu_\downarrow}{2} = \mu \\ \mu_\uparrow - \mu_\downarrow = \Delta\mu \end{cases}$$



Branches are split proportionally to polarization

$$\boxed{E_\pm(0,L_Z) \approx \frac{|\Delta_\infty|^2}{\varepsilon_F \dfrac{r_V}{\xi}\left(\dfrac{r_V}{\xi}+1\right)} \frac{L_Z}{\hbar} \mp \frac{\Delta\mu}{2}}$$

Certain fraction of majority spin particles rotate in the opposite direction!

$$\boxed{L_Z^{\max} \approx \frac{1}{2} \frac{\varepsilon_F}{|\Delta_\infty|^2} \frac{r_V}{\xi}\left(\frac{r_V}{\xi}+1\right) \hbar\Delta\mu}$$

Two consequences of vortex core polarization:

1) Minigap vanishes.

2) Direction of the current in the core reverses.

1) Since the polarization correspond to relative shift of anomalous branches therefore the quasiparticle spectrum of spin-up and spin-down components is asymmetric for $k_z = 0$.

However the symmetry of the spectrum has to be restored in the limit of $k_z \to \infty$. Since for a straight vortex one can decouple the degree of freedom along the vortex line:

$$H = \begin{pmatrix} h_{2D}(\mathbf{r}) + \frac{1}{2}k_z^2 - \mu_\uparrow & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h_{2D}^*(\mathbf{r}) - \frac{1}{2}k_z^2 + \mu_\downarrow \end{pmatrix}$$

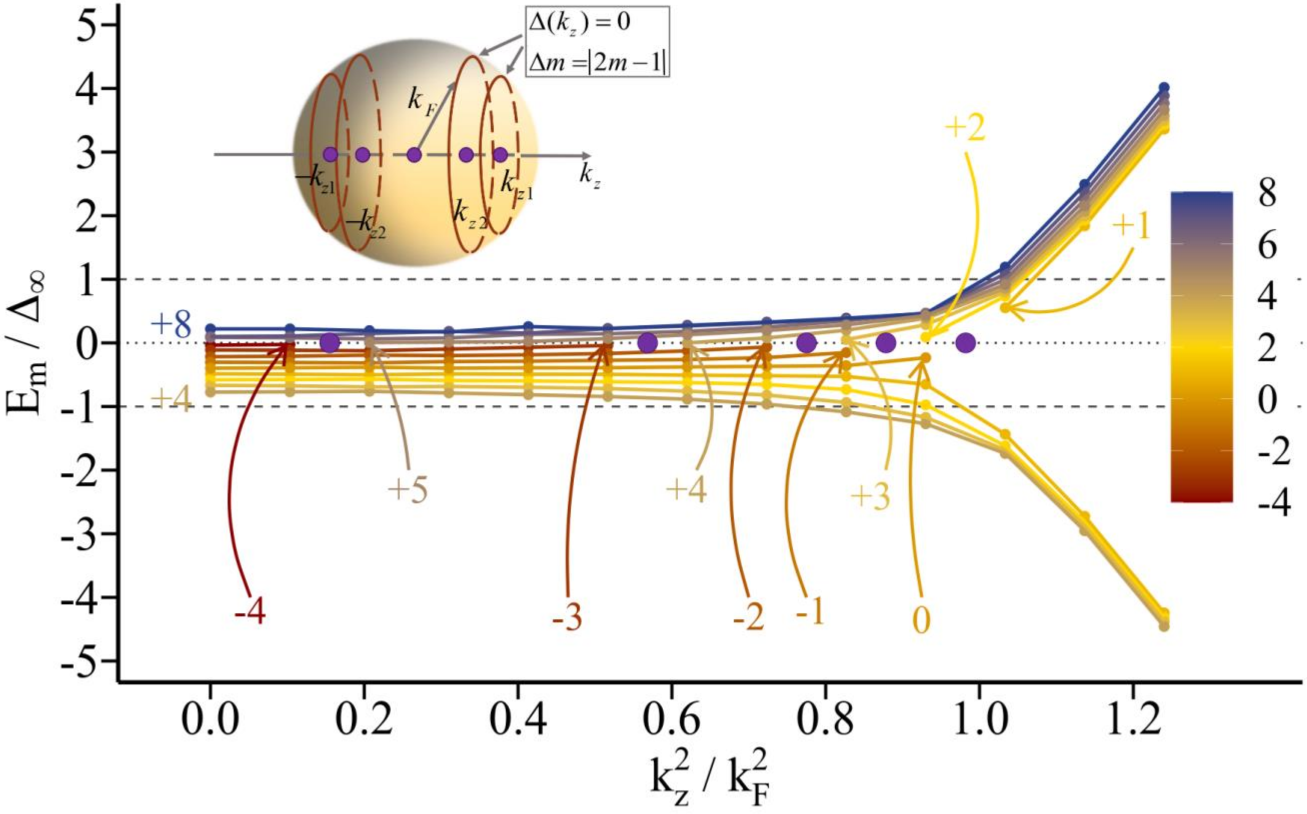
therefore $E(k_z) \propto \pm k_z^2$ when $k_z \to \infty$

