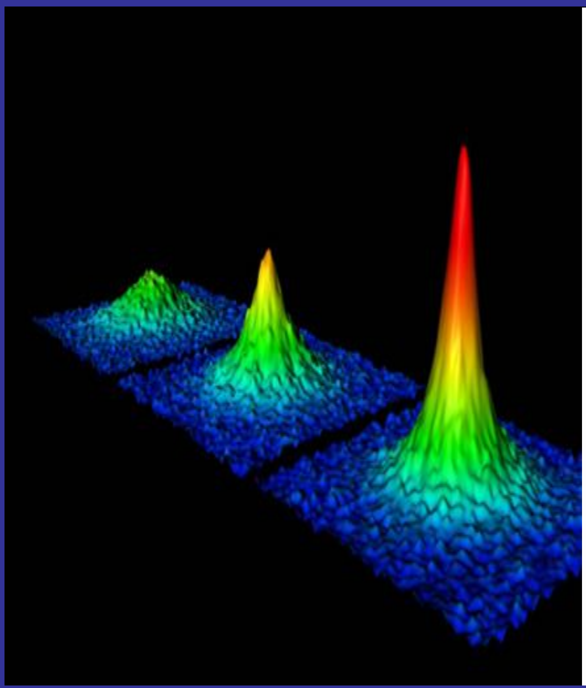
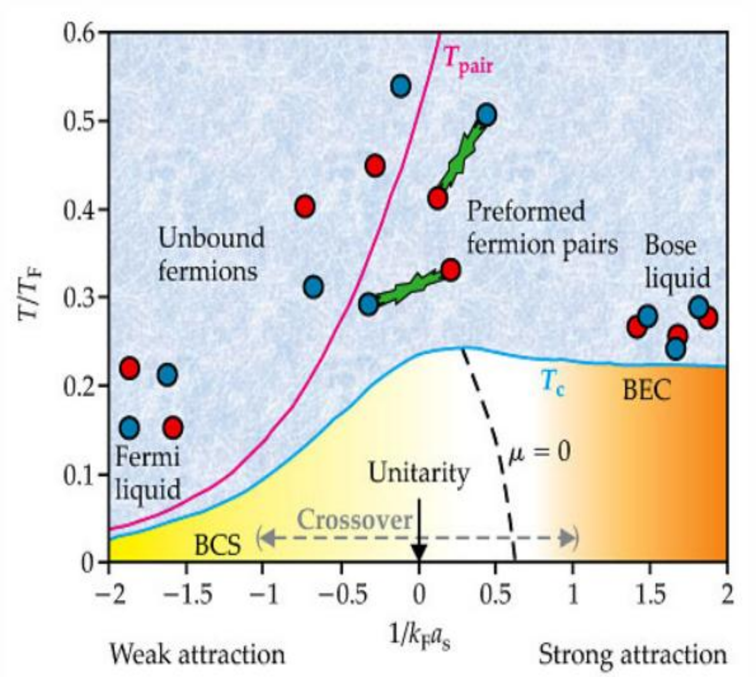
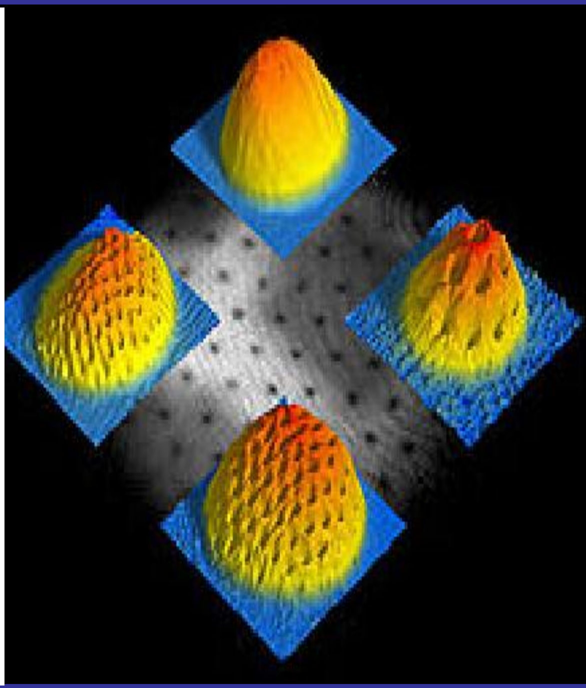
Path Integral (Auxiliary Field) Monte Carlo approach to ultracold atomic gases







Piotr Magierski
Warsaw University of Technology

Collaborators:

A. Bulgac - University of Washington
J.E. Drut - University of North Carolina
K.J. Roche - PNNL
G. Wlazłowski - Univ. of Washington/Warsaw Univ. of Techn.

Outline

- BCS-BEC crossover. Unitary regime.
- Theoretical approach: Path Integral Monte Carlo (QMC)
- Equation of state for the Fermi gas in the unitary regime. Critical temperature.
- Correlation functions from QMC: analytic continuation and inverse problem.
- Pairing gap and pseudogap.
- Viscosity, spin conductivity and diffusion.

What is a unitary gas?

A gas of interacting fermions is in the unitary regime if the average separation between particles is large compared to their size (range of interaction), but small compared to their scattering length.

