Selected aspects of physics of dilute nuclear matter

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Content:

- Cold fermions on the lattice in the unitary regime.

- Neutron localization induced by the pairing field in the inner crust.

➢ What is the unitary regime?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$$\boxed{n\, r_0^3 \ll 1} \qquad \boxed{n\, |a|^3 \gg 1}$$

n - number density

$$\boxed{r_0 \ll n^{-1/3} \approx \lambda_F/2 \ll |a|}$$

r_0 - range of interaction

a - scattering length

The only scale:

$$\boxed{E_{FG}/N = \frac{3}{10}\frac{\hbar^2 k_F^2}{m}}$$

Bertsch Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

Why?

Besides pure theoretical curiosity, this problem is relevant to neutron stars!

What is the *Holy Grail* of this field?

Fermionic superfluidity!

In 1999 it was not yet clear, either theoretically or experimentally, whether such fermion matter is stable or not!

- Baker (winner of the MBX challenge) concluded that the system is stable. See also Heiselberg (entry to the same competition)
- Carlson et al (2003) and Astrakharchik et al. (2004) provided the best theoretical estimates for the ground state energy of such systems.
- Thomas' Duke group (2002) demonstrated experimentally that such systems are stable.

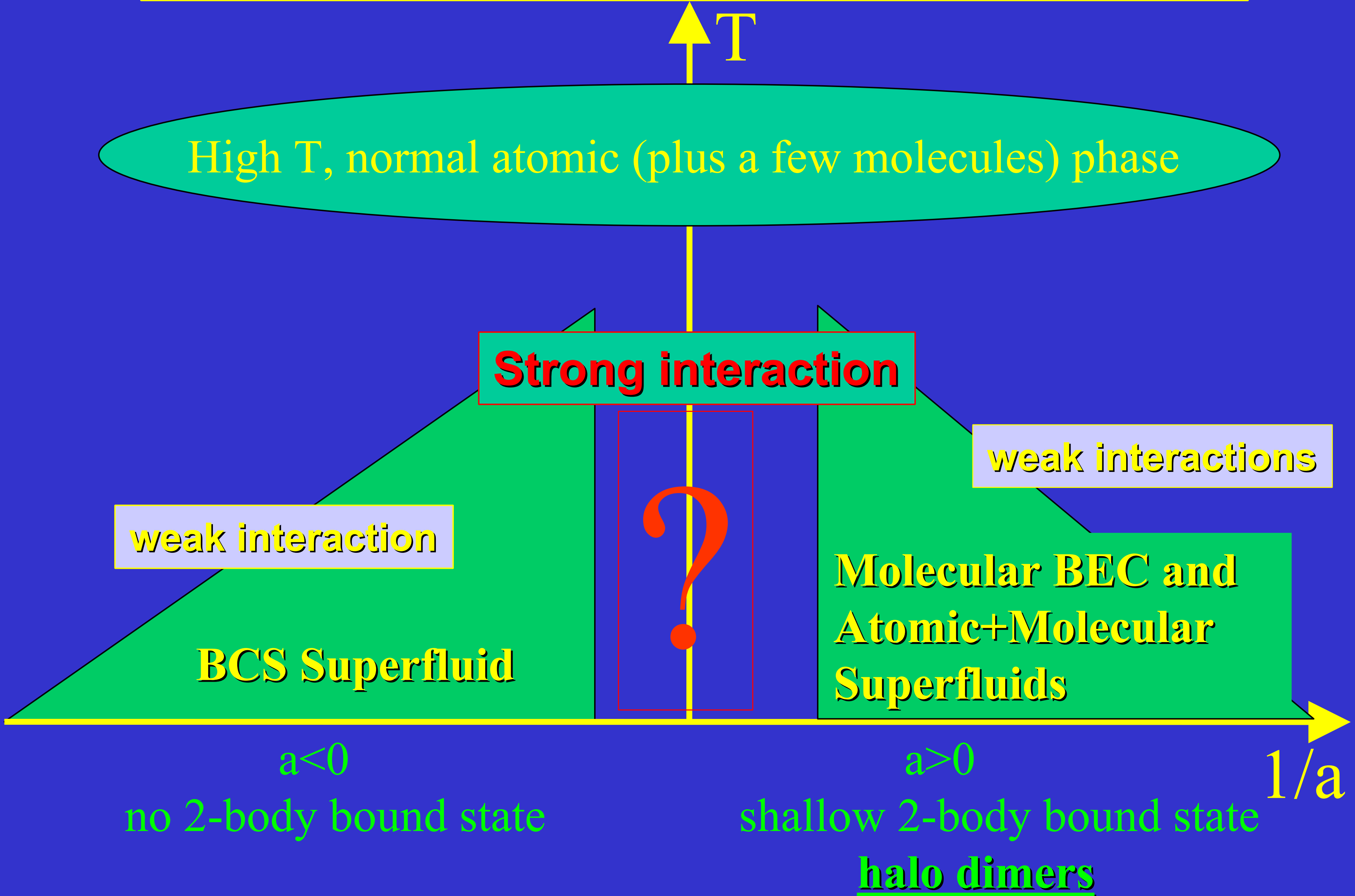
Superconductivity and superfluidity in Fermi systems

20 orders of magnitude over a century of (low temperature) physics

Dilute atomic Fermi gases	$T_c \approx 10^{-12} - 10^{-9}$ eV
Liquid ^{3}He	$T_c \approx 10^{-7}$ eV
Metals, composite materials	$T_c \approx 10^{-3} - 10^{-2}$ eV
Nuclei, neutron stars	$T_c \approx 10^{5} - 10^{6}$ eV
QCD color superconductivity	$T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV $\approx 10^4$ K)

Expected phases of a two species dilute Fermi system
BCS-BEC crossover



T

High T, normal atomic (plus a few molecules) phase

Strong interaction

weak interaction

BCS Superfluid

?

weak interactions

Molecular BEC and
Atomic+Molecular
Superfluids

a<0
no 2-body bound state

a>0
shallow 2-body bound state
halo dimers

1/a

Neutron matter:

