Vortices in ultracold Fermi gases: peculiarity of their structure and impact on dynamics

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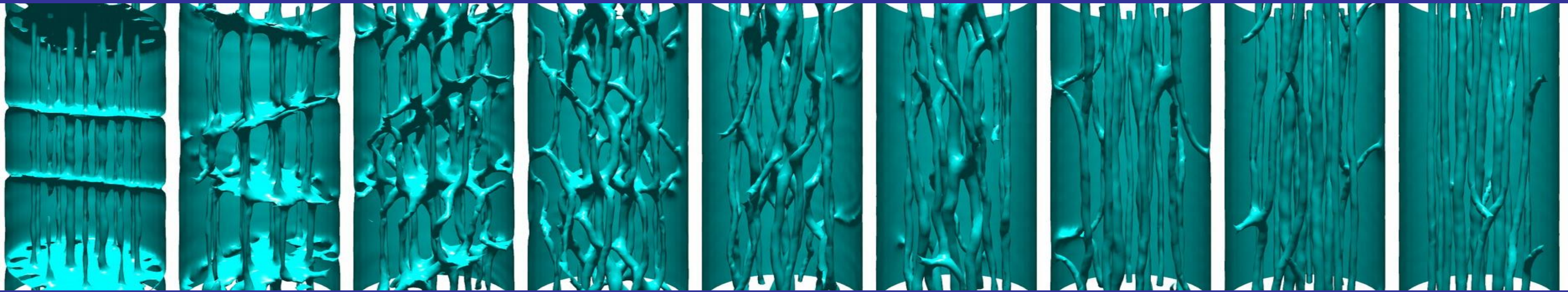

Generation and decay of fermionic turbulence

Collaborators:

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Konrad Kobuszewski (WUT - PhD student)

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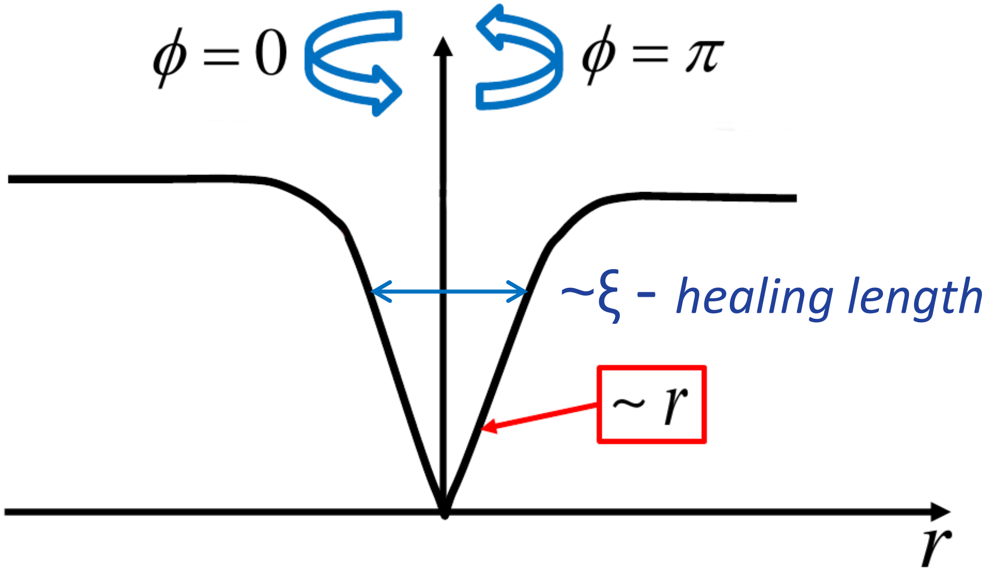
Conference on Quantum-Many-Body Correlations in memory of Peter Schuck (QMBC 2023)

Anatomy of the vortex core

Bosonic vortex structure:
weakly interacting Bose gas at T=0 → Gross-Pitaevskii eq. (GPE)

$$\left[-\frac{1}{2m}\nabla^2 + g|\psi(\vec{r})|^2 + V_{ext}(\vec{r})\right]\psi(\vec{r}) = \mu\psi(\vec{r})$$



Order parameter:

$$\psi(\vec{r}) = \sqrt{n(\vec{r})}e^{i\phi}$$

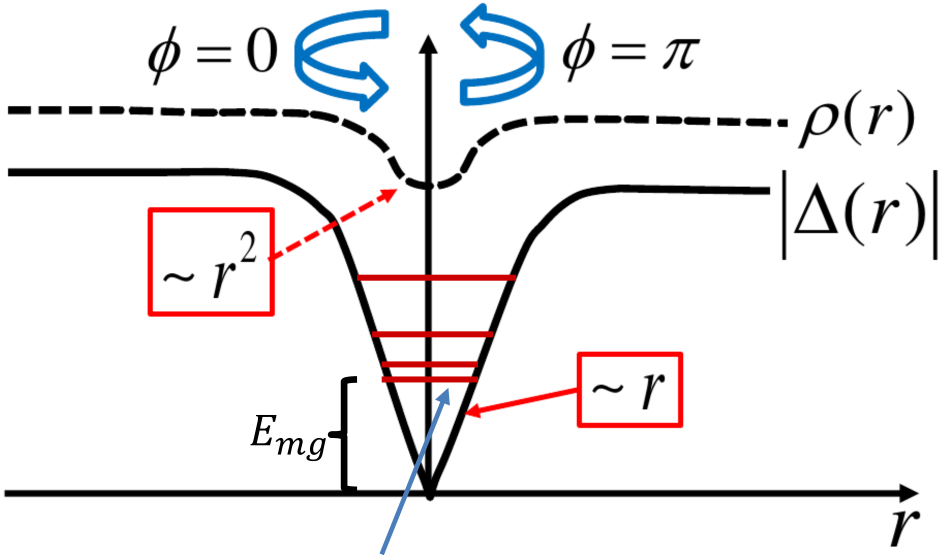
$$v_S = \frac{\hbar}{m}\nabla\phi$$

$$\kappa = \oint d\vec{l}\cdot\vec{v}_S = \frac{\hbar}{m}$$

Fermionic vortex structure:

Weakly interacting Fermi gas → Bogoliubov de Gennes (BdG) eqs.

$$\begin{pmatrix} h_\uparrow & \Delta \\ \Delta^* & -h_\downarrow^* \end{pmatrix} \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix} = \varepsilon_n \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix}$$



Form of the vortex-like solutions:

$$u_\eta(\mathbf{r}) = u_{nmk_z}(\rho) e^{im\varphi} e^{ik_z z}$$
$$v_\eta(\mathbf{r}) = v_{nmk_z}(\rho) e^{i(m+1)\varphi} e^{ik_z z}$$

CdGM (Andreev) states

C. Caroli, P. de Gennes, J. Matricon, Phys. Lett. 9, 307 (1964):

Minigap: $E_{mg} \sim \dfrac{|\Delta_\infty|^2}{\varepsilon_F}$ - energy scale for vortex core excitations.