$n\, r_0^3 \ll 1$ | $n\, |a|^3 \gg 1$

n - particle density
a - scattering length
r_0 - effective range

$$\text{i.e. } r_0 \to 0,\, a \to \pm\infty$$

NONPERTURBATIVE REGIME

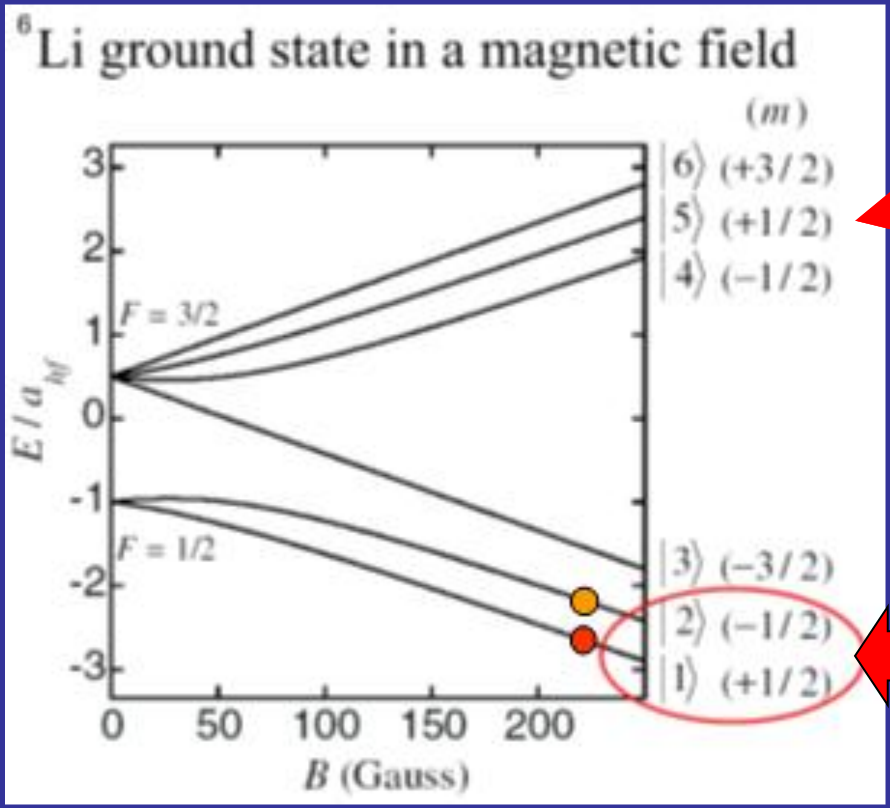
System is dilute but strongly interacting!

Universality: $E(x) = \xi(x) E_{FG}$; $x = T/\varepsilon_F$

$\xi(0) = 0.376(5)$ - **Exp. estimate**

E_{FG} - **Energy of noninteracting Fermi gas**

One fermionic atom in magnetic field



$$\left| F \; m_F \right\rangle$$

$$\vec{F} = \vec{I} + \vec{J} \; ; \; \vec{J} = \vec{L} + \vec{S}$$

Nuclear spin **Electronic spin**

Two hypefine states are populated in the trap

Collision of two atoms: **At low energies (low density of atoms) only L=0 (s-wave) scattering is effective.**

- **Due to the high diluteness atoms in the same hyperfine state do not interact with one another.**
- **Atoms in different hyperfine states experience interactions only in s-wave.**

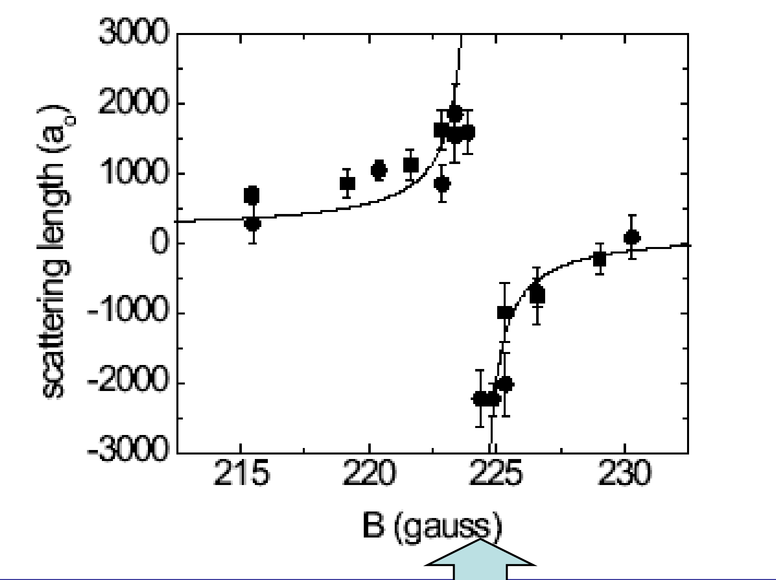
Feshbach resonance

$$H=\frac{\vec{p}^{\,2}}{2\mu}+\sum_{i=1}^{2}(V_i^{hf}+V_i^{Z})+V_0(\vec{r})P_0+V_1(\vec{r})P_1+V^d$$

$$V^{hf}=\frac{a_{hf}}{\hbar^2}\vec{I}\cdot\vec{J},\; V^Z=(\gamma_e J_z-\gamma_n I_z)B$$