Effective range: $r_0 \approx 2.8$ fm

Scattering length: $a \approx -18$ fm

Density range

$$r_0 \ll n^{-1/3} \approx \lambda_F/2 \ll |a|$$

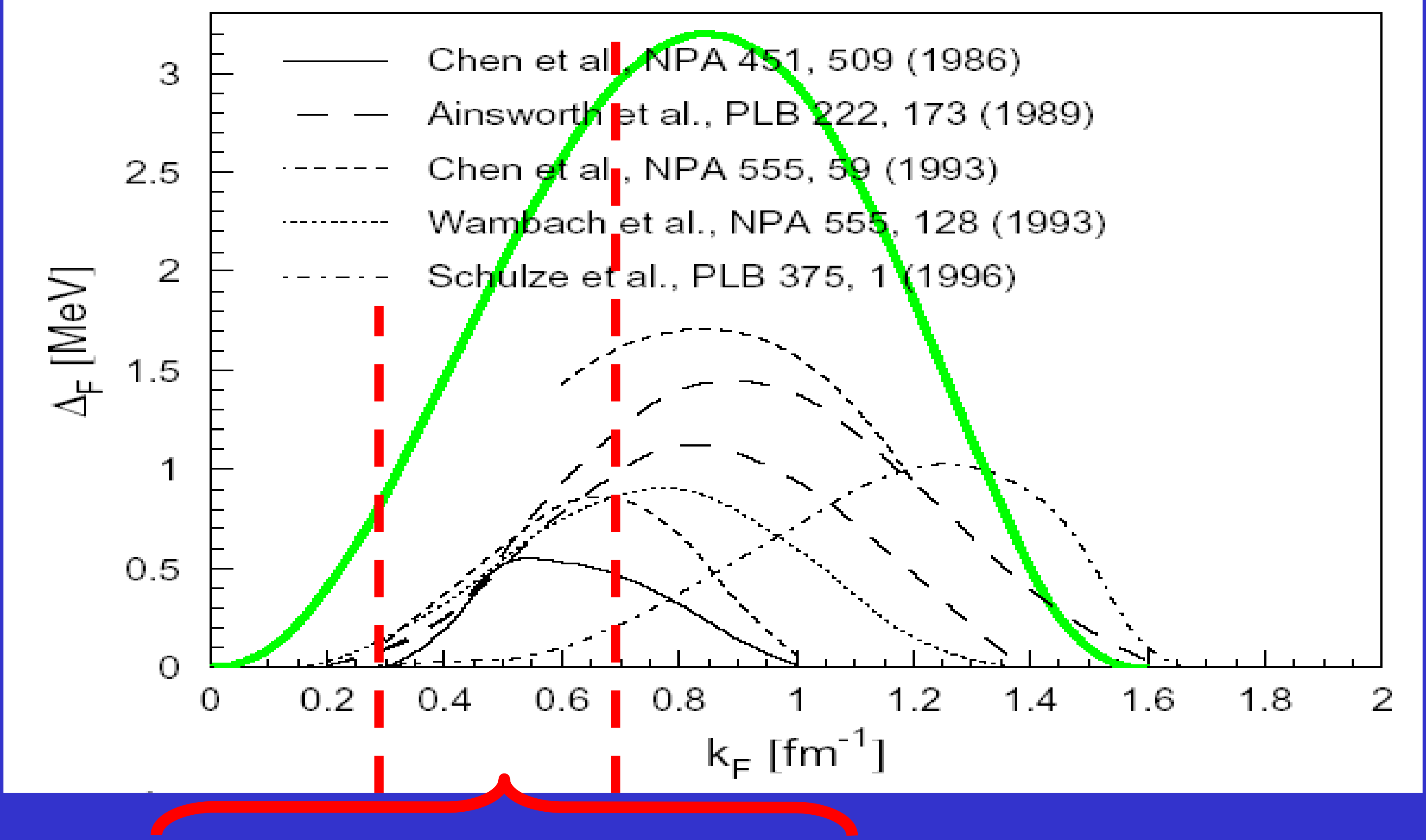
corresponds to

$n \approx 0.001 - 0.01\ \mathrm{fm}^{-3}$

$k_F \approx 0.3 - 0.7\ \mathrm{fm}^{-1}$

Neutron matter

s-wave pairing gap in infinite neutron matter with realistic NN-interactions



$n \approx 0.001 - 0.01\ \mathrm{fm}^{-3}$

$k_F \approx 0.3 - 0.7\ \mathrm{fm}^{-1}$

Grand Canonical Path-Integral Monte Carlo calculations on 4D-lattice

$$H = T + V = \int d^3r\ \hat{\psi}^\dagger(\vec{r})\left(-\frac{\hbar^2\Delta}{2m}\right)\hat{\psi}(\vec{r}) \ - \ g\int d^3r\ \hat{n}_\uparrow(\vec{r})\hat{n}_\downarrow(\vec{r})$$

$$N = \int d^3r\ \left(\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r})\right)$$

$$\frac{1}{g} = -\frac{m}{4\pi\hbar^2 a} + \frac{mk_{cut}}{2\pi^2\hbar^2}$$

Running coupling constant g defined by lattice

Trotter expansion

$$Z(\beta) = Tr\left\{\exp(-\beta(H-\mu N)\right\} \approx Tr\left\{\exp(-\tau(H-\mu N)\right\}^{N_\tau}$$

$$\beta = N_\tau \tau$$

Recast the propagator at each time slice and use FFT

$$\exp(-\tau(H-\mu N)) \approx \exp(-\tau(T-\mu N)/2)\exp(-\tau V)\exp(-\tau(T-\mu N)/2) \ + \ O(\tau^3)$$

Grand Canonical Path-Integral Monte Carlo calculations on 4D-lattice

Discrete Hubbard-Stratonovich transformation

$$\exp(-\tau V) = 2^{-N^3} \sum_{\{\sigma(\vec{r})=\pm 1\}} \exp[\int d^3r \ \left(n_\uparrow(\vec{r}) + n_\downarrow(\vec{r})\right)\left(a + b\sigma(\vec{r})\right) \ ]$$

$$\exp(a) = \sqrt{2 - \exp(\tau g)}$$

$$\exp(b) = \exp(-a)[1 \pm \sqrt{\exp(\tau g) - 1}]$$

$$\tau \in (0, \log(2)/g), \quad N^3 - lattice \quad size$$

Statistical sum:

$$Z(\beta) = const. \times \sum_{\{\sigma(\vec{r},\tau)=\pm 1\}} \det\left\{1 + \prod_{k=1}^{N_\tau} \exp\left[-\tau h\left(\sigma(k\tau)\right)\right]\right\}$$

$$h\left(\sigma(k\tau)\right) \ - \ one - body \quad hamiltonian$$

Grand Canonical Path-Integral Monte Carlo calculations on 4D-lattice

Thermal average of one-body operator

$$\langle a^\dagger(\vec{r})\,a(\vec{r}\,')\rangle = \sum_{\alpha,\beta} \sum_{\{\sigma(\vec{r},\tau)=\pm 1\}} P(\sigma)\ \varphi_\alpha(\vec{r})O_{\alpha\beta}(\sigma)\varphi^*{}_\alpha(\vec{r}\,')$$