As a result there must exist a sequence of values: $k_z = \pm k_{z1}, \pm k_{z2}, \ldots$
for which:

$$\boxed{E(\pm k_{zi}) = 0}$$

Moreover the crossings occur between levels of particular projection of angular momentum on the vortex line.

Namely, the crossing occurs in such a way that the particle state: $v_{\uparrow}$ of ang. momentum ***m*** is converted into a hole $u_{\uparrow}$ of momentum ***–m+1***
Hence the configuration changes by $\Delta m = |2m-1|$

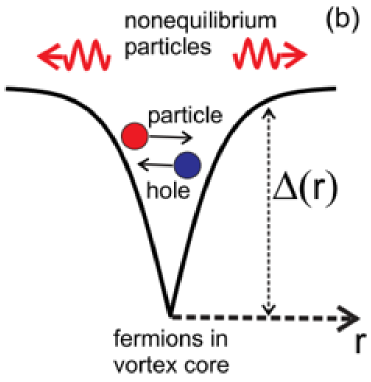


P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski,
Phys. Rev. A 106, 033322 (2022)

How can we measure the influence of core states in ultracold gases?

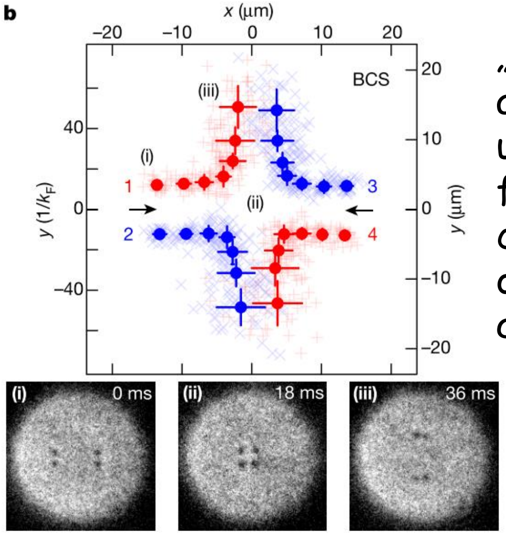
Dissipative processes involving vortex dynamics.

Silaev, Phys. Rev. Lett. 108, 045303 (2012)
Kopnin, Rep. Prog. Phys. 65, 1633 (2002)
Stone, Phys. Rev. B54, 13222 (1996)
Kopnin, Volovik, Phys. Rev. B57, 8526 (1998)
....



Classical treatment of states in the core (Boltzmann eq.).
More applicable in deep BCS limit unreachable in ultracold atoms.