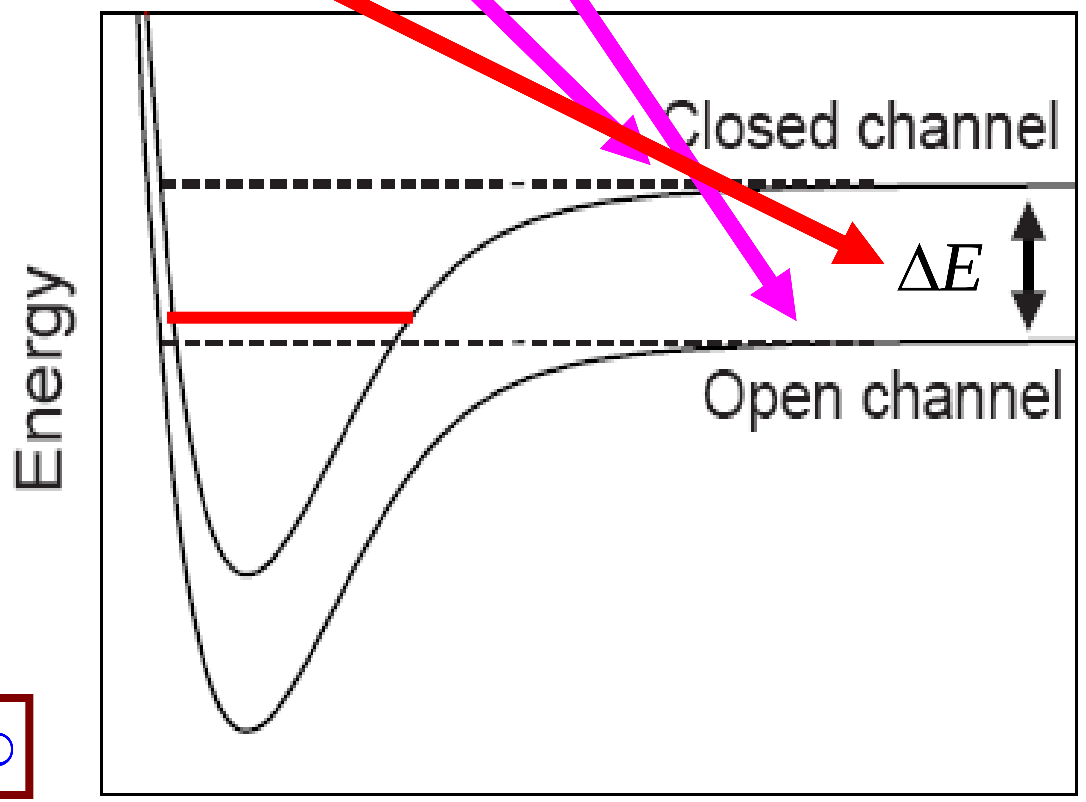
Tiesinga, Verhaar, Stoof, Phys. Rev. A**47**, 4114 (1993)

Channel coupling



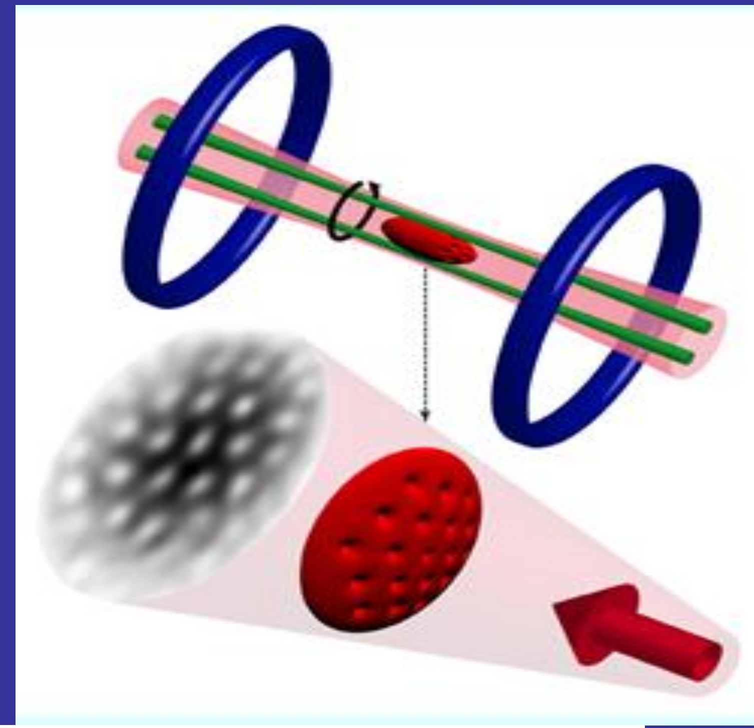
resonance: $a\rightarrow\pm\infty$

Regal and Jin, PRL **90**, 230404 (2003)



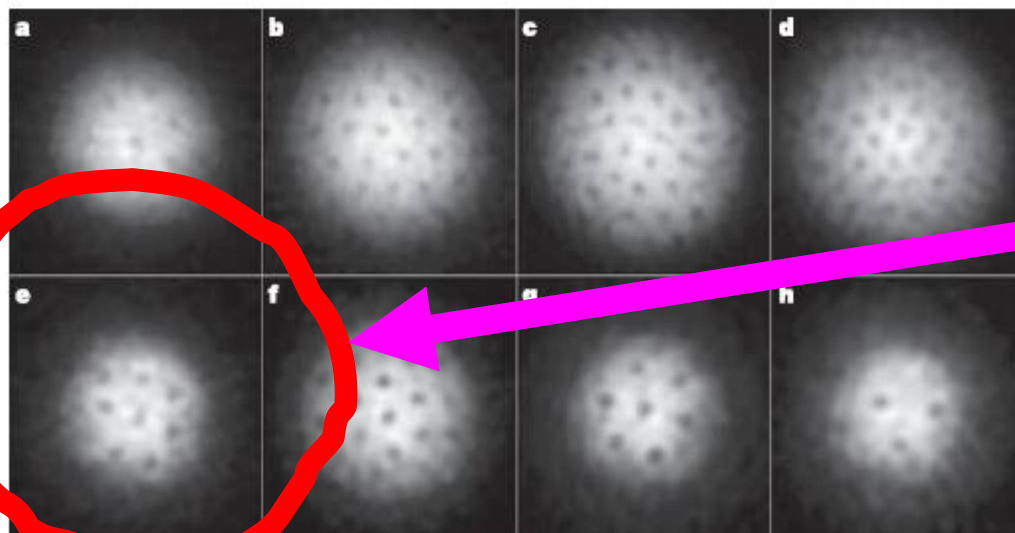
system of fermionic ^{6}Li atoms

Feshbach resonance: B=834G



BEC side: a>0

BCS side: a<0



UNITARY REGIME

M.W. Zwierlein *et al*., Nature, 435, 1047 (2005)

Figure 2 | Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (**a**) or 500 ms (**b**–**h**) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were 740 G (**a**), 766 G (**b**), 792 G (**c**), 812 G (**d**), 833 G (**e**), 843 G (**f**), 853 G (**g**) and 863 G (**h**). The field of view of each image is 880 μm × 880 μm.