Density of states: $g(\varepsilon) \sim \dfrac{\varepsilon_F}{|\Delta_\infty|^2}$; $\varepsilon \ll |\Delta_\infty|$

What happens between deep BEC and BCS limits?

Within pure BdG framework: $$E = \frac{1}{2}\tau + g\nu^{*}\nu$$

one **still gets localized core states in the unitary regime** and they disappear slightly on the BEC side.

Sensarma, Randeria, Ho, Phys. Rev. Lett. 96, 090403 (2006).

This is confirmed within Superfluid Local Density Approximation (SLDA) framework and the extension of SLDA (SLDAE) towards BCS limit.

$$E = \frac{1}{2}A(k_F a)\tau + \frac{3}{5}B(k_F a)n\varepsilon_F + \frac{C(k_F a)}{n^{1/3}}\nu^{*}\nu$$

TABLE II. Properties of superfluid vortices at $T = 0.05\,T_F$ for selected value of the s-wave scattering length obtained using the SLDAE functional. The length scales of the vortex state (density at the center of the vortex core n_v according to the bulk density $n_0 = k_F^3/3\pi^2$, the coherence length l_c obtained with eq. (25), and the vortex core radius r_v) are given by the first block, and the energy scales (the pairing gap in the bulk Δ_0, the mini-gap energy $E_{m.g.}$, and the critical temperature T_c) are provided by the second block. The error bars for the vortex core radius r_v are due to lattice spacing uncertainty.

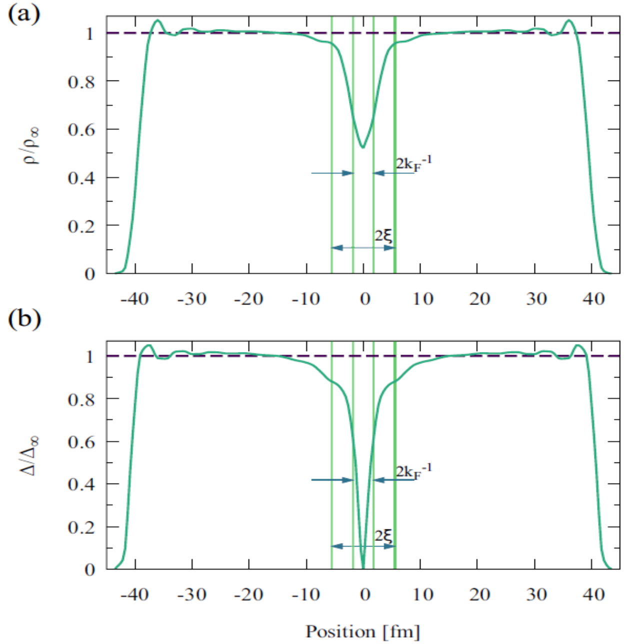
$\lvert ak_F \rvert$		1.00	1.50	2.00	2.50	3.33	5.00	10.0	20.0	50.0	∞
n_v	$[n_0]$	0.963	0.849	0.718	0.623	0.524	0.427	0.337	0.296	0.274	0.262
l_c	$[k_F^{-1}]$	5.519	2.874	2.241	1.950	1.706	1.503	1.342	1.282	1.253	1.238
r_v	$[k_F^{-1}]$	9.4(1)	3.7(1)	2.5(1)	2.1(1)	1.7(1)	1.4(1)	1.2(1)	1.1(1)	1.0(1)	1.0(1)
$\lvert\Delta_0\rvert$	$[\varepsilon_F]$	0.108	0.201	0.251	0.283	0.317	0.351	0.388	0.408	0.422	0.431
$E_{m.g.}$	$[\varepsilon_F]$	0.009	0.018	0.034	0.048	0.066	0.087	0.112	0.127	0.137	0.144
T_c	$[T_F]$	0.085	0.137	0.173	0.199	0.227	0.259	0.291	0.304	0.309	0.311