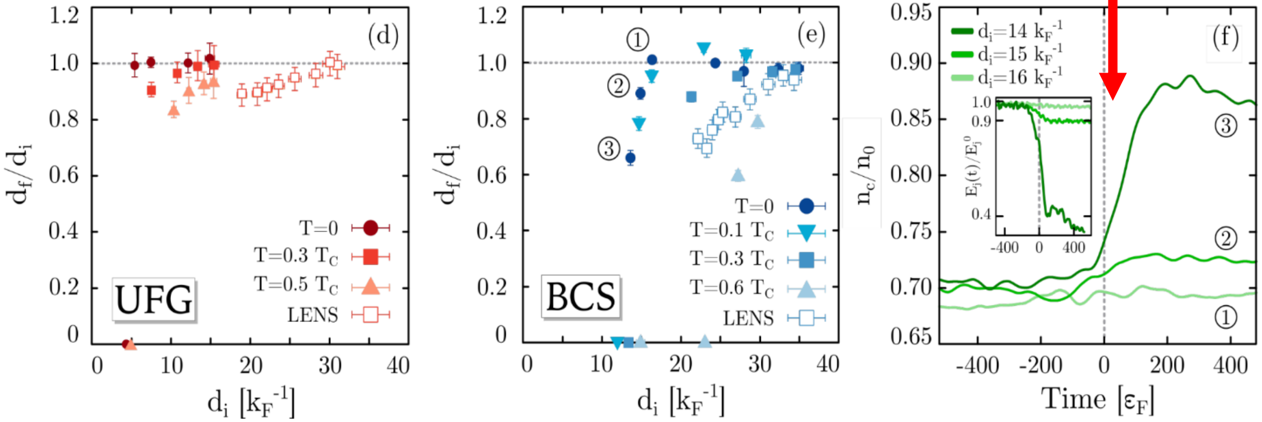
Vortex-antivortex scattering in 2D



„Further, our few-vortex experiments extending across different superfluid regimes reveal non-universal dissipative dynamics, suggesting that fermionic quasiparticles localized inside the vortex core contribute significantly to dissipation, thereby opening the route to exploring new pathways for quantum turbulence decay, vortex by vortex."

W.J. Kwon et al. Nature **600**, 64 (2021)

Exciting quasiparticles in the vortex core



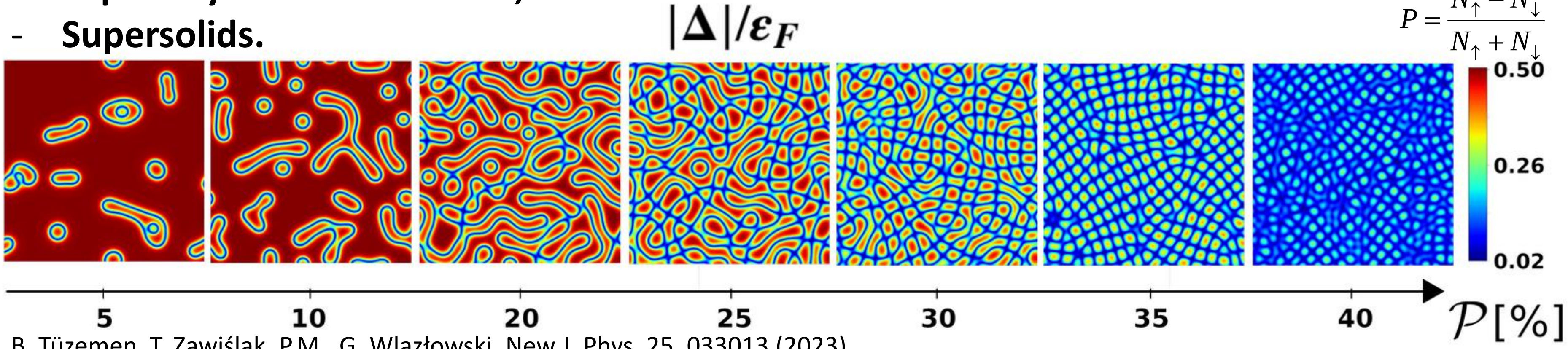
Indeed quasiparticles in the core are excited due to vortex acceleration but the effect is too weak to account for the total dissipation rate.

A. Barresi, A. Boulet, P.M., G. Wlazłowski, Phys. Rev. Lett. 130, 043001 (2023)

What is going to happen if we introduce spin imbalance?

In general it will generate distortions of Fermi spheres locally and triggering the appearance of **pairing field inhomogeneity** leading to various patterns involving:

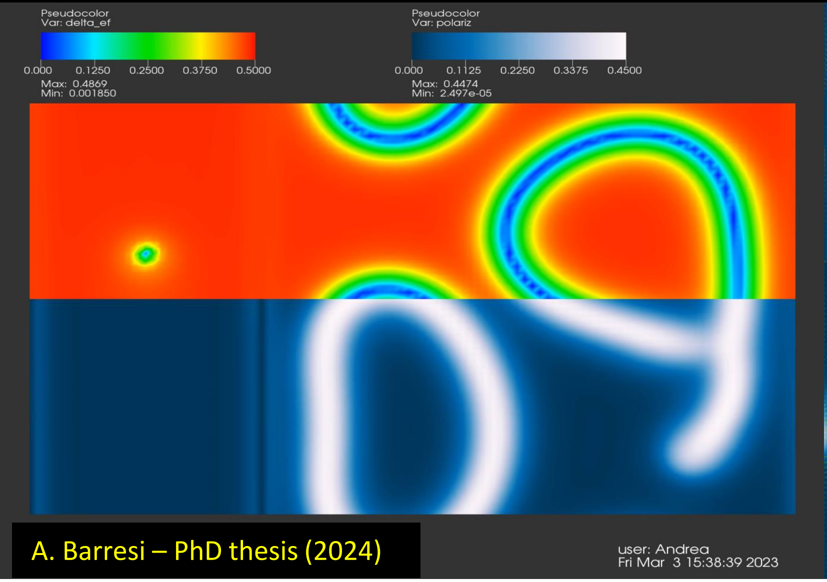
- **Separate impuritites (ferrons),**
- **Liquid crystal-like structure,**
- **Supersolids.**