ENERGY: $E(x) = \frac{3}{5}\xi(x)\varepsilon_F N; \quad x = \frac{T}{\varepsilon_F}$

PRESSURE: $P = -\frac{\partial E}{\partial V} = \frac{2}{5}\xi(x)\varepsilon_F \frac{N}{V}$

$PV = \frac{2}{3}E$

Note the similarity to the ideal Fermi gas

$\lim_{k\to\infty} n(k) = \frac{C}{k^4}, \qquad C$ - contact

$$E = \sum_{\sigma=\pm 1} \int \frac{d^3k}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left(n_\sigma(k) - \frac{C}{k^4} \right)$$

Total energy of the system

$$\left(\frac{dE}{d(1/a_s)} \right)_S = -\frac{\hbar^2}{4\pi m} C$$

Adiabatic relation

C and 1/a are conjugate thermodynamic variables
1/a – „generalized force"
C – „generalize displacement" – capture physics at short length scales.

Shina Tan, Ann.Phys.323,2971(2008), Ann.Phys.323,2952(2008)

Other theory papers: Tan, Leggett, Braaten, Combescot, Baym, Blume, Werner, Castin, Randeria,Strinati,…

Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \hat{\psi}_s(\vec{r}) \; - \; g \int d^3r \; \hat{n}_\uparrow(\vec{r}) \hat{n}_\downarrow(\vec{r})$$

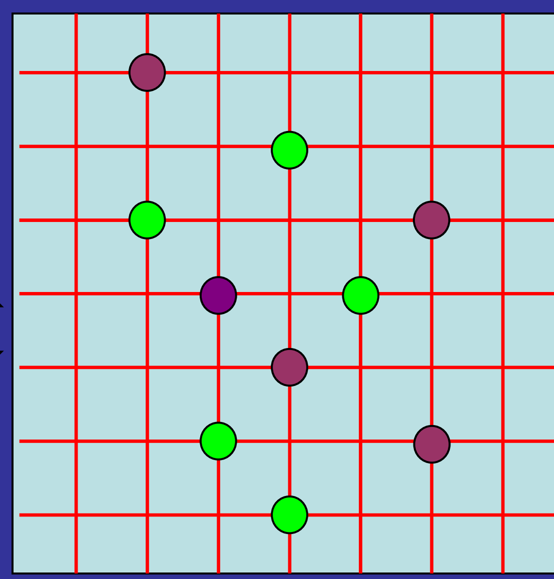
$$\hat{N} = \int d^3r \; \left(\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r}) \right); \;\; \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r}) \hat{\psi}_s(\vec{r})$$

Path Integral Monte Carlo for fermions on 3D lattice

Coordinate space

L –limit for the spatial correlations in the system

$k_{cut} = \frac{\pi}{\Delta x} \; ; \; \Delta x$



$Volume = L^3$

$lattice \; spacing = \Delta x$

- Spin up fermion:

- Spin down fermion:

External conditions:

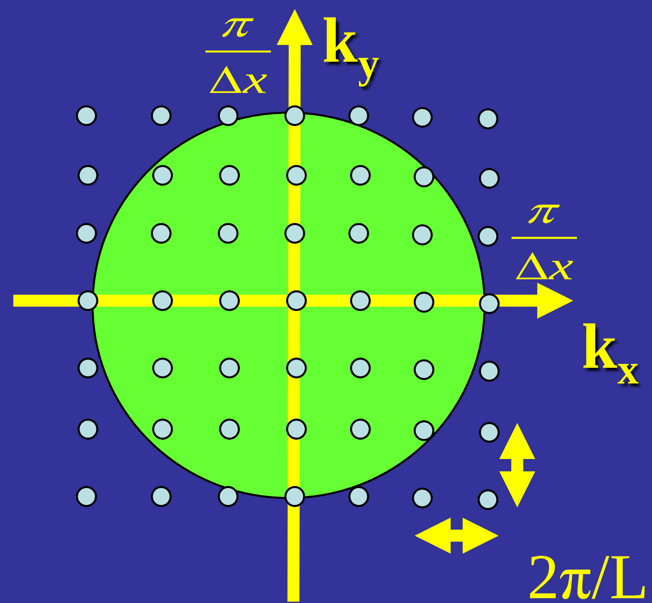
T - temperature

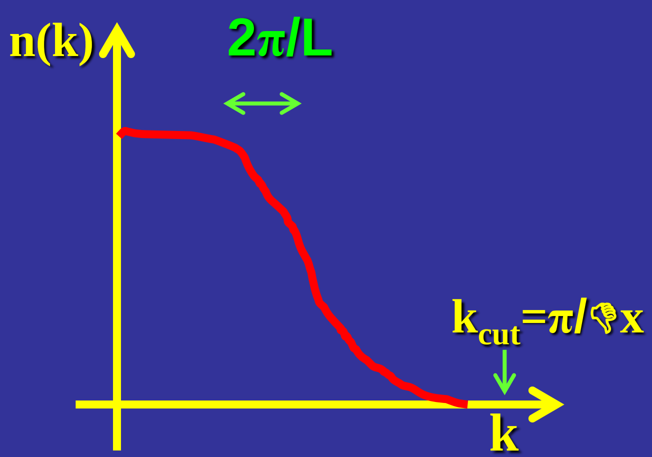
μ - chemical potential

Periodic boundary conditions imposed

Path Integral Monte Carlo for Fermions on 3D lattice

Momentum space





UV momentum cutoff $\Lambda_{\text{UV}} = \dfrac{\pi}{\Delta x}$

IR momentum cutoff $\Lambda_{\text{IR}} = \dfrac{2\pi}{L}$

$$\frac{\hbar^2 {\Lambda_{IR}}^2}{2m} << \varepsilon_F,\ \Delta << \frac{\hbar^2 {\Lambda_{UV}}^2}{2m}$$