UFG

A. Boulet, G. Wlazłowski, P.M., Phys. Rev A106, 013306 (2022)

Example: vortices across the neutron star crust

Section through the vortex core

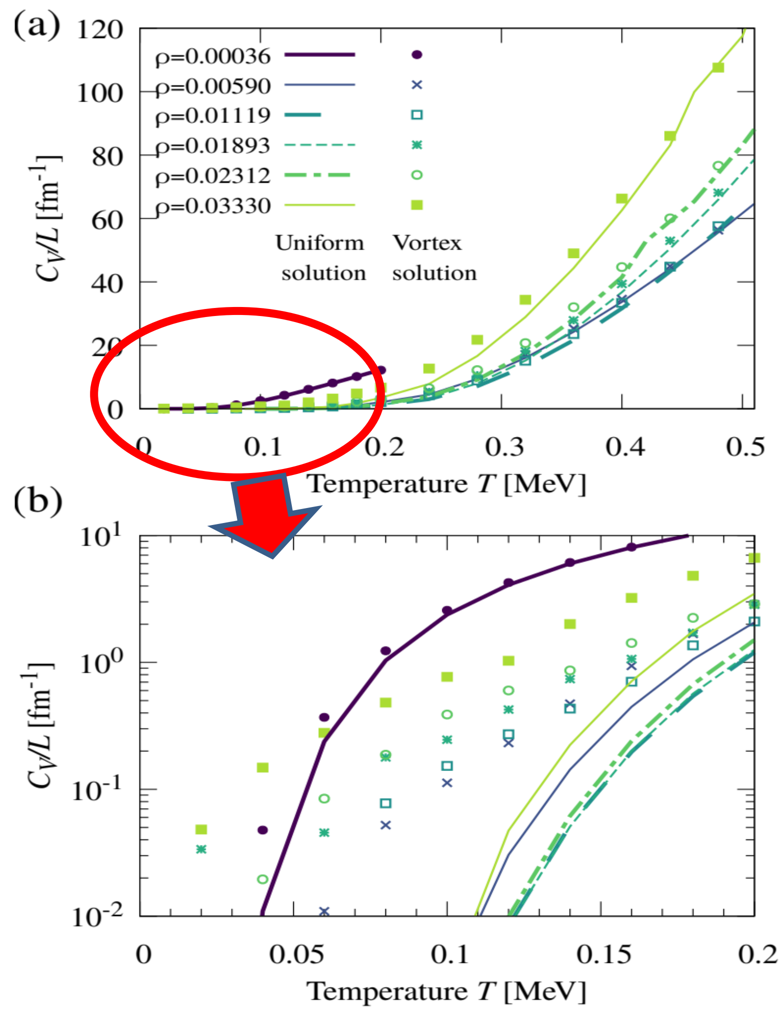


Normal density

Pairing field

Note two different length scales inside the core as explained by:
Sensarma, Randeria, Ho, Phys. Rev. Lett. 96, 090403 (2006)

Specific heat contribution vs uniform matter



ρ_∞ (fm^{-3})	0.00036	0.0059	0.0112	0.0189	0.0231	0.0333
k_F^{-1} (fm)	4.52	1.79	1.45	1.21	1.14	1.01
ξ (fm)	8.44	5.53	5.97	7.00	7.78	10.28
R_{VFM} (fm)	15.0	10.5	10.5	12.0	13.5	16.5
Δ_∞ (MeV)	0.35	1.33	1.53	1.55	1.50	1.28
T_{crit} (MeV)	0.20	0.76	0.87	0.88	0.85	0.73
ε_{F} (MeV)	1.01	6.48	9.93	14.09	16.10	20.53
μ (MeV)	0.80	4.21	5.80	7.30	7.91	9.09
E_{mg} (MeV)	0.090	0.308	0.261	0.199	0.152	0.009
B_{crit} (10^{15} G)	7.76	26.5	22.5	17.2	13.1	0.82

Minigap values

Magnetic field needed to polarize the core

D. Pęcak, N. Chamel, P.M., G. Wlazłowski, Phys. Rev. C**104**, 055801 (2021)

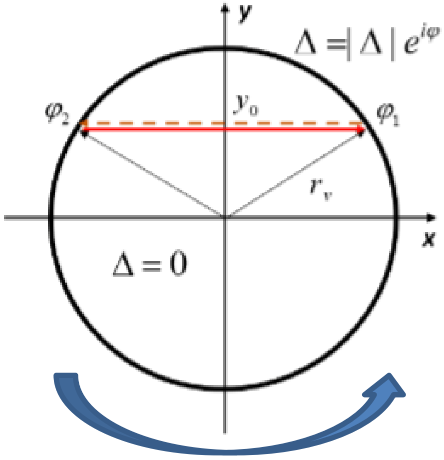
Vortex core structure in Andreev approximation:

$$\frac{E(0,L_z)}{\varepsilon_F}k_F r_V\sqrt{1-\left(\frac{L_z}{k_F r_V}\right)^2}+\arccos\left(\frac{-L_z}{k_F r_V}\right)-\arccos\left(\frac{E(0,L_z)}{|\Delta_\infty|}\right)=0$$

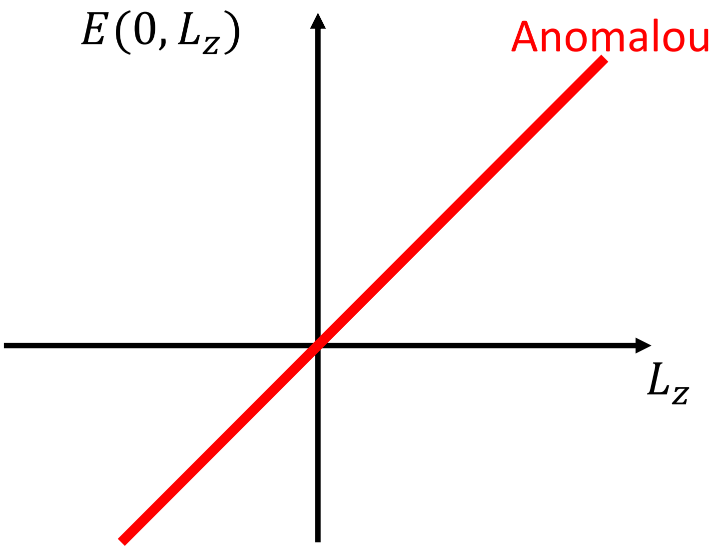
$$E(0,L_z)=E(0)L_z,\ E\ll|\Delta_\infty|$$

$$E(0,L_z)\approx\frac{|\Delta_\infty|^2}{\varepsilon_F\frac{r_V}{\xi}\left(\frac{r_V}{\xi}+1\right)}\frac{L_z}{\hbar},\qquad \xi=\frac{\varepsilon_F}{k_F|\Delta_\infty|}$$

Schematic section of the core



Spectrum of in-gap states



Works well in deep BCS limit: $\frac{1}{k_F a_s}\ll 0$

M. Stone, Phys. Rev. B 54, 13222 (1996)
P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

Quasiparticle mobility along the vortex line

$$E(k_z) = \frac{E(0)}{\sqrt{1-\left(\frac{k_z}{k_F}\right)^2}} \; ; \; k_z < k_F$$