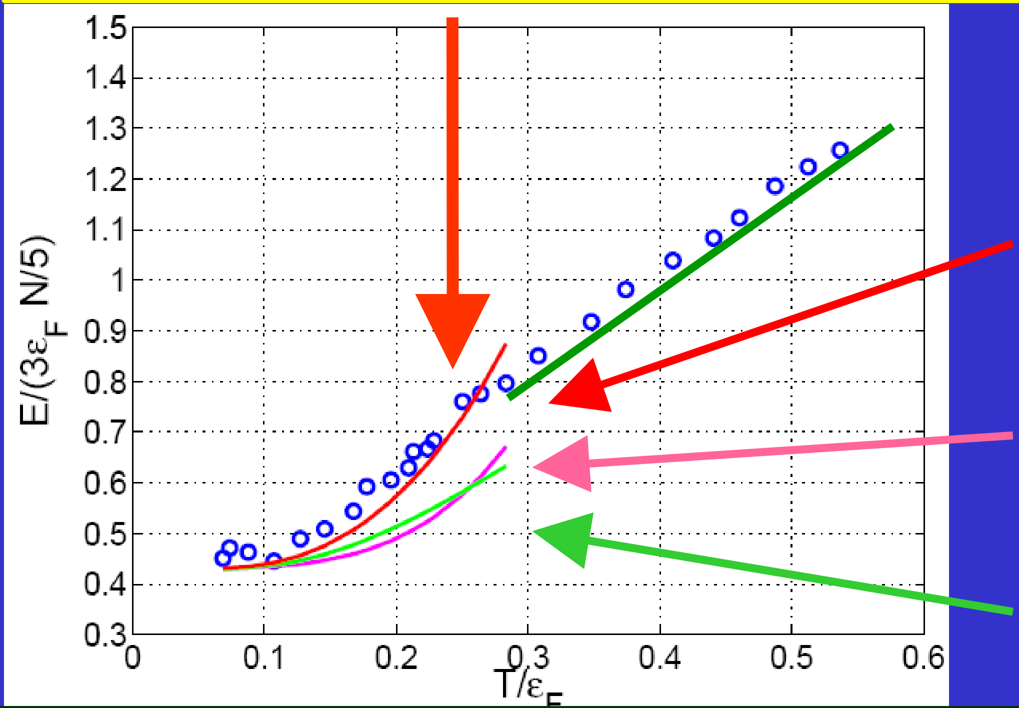
$$P(\sigma) = \det[1+\exp(-K(\sigma))] \Big/ \sum_{\{\sigma(\vec{r},\tau)=\pm 1\}} \det[1+\exp(-K(\sigma))]$$

$$O_{\alpha\beta}(\sigma) = \left[\exp(-K(\sigma)) \Big/ 1+\exp(-K(\sigma))\right]_{\alpha\beta}$$

$$\exp(-K(\sigma)) = \prod_{k=1}^{N_\tau} \exp[-\tau h(\sigma(k\tau))]$$

$$Action: -Log\left(\det[1+\exp(-K(\sigma))]\right)$$

Superfluid to Normal Fermi Liquid Transition: $T_c \approx 0.24\varepsilon_F$



Bogoliubov-Anderson phonons and quasiparticle contribution (red line)

Bogoliubov-Anderson phonons contribution only (magenta line)

Quasi-particles contribution only (green line)

$$E_{phonons}(T) = \frac{3}{5}\varepsilon_F N\left[\xi_s + \frac{\sqrt{3}}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4\right] \quad ;\xi_s = 0.44$$

$$E_{qp}(T) = \frac{3}{5}\varepsilon_F N\left[\xi_s + \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F{}^4}}\exp\left(-\frac{\Delta}{T}\right)\right]$$

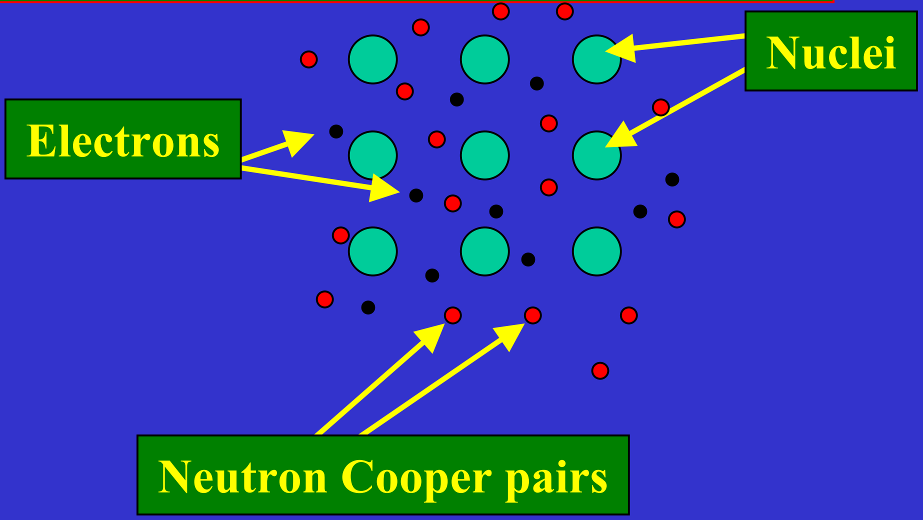
$$\Delta \approx \left(\frac{2}{e}\right)^{7/3}\varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right); \qquad \varepsilon_F = \frac{\hbar^2 k_F{}^2}{2m}$$