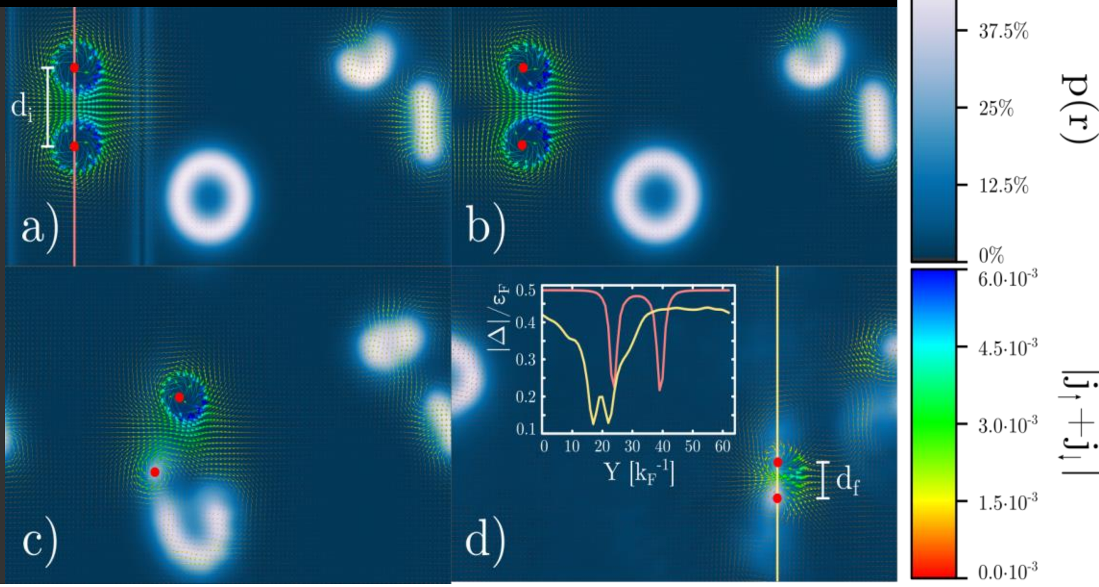
$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

B. Tüzemen, T. Zawiślak, P.M., G. Wlazłowski, New J. Phys. 25, 033013 (2023).

Dynamics of a vortex dipole in spin imbalanced Fermi superfluid.
Strong enhancement of vortex dipole energy dissipation.



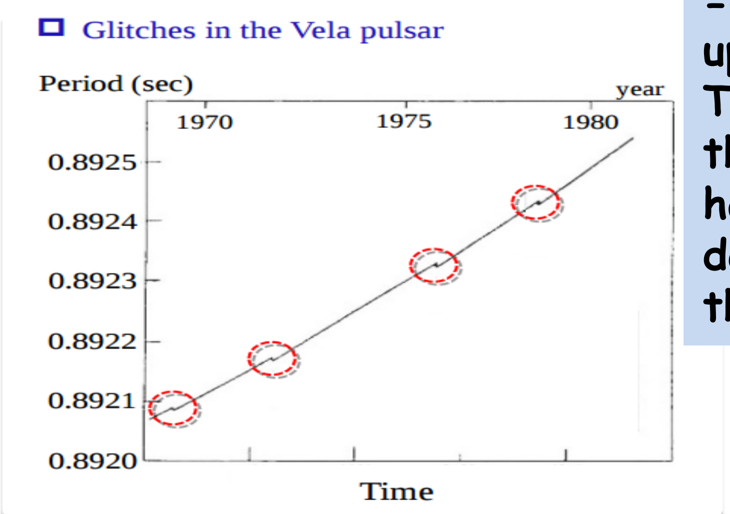
A. Barresi – PhD thesis (2024)