C. Caroli, P. de Gennes, J. Matricon, Phys. Lett. 9, 307 (1964):

In Andreev approximation:

$$\sqrt{\varepsilon_F + E}\,\sin\alpha = \sqrt{\varepsilon_F - E}\,\sin\beta$$

$$k_h = \sqrt{2(\varepsilon_F - E)}$$

$$k_p = \sqrt{2(\varepsilon_F + E)}$$

$$v_z = k_z \frac{\sqrt{k_p^2 - k_z^2} - \sqrt{k_h^2 - k_z^2}}{\sqrt{k_p^2 - k_z^2} + \sqrt{k_h^2 - k_z^2}}$$

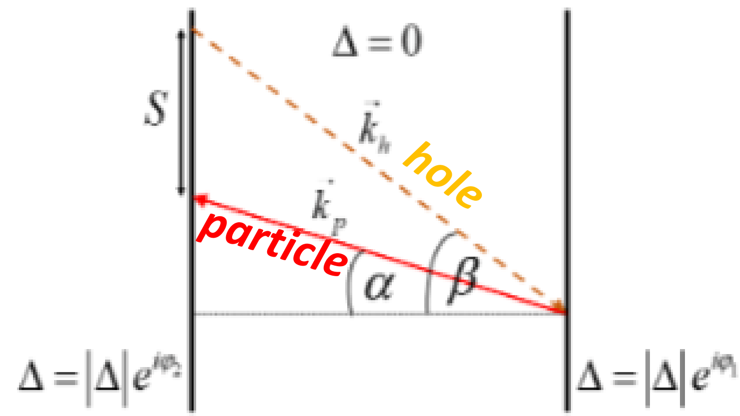
Velocity component along the vortex line

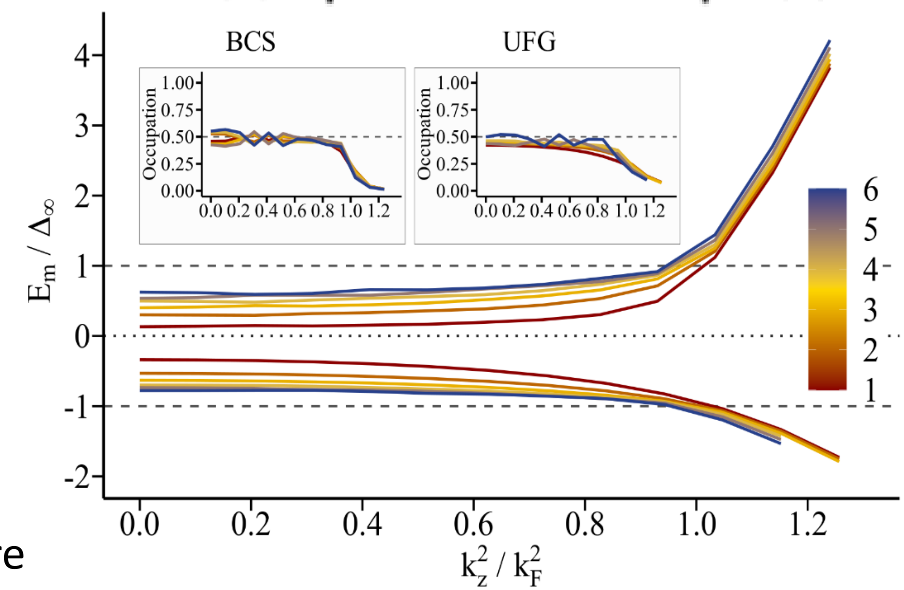
It gives the same dispersion relations as above up to the second order.

$$M_{eff}^{-1}(L_z) \approx \frac{2}{3}\left(\frac{|\Delta_\infty|}{\varepsilon_F}\right)^2 \frac{L_z}{\hbar}$$

Effective mass of quasiparticle in the core carrying ang. mom. Lz

Schematic picture of Andreev reflection of particle-hole moving along the vortex line





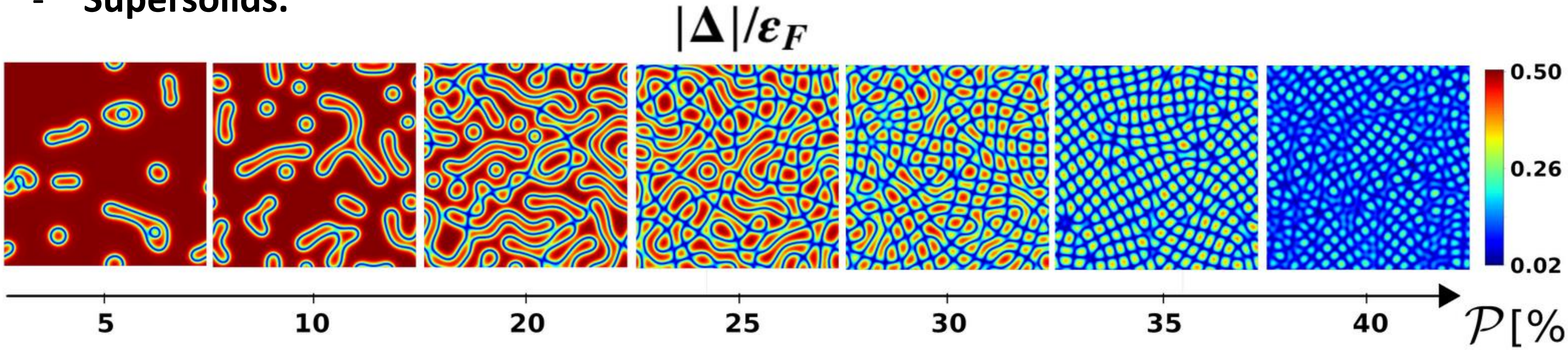
P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

Note that large value of effective mass along the vortex line originate from the fact that the occupations of hole and particle states below the gap are approximately equal.

What is going to happen if we introduce spin imbalance?