Lattice size:
from 6^3 x 112 at low T
to 6^3 x 30 at high T

Number of samples:
Several 10^5 for T

Limited results for 8^3 lattices

Structure of the inner crust of neutron stars

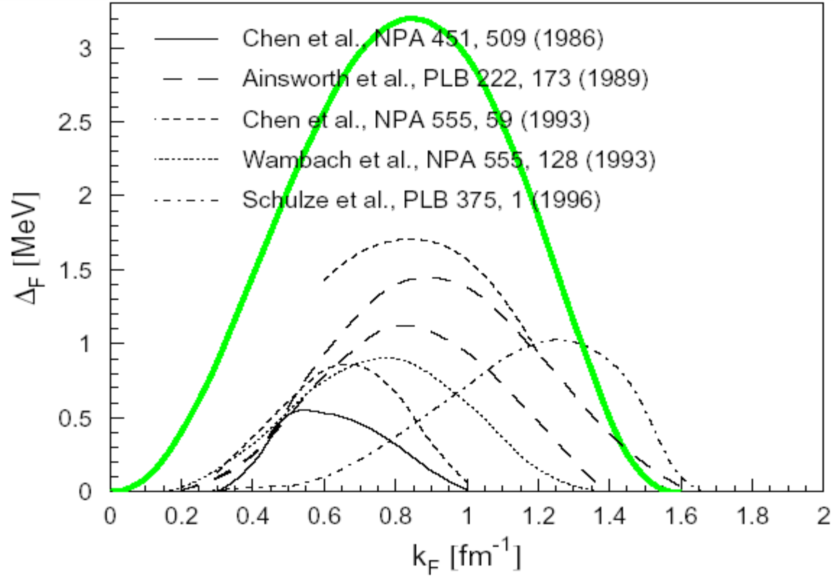


Nuclei

Electrons

Neutron Cooper pairs

s-wave pairing gap in infinite neutron matter with realistic NN-interactions

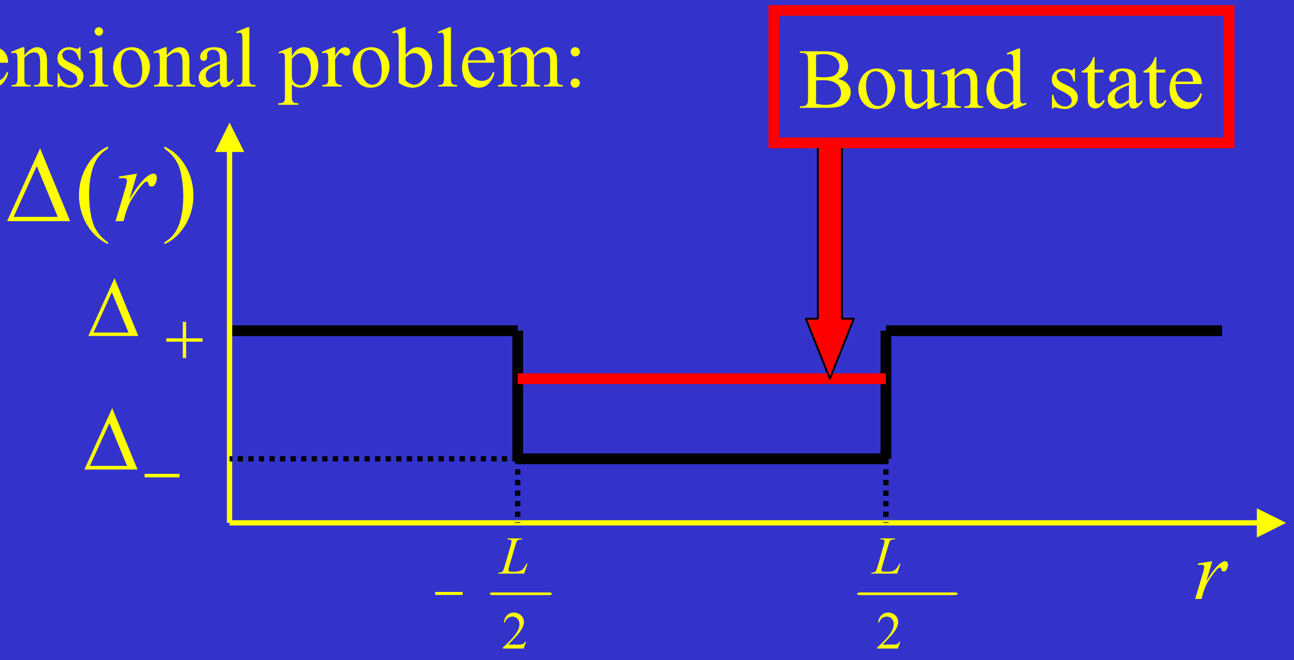


$$\begin{pmatrix} h-\mu & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & -h+\mu \end{pmatrix}\begin{pmatrix} u(\vec{r}) \\ v(\vec{r}) \end{pmatrix} = E\begin{pmatrix} u(\vec{r}) \\ v(\vec{r}) \end{pmatrix}$$

BdG eqs.

$$h = -\frac{\hbar^2}{2m}\nabla^2; \quad \Delta(\vec{r}+\vec{a}) = \Delta(\vec{r})$$

1-dimensional problem:



Andreev approximation: $\begin{pmatrix} u(r) \\ v(r) \end{pmatrix} = \begin{pmatrix} \bar{u}(r) \\ \bar{v}(r) \end{pmatrix} e^{ik_F r}$

Quantization condition: $\boxed{A(\varphi,\psi)e^{2iqL}=1}$

$$A(\varphi,\psi)=\frac{(e^{-\varphi}-e^{-i\psi})(e^{\varphi}-e^{i\psi})}{(e^{-\varphi}-e^{i\psi})(e^{\varphi}-e^{-i\psi})};\qquad \cos\psi=\frac{E}{\Delta_+}$$

$$\cosh\varphi=\frac{E}{\Delta\_}$$

$$q=\frac{m}{\hbar^2 k_F}\sqrt{E^2-\Delta\_{}^2}$$

There is always at least one bound state!