Modelling neutron star interior

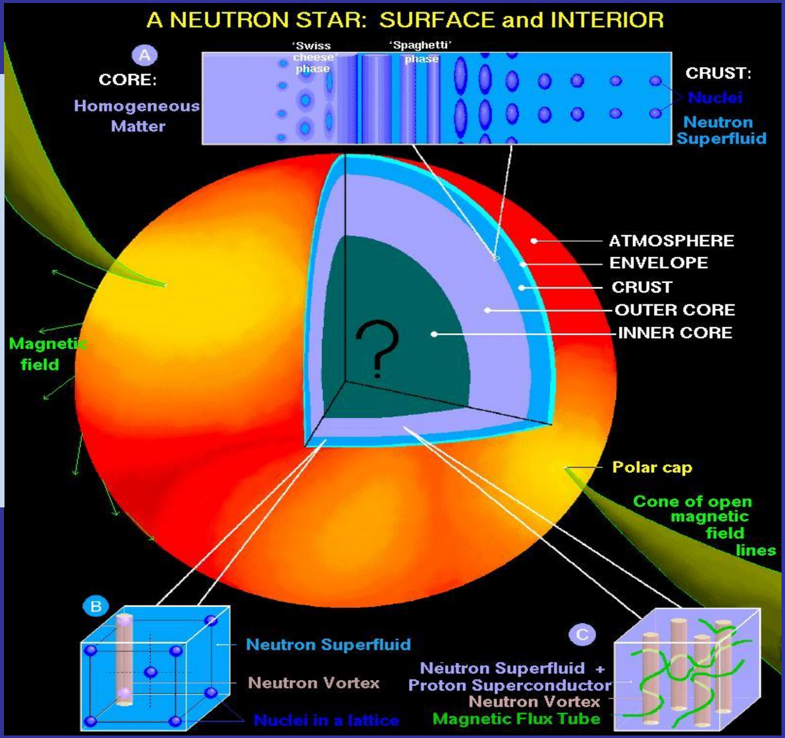
Neutron star is a huge superfluid

Glitch: a sudden increase of the rotational frequency



V.B. Bhatia, A Textbook of Astronomy and Astrophysics with Elements of Cosmology, Alpha Science, 2001.

glitch phenomenon =a sudden speed up of rotation. To date more than 300 glitches have been detected in more than 100 pulsars



Glitch phenomenon is commonly believed to be related to rearrangement of vortices in the interior of neutron stars (Anderson, Itoh, Nature 256, 25 (1975)) It would require however a correlated behavior of huge number of quantum vortices and the mechanism of such collective rearrangement is still a mystery.

Large scale dynamical model of neutron star interior (in particular neutron star crust), based on microscopic input from nuclear theory, is required.
In particular: vortex-impurity interaction, deformation modes of nuclear lattice, effective masses of nuclear impurities and couplings between lattice vibrations and neutron superfluid medium, need to be determined.

Properties of a vortex across the neutron star crust

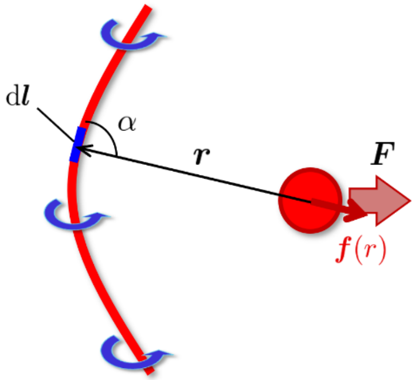
ρ_∞ (fm^{-3})	0.00036	0.0059	0.0112	0.0189	0.0231	0.0333
k_F^{-1} (fm)	4.52	1.79	1.45	1.21	1.14	1.01
ξ (fm)	8.44	5.53	5.97	7.00	7.78	10.28
$R_{\rm VFM}$ (fm)	15.0	10.5	10.5	12.0	13.5	16.5
Δ_∞ (MeV)	0.35	1.33	1.53	1.55	1.50	1.28
$T_{\rm crit}$ (MeV)	0.20	0.76	0.87	0.88	0.85	0.73
$\varepsilon_{\rm F}$ (MeV)	1.01	6.48	9.93	14.09	16.10	20.53
μ (MeV)	0.80	4.21	5.80	7.30	7.91	9.09
$E_{\rm mg}$ (MeV)	0.090	0.308	0.261	0.199	0.152	0.009
$B_{\rm crit}$ (10^{15} G)	7.76	26.5	22.5	17.2	13.1	0.82

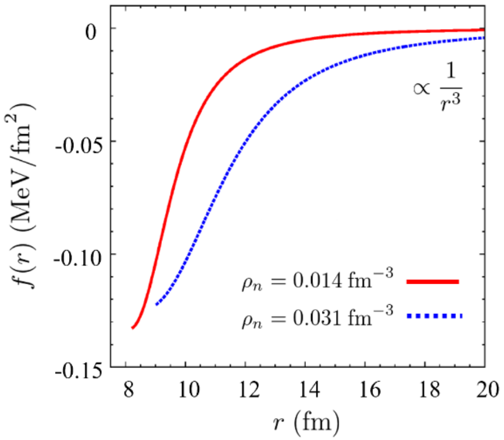
Minigap values

Magnetic field needed to polarize the core

D. Pęcak, N. Chamel, P.M., G. Wlazłowski, Phys. Rev. C**104**, 055801 (2021)

Vortex – impurity interaction (pinning force)