In general it will generate distortions of Fermi spheres locally and triggering the appearance of **pairing field inhomogeneity** leading to various patterns involving:

- **Separate impuritites (ferrons),**
- **Liquid crystal-like structure,**
- **Supersolids.**



B. Tüzemen, T. Zawiślak, P.M., G. Wlazłowski, New J. Phys. 2023 (in press) - see also poster

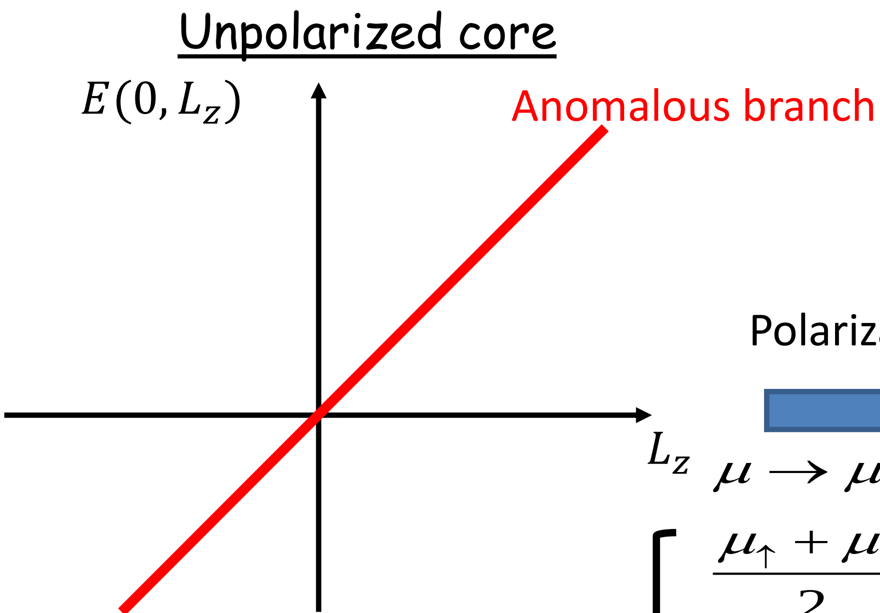
$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

Suppose however that polarization is weak enough, so it does not affect the bulk and only the core of the vortex.

Note that due to the fact that: $E_{mg} < \Delta_\infty$

the core will always be affected by polarization before the bulk will respond.
It implies also that the vortex core will „suck in" the majority spin particles from the bulk whenever such possibility occurs.

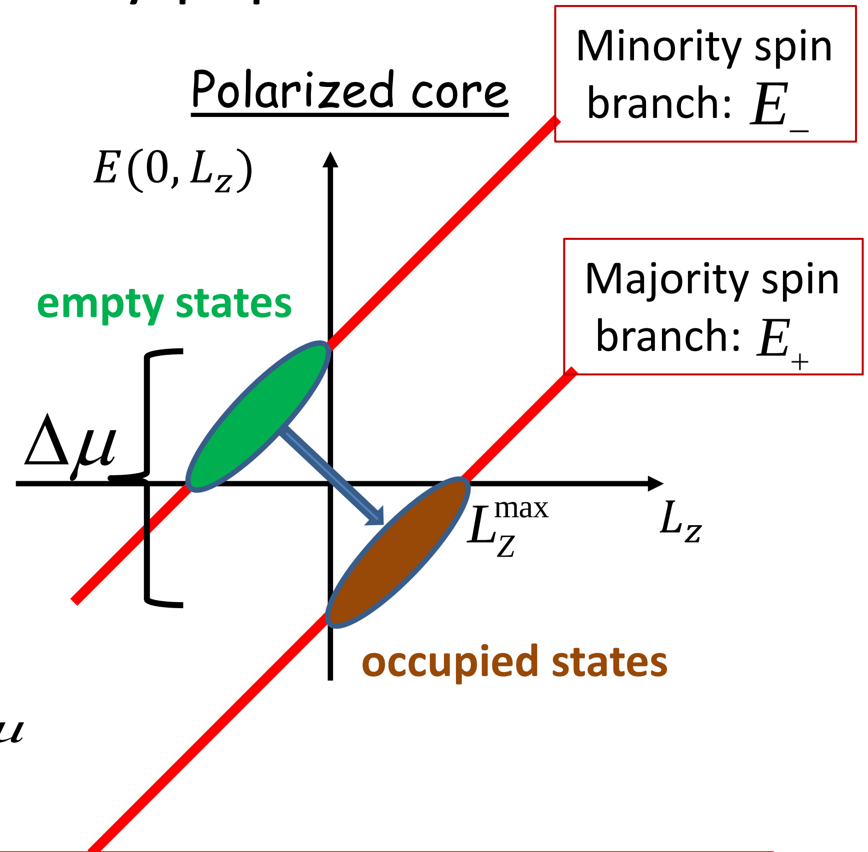
Changes of the core structure induced by spin polarization



Polarization

$$\mu \to \mu_\uparrow, \mu_\downarrow$$

$$\begin{cases} \dfrac{\mu_\uparrow + \mu_\downarrow}{2} = \mu \\ \mu_\uparrow - \mu_\downarrow = \Delta\mu \end{cases}$$



Branches are split proportionally to polarization

$$E_\pm(0, L_Z) \approx \frac{|\Delta_\infty|^2}{\varepsilon_F \dfrac{r_V}{\xi}\left(\dfrac{r_V}{\xi}+1\right)} \frac{L_Z}{\hbar} \mp \frac{\Delta\mu}{2}$$

Certain fraction of majority spin particles rotate in the opposite direction!

$$L_Z^{\max} \approx \frac{1}{2} \frac{\varepsilon_F}{|\Delta_\infty|^2} \frac{r_V}{\xi}\left(\frac{r_V}{\xi}+1\right) \hbar\Delta\mu$$

Two consequences of vortex core polarization:

1) Minigap vanishes.

2) Direction of the current in the core reverses.

1) Since the polarization correspond to relative shift of anomalous branches therefore the quasiparticle spectrum of spin-up and spin-down components is asymmetric for $k_Z = 0$.