Penetration length inside a barrier Δ_+

$$\xi=\hbar^2 k_F/(m\sqrt{\Delta_+{}^2-E^2})$$

Localization condition: $\boxed{\xi < R_C - R_N}$

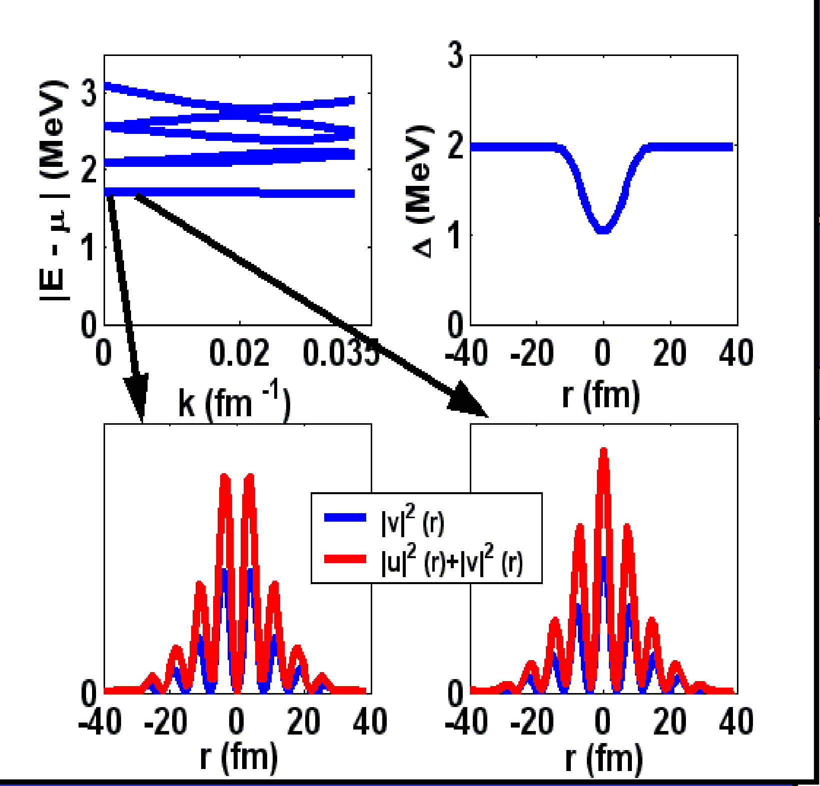
R_C – Wigner-Seitz cell radius

R_N – Nuclear radius

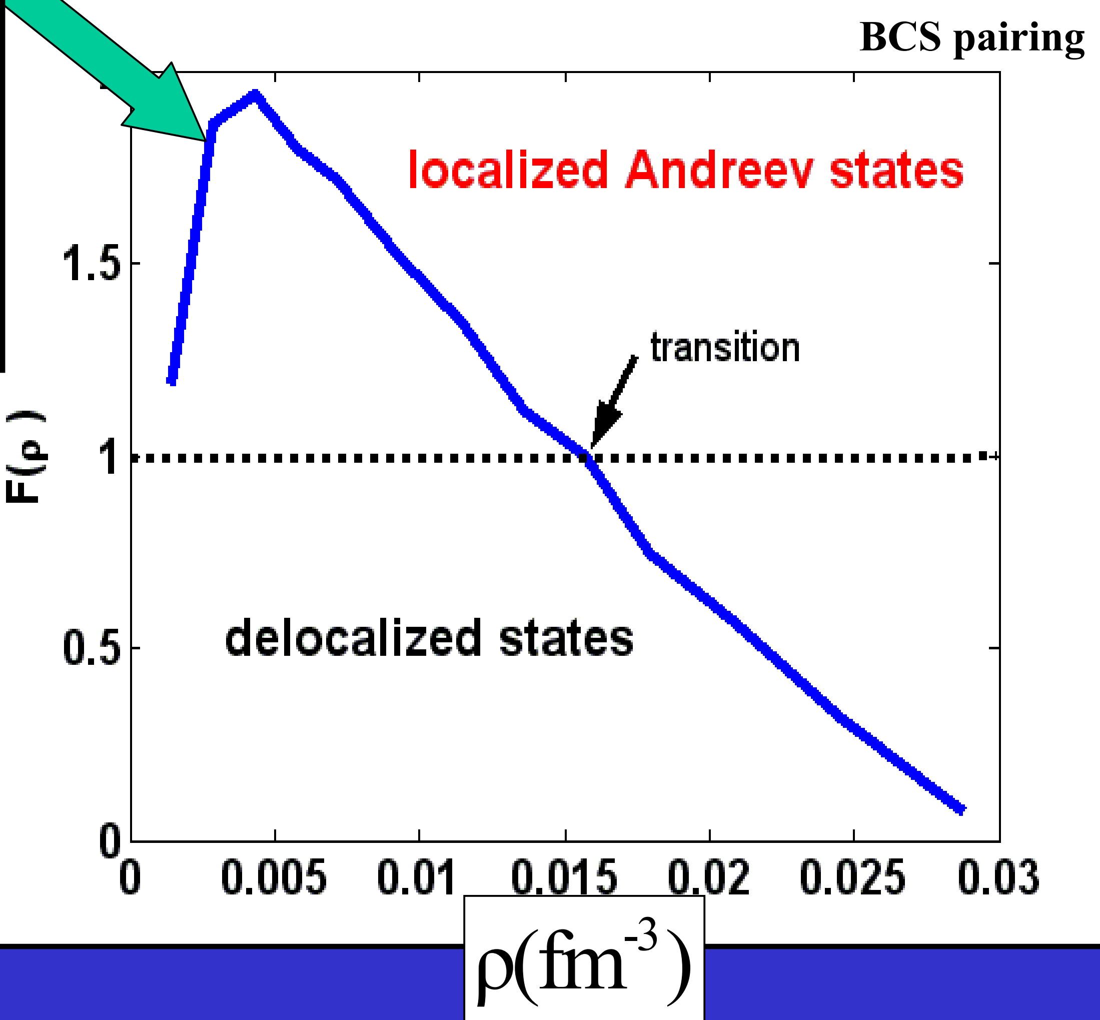
Localization condition: $\boxed{F(\rho) > 1}$

where:

$$F(\rho) = \frac{1}{2} k_F R_N \sqrt{\left(\frac{\Delta_+}{\mu}\right)^2 - \left(\frac{E}{\mu}\right)^2} \left(\frac{R_C}{R_N} - 1\right)$$