G. Wlazłowski, K. Sekizawa, P. Magierski, A. Bulgac, M.M. Forbes, Phys. Rev. Lett. 117, 232701(2016)

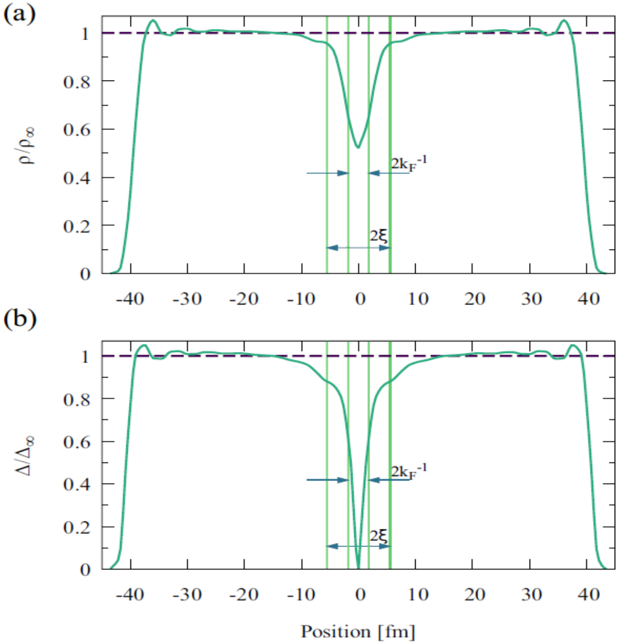
Is neutron star a turbulent system?

- What are differences and similarities of turbulence and its decay in Fermi and Bose superfluids?

A. Bulgac, A. Luo, P. Magierski, K.Roche, Y. Yu, Science 332, 1288 (2011).
M. Tylutki, G. Wlazłowski, Phys. Rev. A103, 051302 (2021).
K.Hossain, K.Kobuszewski, M.M.Forbes, P. Magierski, K.Sekizawa, G.Wlazłowski Phys. Rev. A 105, 013304 (2022).
G. Wlazłowski, M.M. Forbes, S.R. Sarkar, A. Marek, M. Szpindler, PNAS Nexus 3, 160 (2024).