However the symmetry of the spectrum has to be restored in the limit of $k_Z \to \infty$.
Since for a straight vortex one can decouple the degree of freedom along the vortex line:

$$H = \begin{pmatrix} h_{2D}(\mathbf{r}) + \frac{1}{2}k_z^2 - \mu_\uparrow & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h^*_{2D}(\mathbf{r}) - \frac{1}{2}k_z^2 + \mu_\downarrow \end{pmatrix}$$

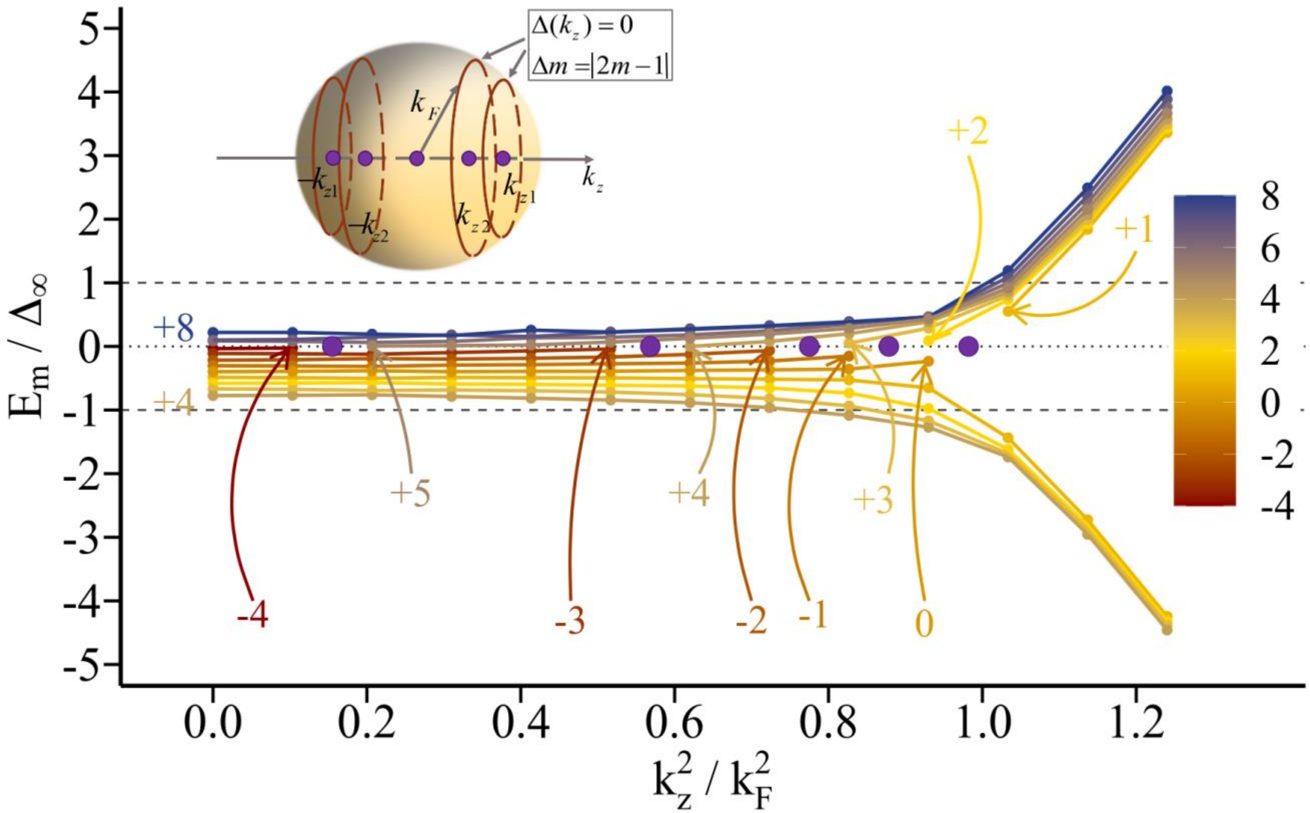
therefore $E(k_Z) \propto \pm k_Z^2$ when $k_Z \to \infty$

As a result there must exist a sequence of values: $k_Z = \pm k_{Z1}, \pm k_{Z2}, \ldots$
for which:

$$E(\pm k_{Zi}) = 0$$

Moreover the crossings occur between levels of particular projection of angular momentum on the vortex line.

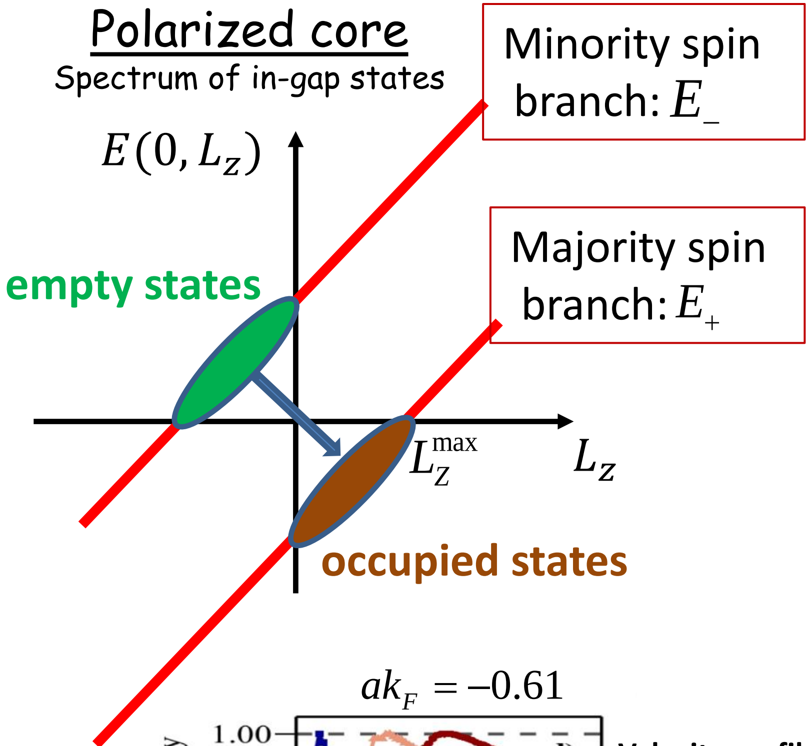
Namely, the crossing occurs in such a way that the particle state: $v_{\uparrow}$ of ang. momentum ***m*** is converted into a hole $u_{\uparrow}$ of momentum ***–m+1***
Hence the configuration changes by $\Delta m = |2m-1|$