REAL SPACE ⟺ *FFT* ⟺ **MOMENTUM SPACE**

Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^\dagger(\vec{r}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \hat{\psi}_s(\vec{r}) \; - \; g \int d^3r \; \hat{n}_\uparrow(\vec{r}) \hat{n}_\downarrow(\vec{r})$$

$$\hat{N} = \int d^3r \; \left(\hat{n}_\uparrow(\vec{r}) + \hat{n}_\downarrow(\vec{r}) \right); \quad \hat{n}_s(\vec{r}) = \hat{\psi}_s^\dagger(\vec{r}) \hat{\psi}_s(\vec{r})$$

$$\frac{1}{g} = -\frac{m}{4\pi\hbar^2 a} + \frac{mk_{cut}}{2\pi^2\hbar^2}$$

Running coupling constant g defined by lattice

$$\frac{1}{g} = \frac{m}{2\pi\hbar^2 \Delta x}$$ - UNITARY LIMIT

Trotter expansion (*trotterization* of the propagator)

$$Z(\beta) = \mathrm{Tr}\; \exp\left[-\beta \left(\hat{H} - \mu \hat{N} \right) \right] = \mathrm{Tr}\; \left\{ \exp\left[-\tau \left(\hat{H} - \mu \hat{N} \right) \right] \right\}^{N_\tau}, \qquad \beta = \frac{1}{T} = N_\tau \tau$$

$$E(T) = \frac{1}{Z(T)} \mathrm{Tr}\; \hat{H} \exp\left[-\beta \left(\hat{H} - \mu \hat{N} \right) \right]$$

$$N(T) = \frac{1}{Z(T)} \mathrm{Tr}\; \hat{N} \exp\left[-\beta \left(\hat{H} - \mu \hat{N} \right) \right]$$

$$\exp\left[-\tau\left(\hat{H}-\mu\hat{N}\right)\right]\approx\exp\left[-\tau\left(\hat{T}-\mu\hat{N}\right)/2\right]\exp(-\tau\hat{V})\exp\left[-\tau\left(\hat{T}-\mu\hat{N}\right)/2\right]$$

$$+O(\tau^3)$$

Discrete Hubbard-Stratonovich transformation

$$\exp(-\tau\hat{V})=\prod_{\vec{r}}\sum_{\sigma(\vec{r})=\pm 1}\frac{1}{2}\left[1+\sigma(\vec{r})A\hat{n}_{\uparrow}(\vec{r})\right]\left[1+\sigma(\vec{r})A\hat{n}_{\downarrow}(\vec{r})\right],\quad A=\sqrt{\exp(\tau g)-1}$$

σ-fields fluctuate both in space and imaginary time

$$\hat{U}(\sigma)=\prod_{j=1}^{N_\tau}\hat{W}_j(\sigma);$$

$$\hat{W}_j(\sigma)=\exp\left[-\tau\left(\hat{T}-\mu\hat{N}\right)/2\right]\prod_{\vec{r}}\left[1+\sigma(\vec{r})A\hat{n}_{\uparrow}(\vec{r})\right]\left[1+\sigma(\vec{r})A\hat{n}_{\downarrow}(\vec{r})\right]\exp\left[-\tau\left(\hat{T}-\mu\hat{N}\right)/2\right]$$

$$Z(T) = \int D\sigma(\vec{r},\tau)\ \mathrm{Tr}\,\hat{U}(\{\sigma\});$$

$$\int D\sigma(\vec{r},\tau) \equiv \sum_{\{\sigma(\vec{r},1)=\pm1\}} \sum_{\{\sigma(\vec{r},2)=\pm1\}} \cdots \sum_{\{\sigma(\vec{r},N_\tau)=\pm1\}} \ ; \quad N_\tau \tau = \frac{1}{T}$$

$$\hat{U}(\{\sigma\}) = T_\tau \exp\{-\int_0^\beta d\tau[\hat{h}(\{\sigma\}) - \mu]\}$$

One-body evolution operator in imaginary time

$$E(T) = \int \frac{D\sigma(\vec{r},\tau)\mathrm{Tr}\,\hat{U}(\{\sigma\})}{Z(T)} \frac{\mathrm{Tr}\left[\hat{H}\hat{U}(\{\sigma\})\right]}{\mathrm{Tr}\,\hat{U}(\{\sigma\})}$$

$$\mathrm{Tr}\,\hat{U}(\{\sigma\}) = \{\det[1 + \hat{U}_\uparrow(\sigma)]\}^2 = \exp[-S(\{\sigma\})] > 0$$

No sign problem!

$$n_\uparrow(\vec{x},\vec{y}) = n_\downarrow(\vec{x},\vec{y}) = \sum_{k,l<k_c} \psi_{\vec{k}}(\vec{x}) \left[\frac{U(\{\sigma\})}{1 + U(\{\sigma\})}\right]_{\vec{k}\,\vec{l}} \psi^*_{\vec{l}}(\vec{y}), \qquad \psi_{\vec{k}}(\vec{x}) = \frac{\exp(i\vec{k}\cdot\vec{x})}{\sqrt{L^3}}$$

All traces can be expressed through these single-particle density matrices

$$\hat{U}(\{\sigma\}) = T_\tau \exp\{-\int_0^\beta d\tau[\hat{h}(\{\sigma\}) - \mu]\};\ \hat{h}(\{\sigma\}) \text{ – one-body operator}$$