Example: vortices across the neutron star crust

Section through the vortex core



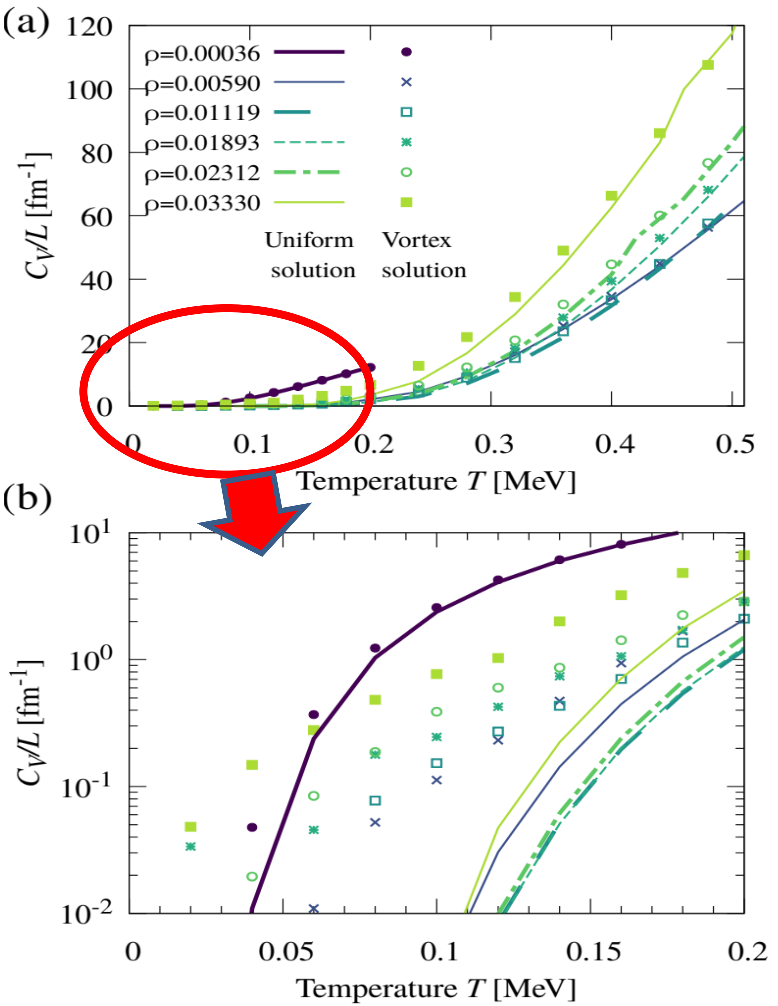
Normal density

Pairing field

Note two different length scales inside the core as explained by:

Sensarma, Randeria, Ho, Phys. Rev. Lett. 96, 090403 (2006)

Specific heat contribution vs uniform matter



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Minigap values

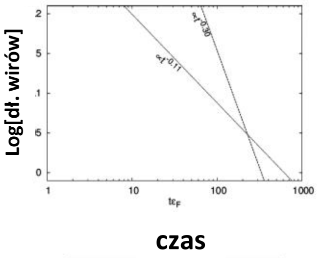
Magnetic field needed to polarize the core

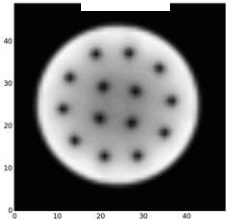
D. Pęcak, N. Chamel, P.M., G. Wlazłowski, Phys. Rev. C**104**, 055801 (2021)

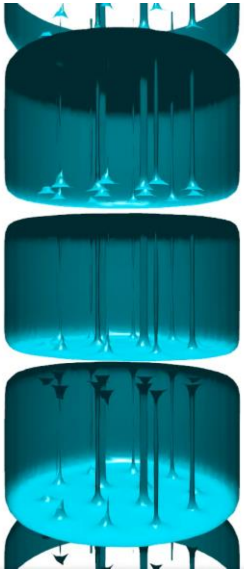
Superfluid turbulence (quantum turbulence):
disordered set of quantized vortices (vortex tangle).

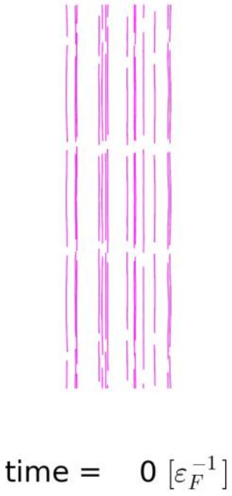
Interesting questions:

- What are differences and similarities of turbulence in Fermi and Bose superfluids?
- Characteristics of turbulence in spin imbalanced systems?









Creation and evolution of disordered vortex tangle – microscopic simulation (TDDFT)

K.Hossain, K.Kobuszewski, M.M.Forbes, PM, K.Sekizawa, G.Wlazłowski
Phys. Rev. A 105, 013304 (2022)

Vortex reconnections, Kelvin waves and one body dissipation are crucial for decay of turbulent state.

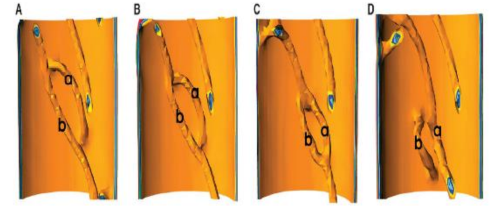
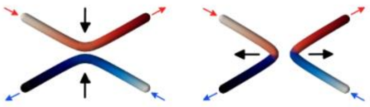
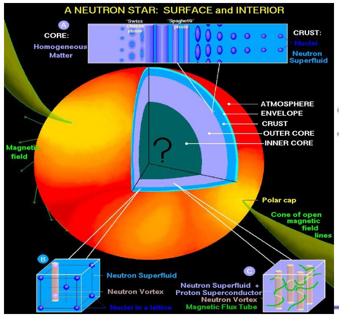


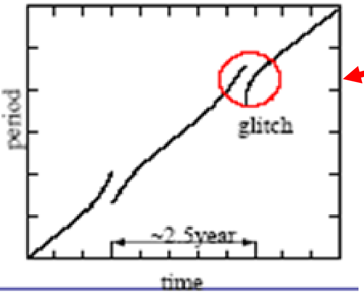
Fig. 3. (**A** to **D**) Two vortex lines approach each other, connect at two points, form a ring and exchange between them a portion of the vortex line, and subsequently separate. Segment (a), which initially belonged to the vortex line attached to the wall, is transferred to the long vortex line (b) after reconnection and vice versa.