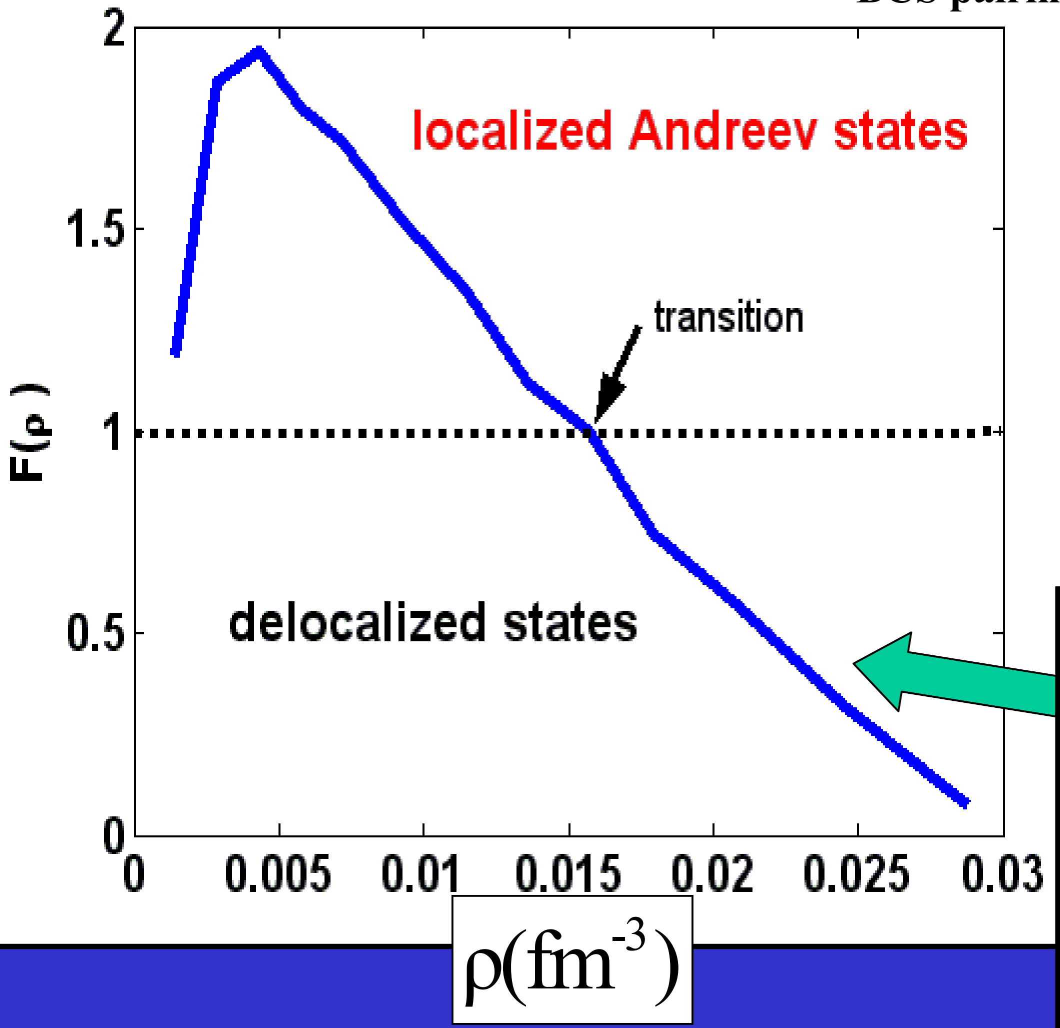
$$F(\rho)>1$$



$$\Delta(r+a)=\Delta(r)$$

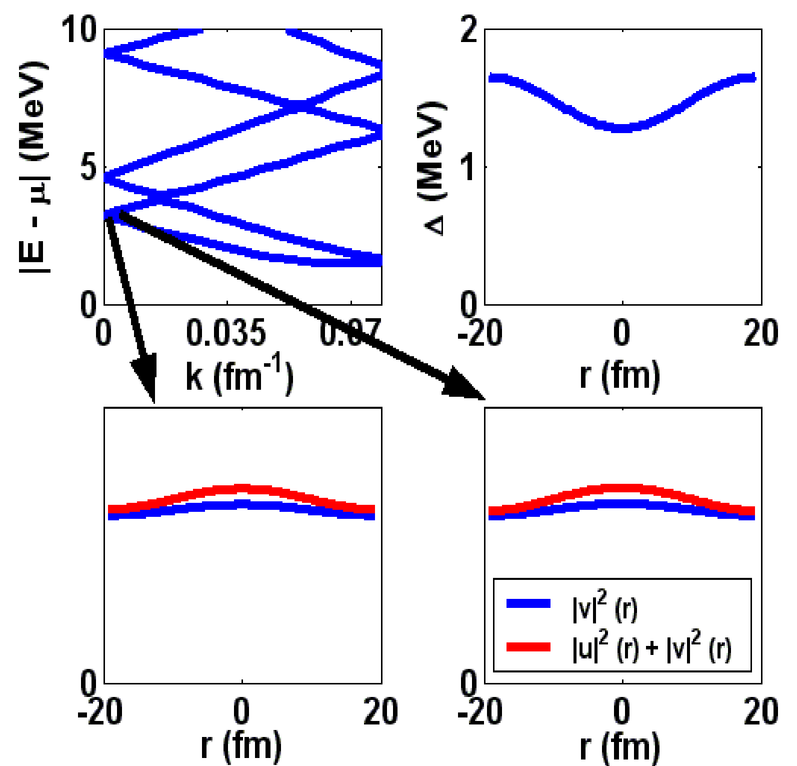
$$a=80\,fm$$

BCS pairing



$$\Delta(r+a)=\Delta(r)$$

$$a=40\,fm$$

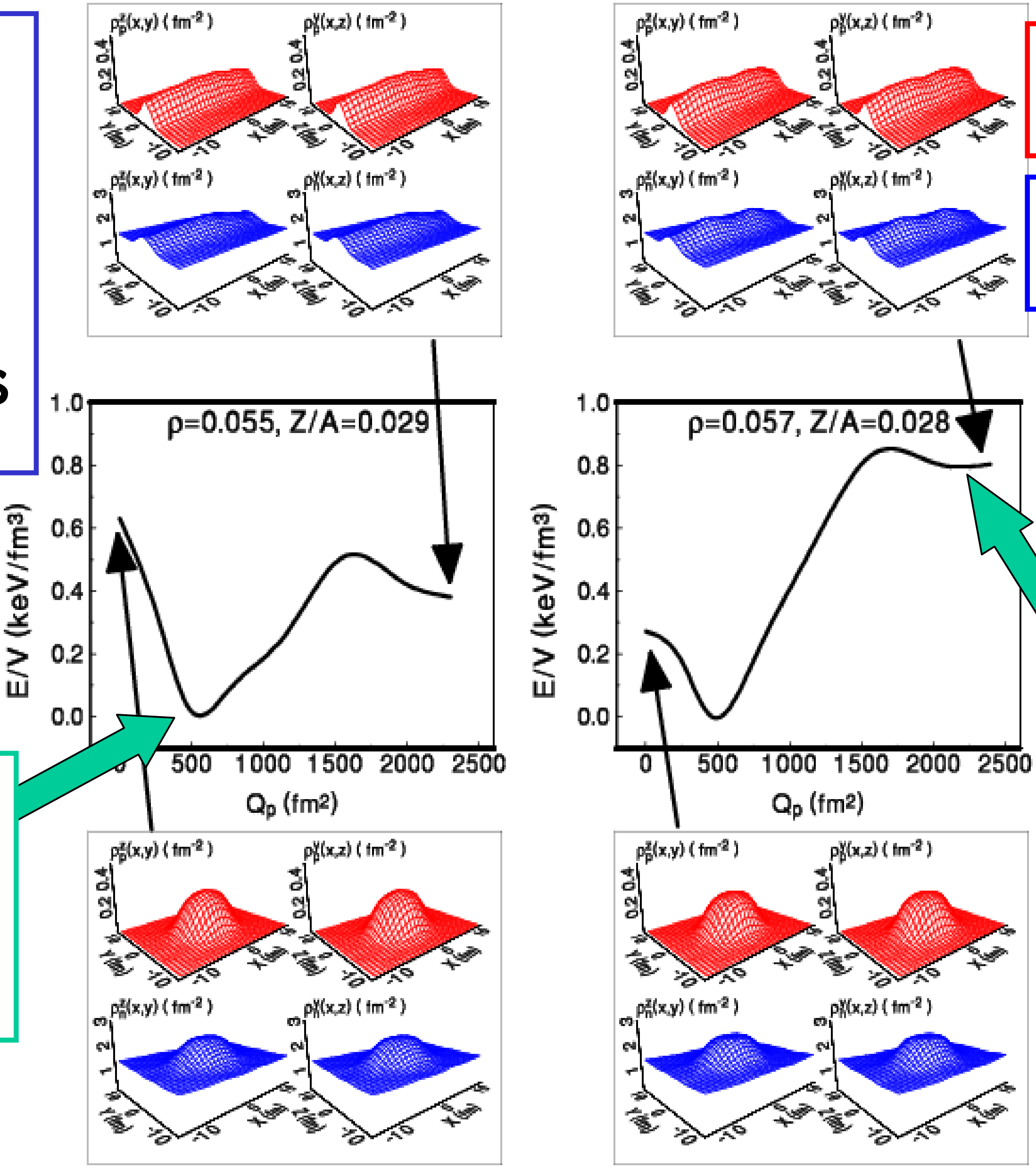


$$F(\rho)<1$$

Hartree-Fock + BCS results on the lattice

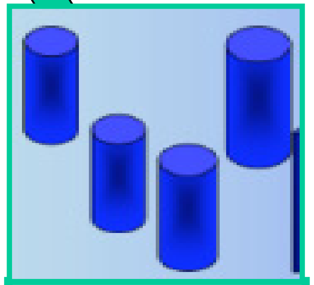
Hartree-Fock calcs. In coordinate space for 1000 nucleons in the Wigner-Seitz (WS) cell!

Coulomb interaction treated beyond the WS approximation!