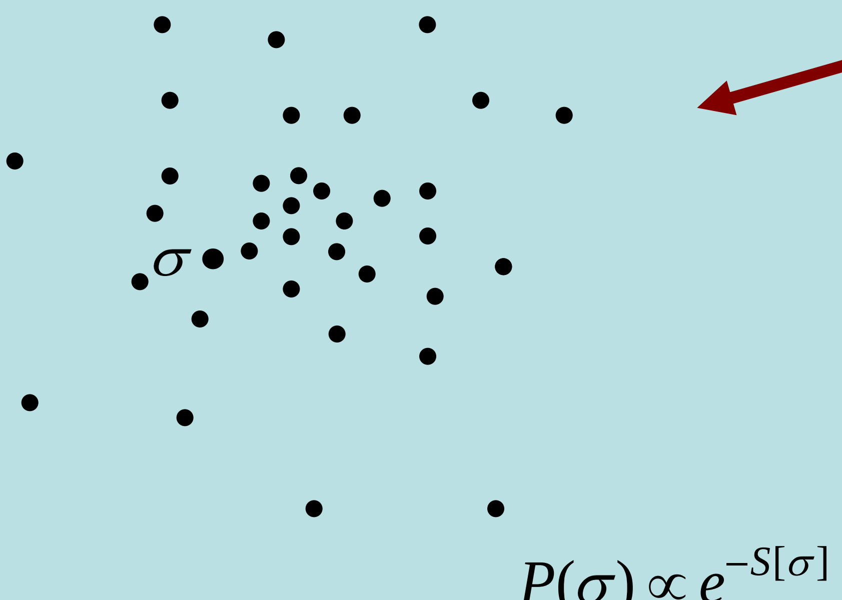
$$U(\{\sigma\})_{kl} = \langle \psi_k | \hat{U}(\{\sigma\}) | \psi_l \rangle; \quad |\psi_l\rangle \text{- single-particle wave function}$$

$$E(T) = \langle \hat{H} \rangle = \int \frac{D[\sigma(\vec{r},\tau)] e^{-S[\sigma]}}{Z(T)} E[U(\{\sigma\})]$$

$E[U(\{\sigma\})]$- energy associated with a given sigma field

Quantum Monte-Carlo:

Sigma space sampling



$$P(\sigma) \propto e^{-S[\sigma]}$$

$$\bar{E}(T) = \frac{1}{N_\sigma} \sum_{k=1}^{N_\sigma} E\left(U(\{\sigma_k\})\right)$$

$\bar{E}(T)$ - stochastic variable

$\langle \bar{E}(T) \rangle = E(T)$

$$\sqrt{\langle \bar{E}(T)^2 \rangle - \langle \bar{E}(T) \rangle^2} \propto 1/\sqrt{N_\sigma}$$

N_σ - number of uncorrelated samples

Details of calculations, improvements and problems

- Currently we can reach 14^3 lattice and perform calcs. down to x = 0.06 (x – temperature in Fermi energy units) at the densities of the order of 0.03.

- Effective use of FFT(W) makes all imaginary time propagators diagonal (either in real space or momentum space) and there is no need to store large matrices.

- Update field configurations using the Metropolis importance sampling algorithm. QMC calculations can be split into two independent processes:
  1) sample generation (generation of sigma fields),
  2) calculations of observables.

- Change randomly at a fraction of all space and time sites the signs the auxiliary fields $\sigma(r,\tau)$ so as to maintain a running average of the acceptance rate between 0.4 and 0.6 .

- At low temperatures use Singular Value Decomposition of the evolution operator $U(\{\sigma\})$ to stabilize the numerics.

- MC correlation "time" ≈ 200 time steps at $T \approx T_c$ for lattices 10^3 . Unfortunately when increasing the lattice size the correlation time also increases. One needs few thousands uncorrelated samples (we usually take about 10 000) to decrease the statistical error to the level of 1%.

Finite size scaling:

$$|T_c - T_{ij}| = \kappa g(L_i, L_j),$$

$$g(L_i, L_j) = L_j^{-(\omega+1/\nu)} \left[\frac{\left(\frac{L_j}{L_i}\right)^{\omega} - 1}{1 - \left(\frac{L_i}{L_j}\right)^{1/\nu}} \right]$$

$$\nu = 0.671$$

$$\omega = 0.8$$

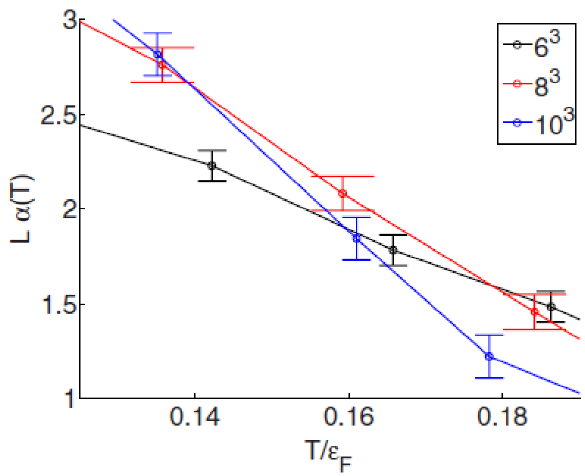


FIG. 11. (Color online) Condensate fraction $\alpha(T)$ scaled with the lattice size L, black squares for 6^3, red stars for 8^3, and blue circles for 10^3 (highest on the left) lattices, respectively, for the case $1/k_F a = -0.1$. The error bars correspond to the statistical errors.

Effective range corrections: $k_F r_{eff} \approx 0.4 - 0.5$

CM corrections:

$$-\frac{1}{g} = \frac{m}{4\pi\hbar^2 a} - \frac{m\Lambda}{2\pi^2\hbar^2} + \frac{m|K|}{8\pi^2\hbar^2}$$
$$= \frac{m}{4\pi\hbar^2 a} - \frac{m\Lambda}{2\pi^2\hbar^2}\left(1 - \frac{|K|}{4\Lambda}\right).$$

CM momentum of a fermion pair

Total systematic error does not exceed 10-15%

Bulgac, Drut, Magierski, PRA78, 023625(2008),
Wlazłowski, Magierski, Bulgac, Drut, Roche, arXive:1212.1503 (to appear in Phys. Rev. Lett.)