Bulgac, Luo, Magierski, Roche, Yu, Science 332, 1288 (2011)

Is neutron star a turbulent system?

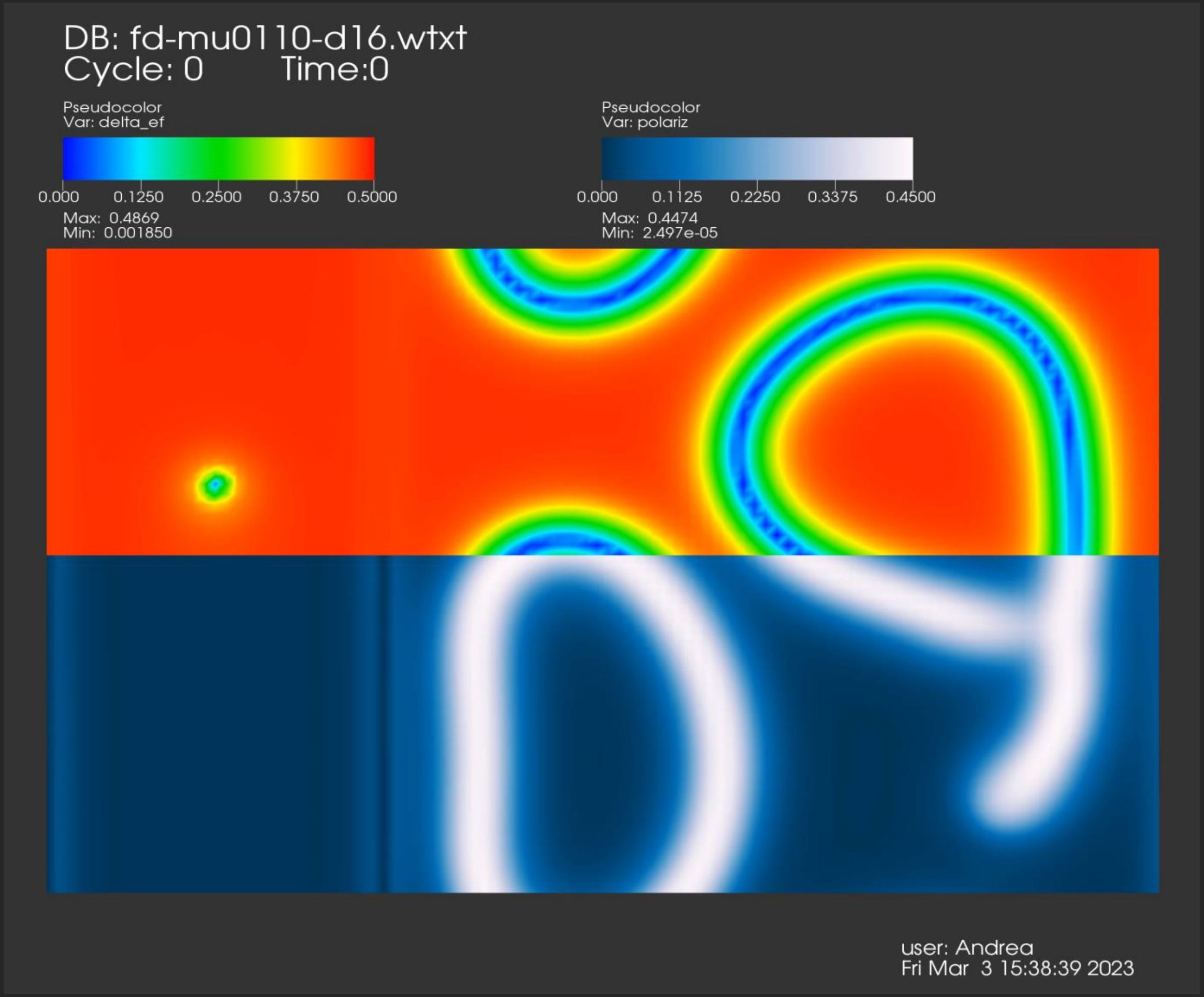




Periodic increase of rotational frequency of neutron star is observed (glitch phenomenon)

Since 70's the effect is associated with rapid rearrangement of quantum vortices inside neutron star caused by its inhomogeneous structure.
To date there is no theory which would explain the effect quantitatively.

Complex dynamics (strongly damped) of vortices in the spin imbalanced environment



Thanks to A. Barresi *et al.*

THANK YOU