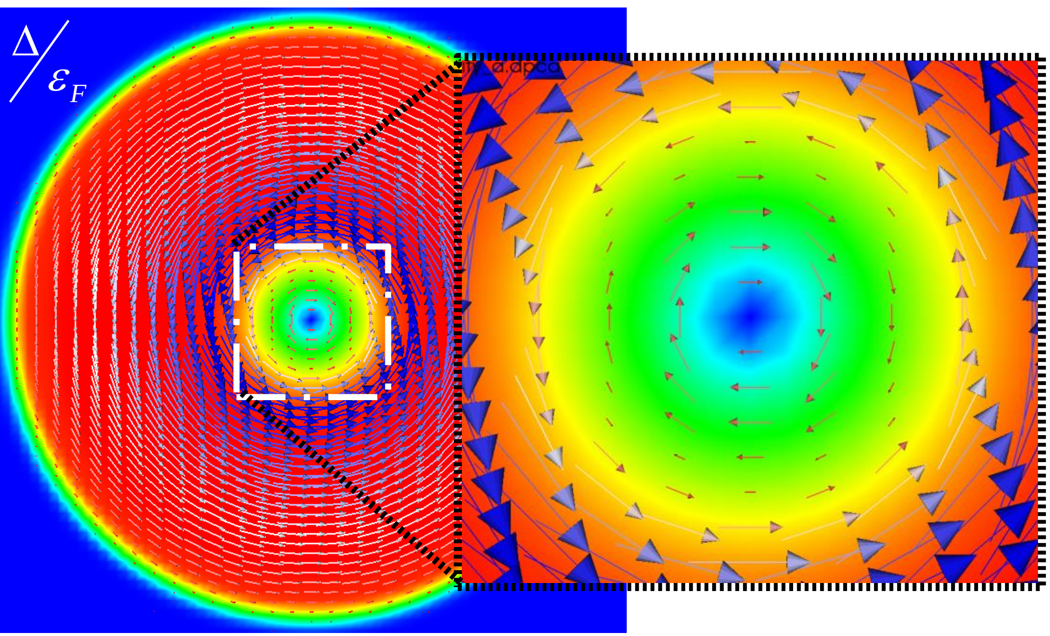


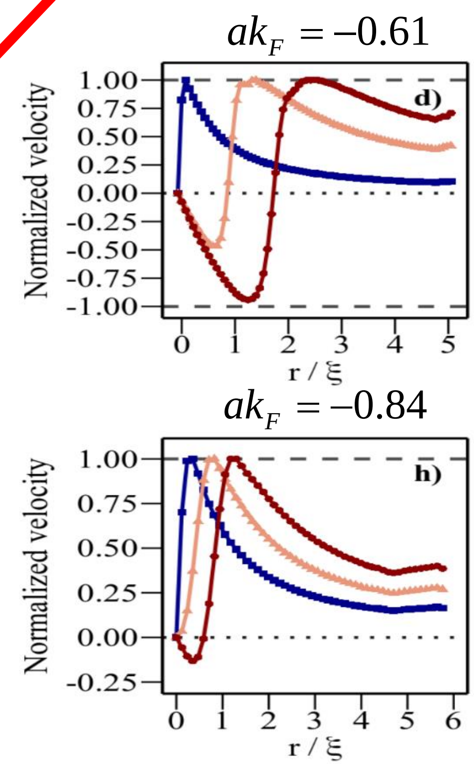
P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

2) Since fraction of majority spin particles **rotate in opposite direction to the vortex circulation** therefore they cancel contribution of low energy minority spin particles occupying anomalous branch.

As a result the flow in the core, originating from anomalous branch, is almost zero. What is left comes from other states and produce **net reversed current in the center**







Velocity profiles in the core for 3 different polarizations:

$\Delta\mu / \varepsilon_F$
0.00
0.12
0.15

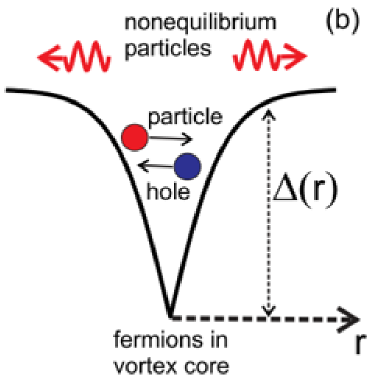
P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

How can we measure the influence of core states in ultracold gases?

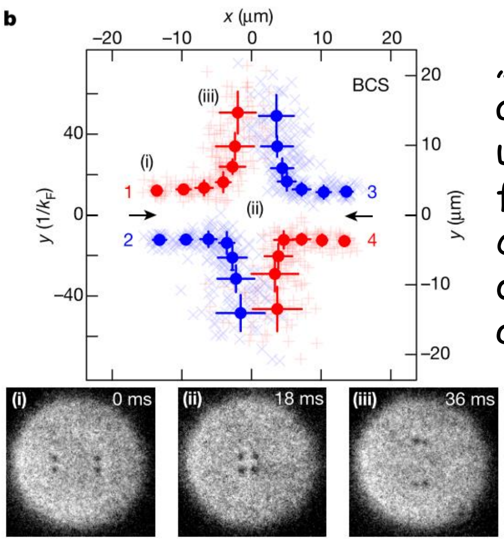
Dissipative processes involving vortex dynamics.

Silaev, Phys. Rev. Lett. 108, 045303 (2012)
Kopnin, Rep. Prog. Phys. 65, 1633 (2002)
Stone, Phys. Rev. B54, 13222 (1996)
Kopnin, Volovik, Phys. Rev. B57, 8526 (1998)
....

Classical treatment of states in the core (Boltzmann eq.).
More applicable in deep BCS limit unreachable in ultracold atoms.