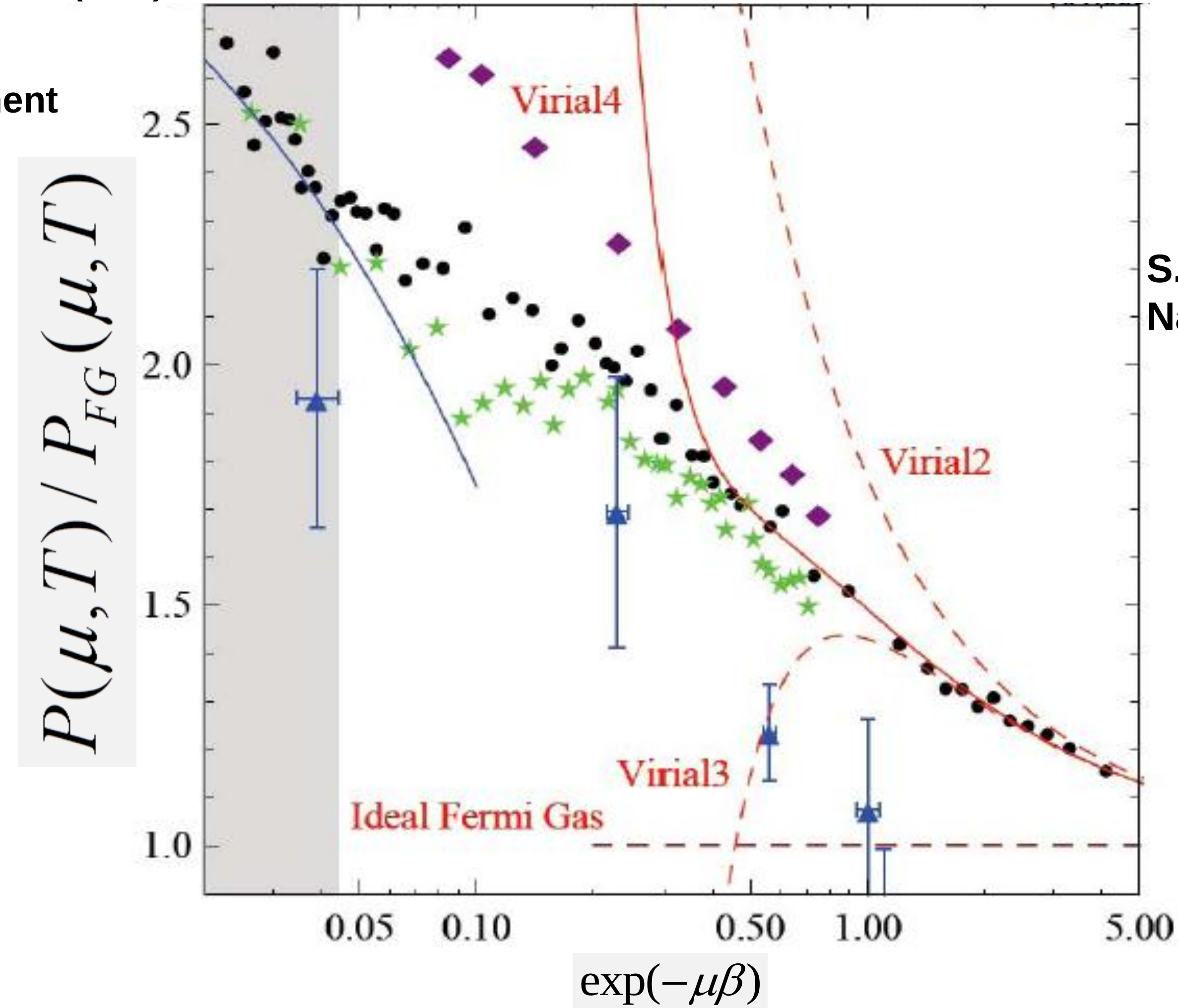
Comparison with Many-Body Theories (1)

▲ **Diagram. MC**
Burovski et al.
PRL96, 160402(2006)

★ **QMC**
Bulgac, Drut, Magierski,
PRL99, 120401(2006)

◆ **Diagram. + analytic**
Haussmann et al.
PRA75, 023610(2007)