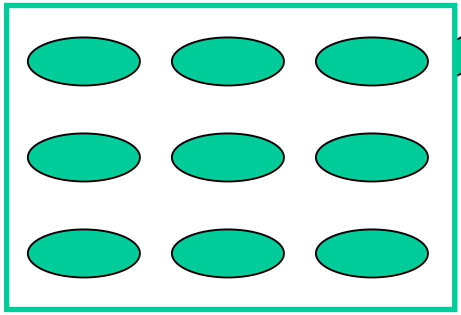


Proton density distribution

Neutron density distribution

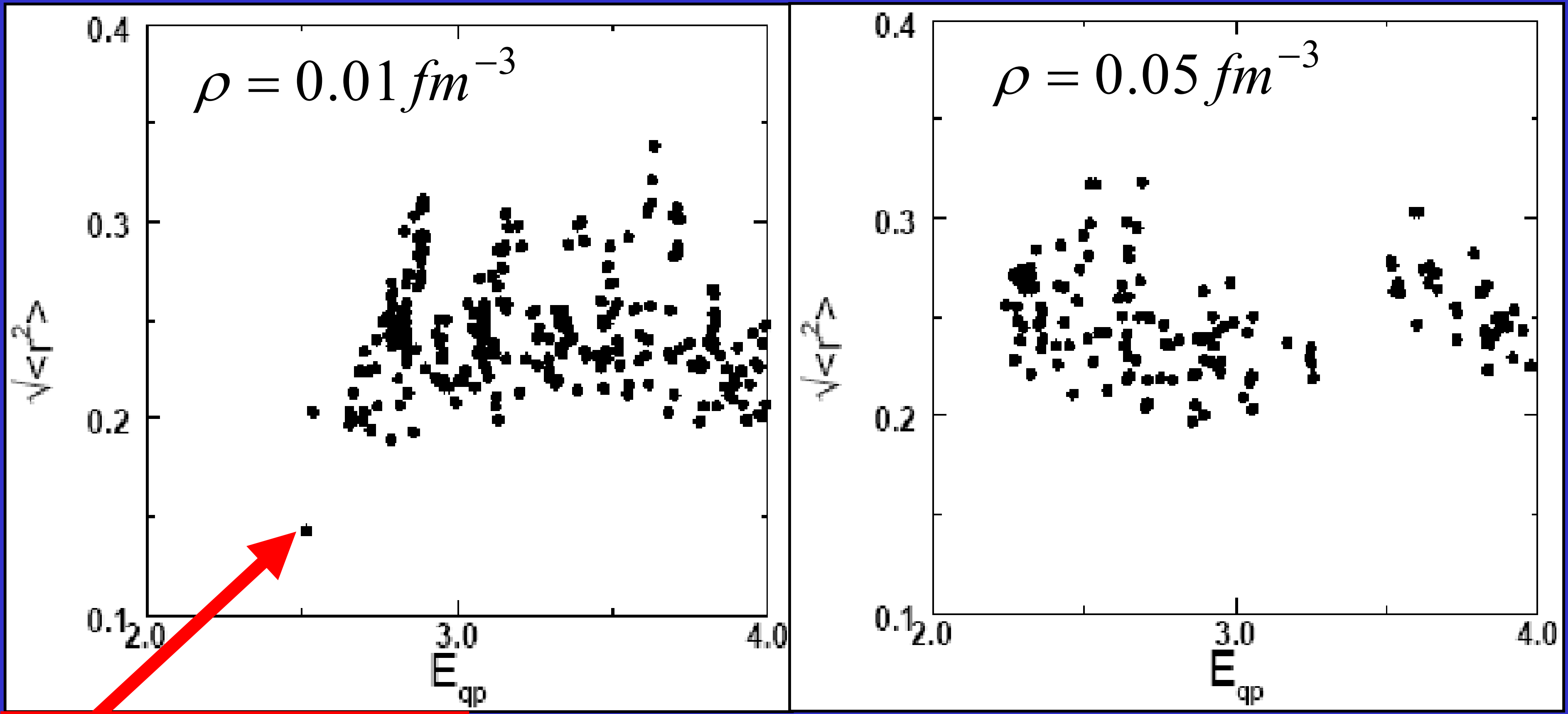


‘Spaghetti’ phase



Hartree-Fock + BCS results on the lattice

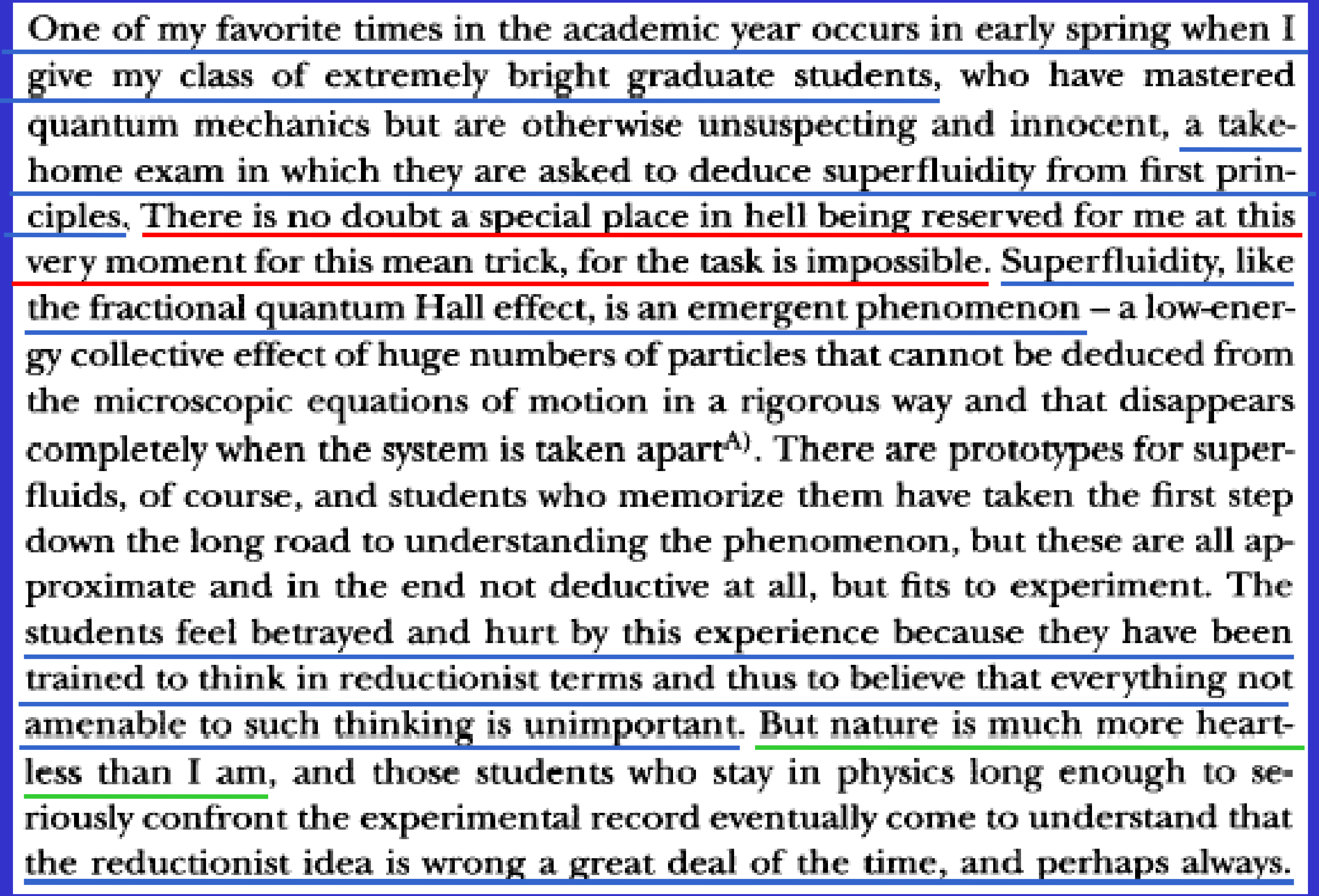
Periodic boundary condition: pseudomomentum=0



Localized state

Conclusions

- **At low densities in the inner crust neutrons around the Fermi level may be localized due to the inhomogeneity of the pairing field.**
- **Bragg scattering on the pairing field is important!**
- **Influence on the transport properties (thermal conductivity) across the crust? Nucleon effective mass?**



One of my favorite times in the academic year occurs in early spring when I give my class of extremely bright graduate students, who have mastered quantum mechanics but are otherwise unsuspecting and innocent, a take-home exam in which they are asked to deduce superfluidity from first principles. There is no doubt a special place in hell being reserved for me at this very moment for this mean trick, for the task is impossible. Superfluidity, like the fractional quantum Hall effect, is an emergent phenomenon – a low-energy collective effect of huge numbers of particles that cannot be deduced from the microscopic equations of motion in a rigorous way and that disappears completely when the system is taken apart[A]. There are prototypes for superfluids, of course, and students who memorize them have taken the first step down the long road to understanding the phenomenon, but these are all approximate and in the end not deductive at all, but fits to experiment. The students feel betrayed and hurt by this experience because they have been trained to think in reductionist terms and thus to believe that everything not amenable to such thinking is unimportant. But nature is much more heartless than I am, and those students who stay in physics long enough to seriously confront the experimental record eventually come to understand that the reductionist idea is wrong a great deal of the time, and perhaps always.

Robert B. Laughlin, Nobel Lecture, December 8, 1998