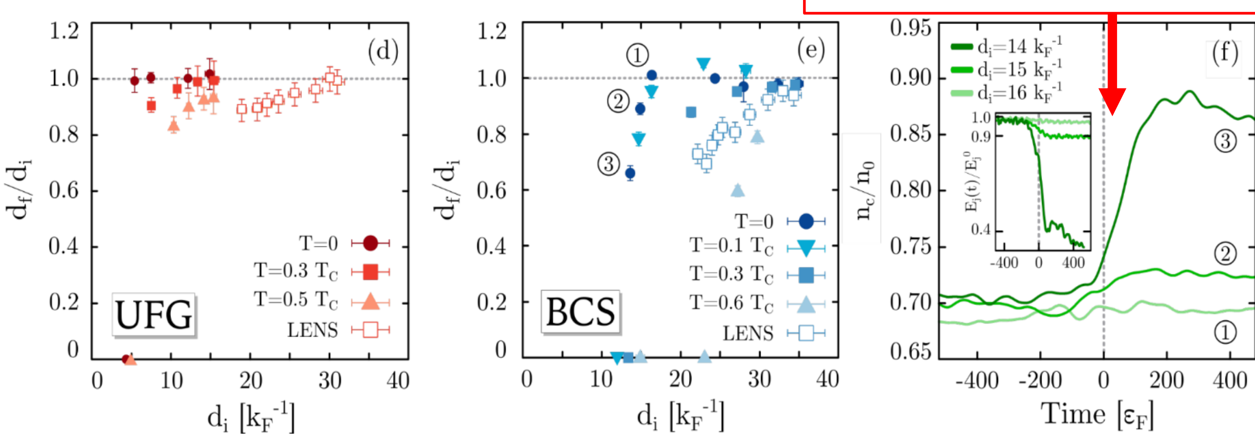
Vortex-antivortex scattering in 2D



„Further, our few-vortex experiments extending across different superfluid regimes reveal non-universal dissipative dynamics, suggesting that fermionic quasiparticles localized inside the vortex core contribute significantly to dissipation, thereby opening the route to exploring new pathways for quantum turbulence decay, vortex by vortex."

W.J. Kwon et al. Nature **600**, 64 (2021)

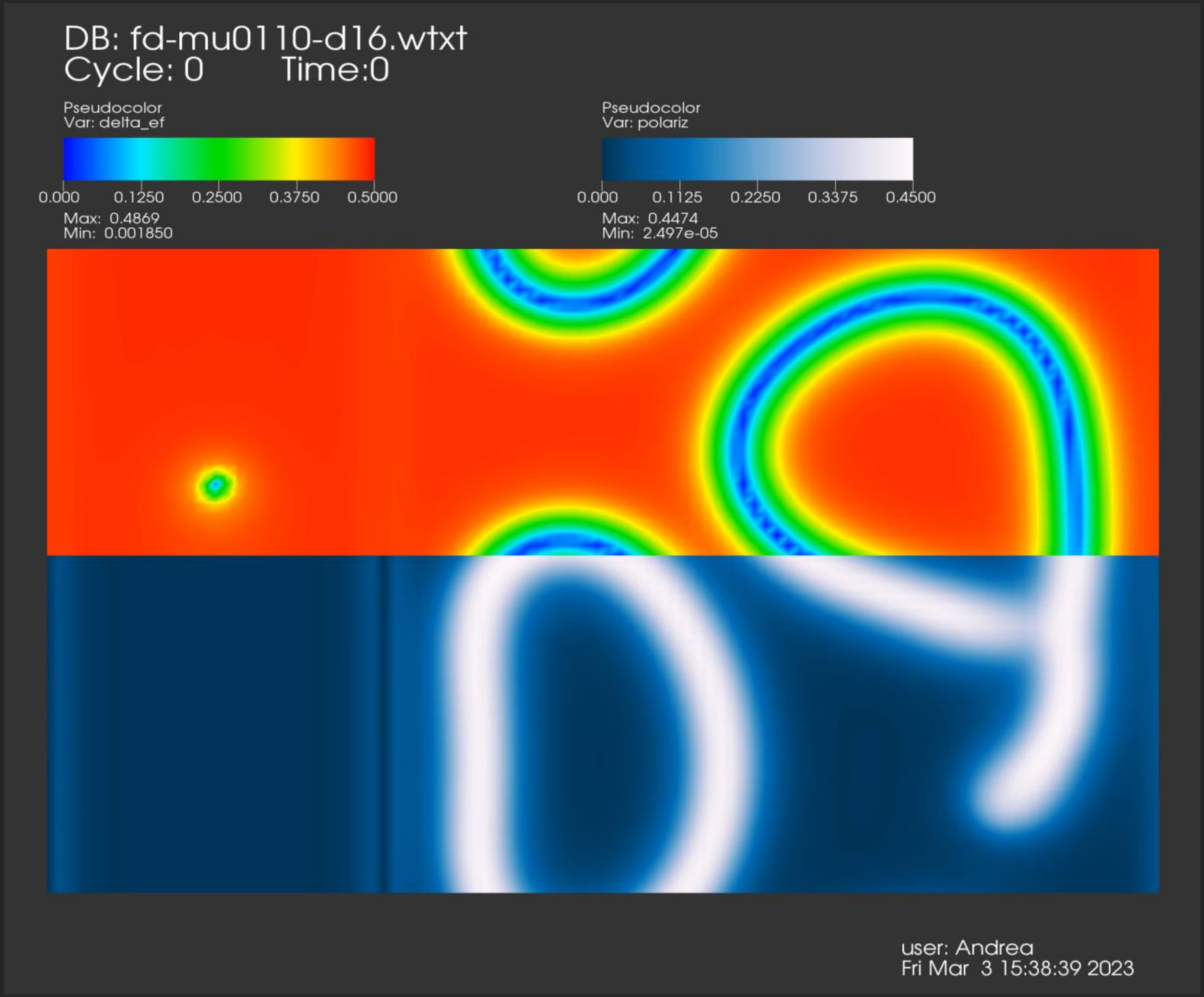
Exciting quasiparticles in the vortex core



Indeed quasiparticles in the core are excited due to vortex acceleration but the effect is too weak to account for the total dissipation rate.

A. Barresi, A. Boulet, P.M., G. Wlazłowski, Phys. Rev. Lett. 130, 043001 (2023)

Complex dynamics (strongly damped) of vortices in the spin imbalanced environment



Thanks to A. Barresi *et al.*

THANK YOU