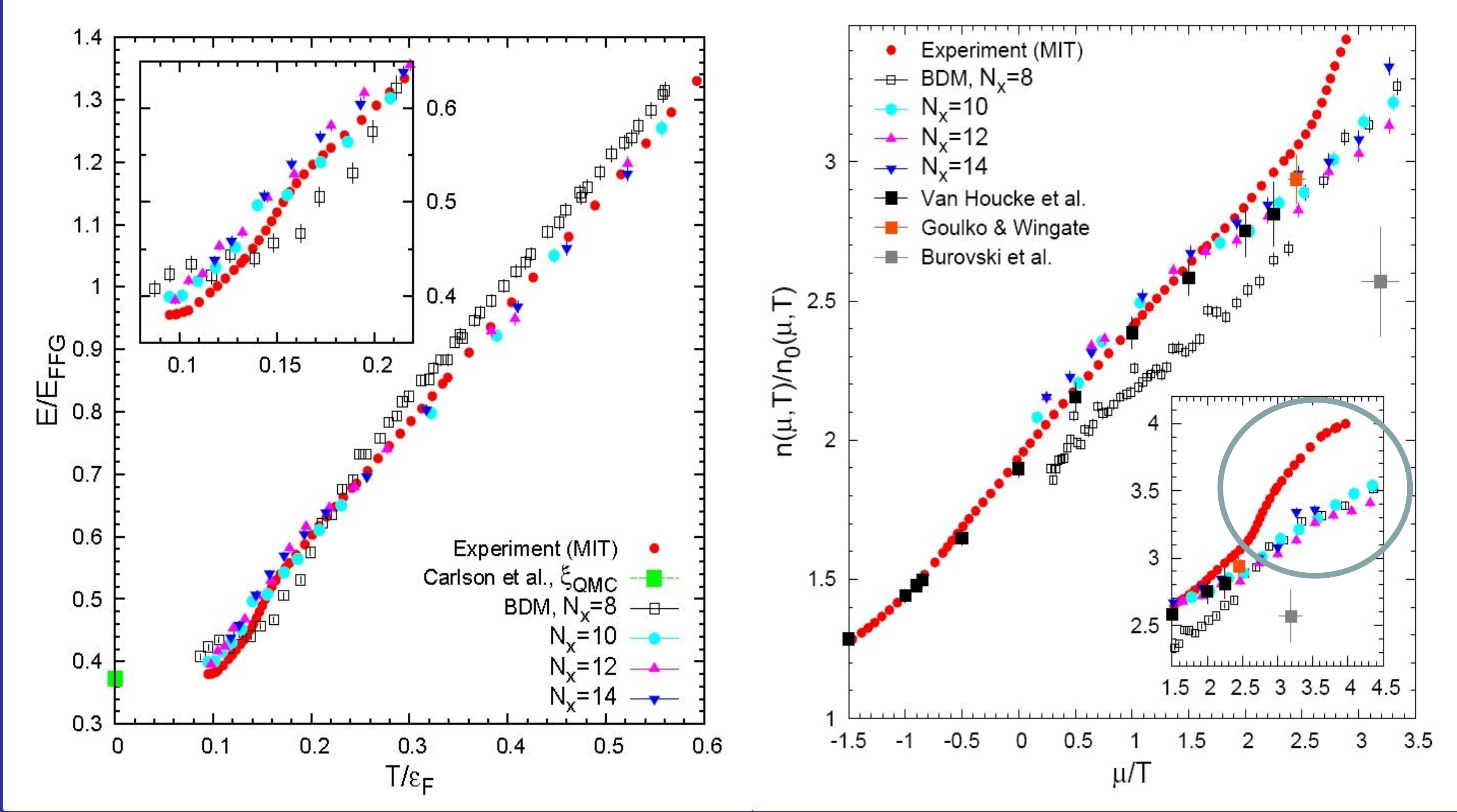
● **Experiment**



S. Nascimbene et al.
Nature 463, 1057 (2010)

Courtesy of C. Salomon

Equation of state of the unitary Fermi gas - current status



Experiment: M.J.H. Ku, A.T. Sommer, L.W. Cheuk, M.W. Zwierlein , Science 335, 563 (2012)

QMC (PIMC + Hybrid Monte Carlo): J.E.Drut, T.Lähde, G.Wlazłowski, P.Magierski, Phys. Rev. A 85, 051601 (2012)

Local density approximation (LDA) from QMC

Uniform system

$$\Omega = F - \lambda N = \frac{3}{5}\varphi(x)\varepsilon_F N - \lambda N$$

Nonuniform system *(gradient corrections neglected)*

$$\Omega = \int d^3 r \left[\frac{3}{5}\varepsilon_F(\vec{r})\varphi(x(\vec{r})) + U(\vec{r}) - \lambda \right] n(\vec{r})$$

$$x(\vec{r}) = \frac{T}{\varepsilon_F(\vec{r})}; \quad \varepsilon_F(\vec{r}) = \frac{\hbar^2}{2m}\left[3\pi^2 n(\vec{r})\right]^{2/3}$$

The overall chemical potential λ and the temperature *T* are constant throughout the system. The density profile will depend on the shape of the trap as dictated by:

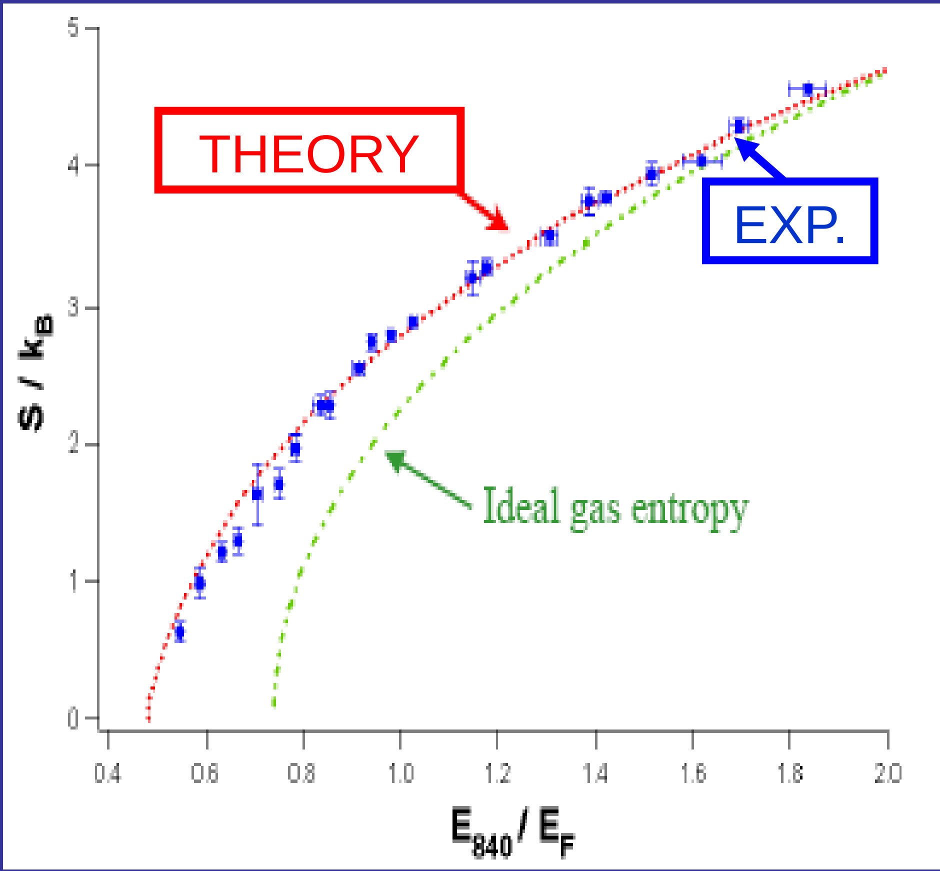
$$\frac{\delta\Omega}{\delta n(\vec{r})} = \frac{\delta(F - \lambda N)}{\delta n(\vec{r})} = \mu(x(\vec{r})) + U(r) - \lambda = 0$$

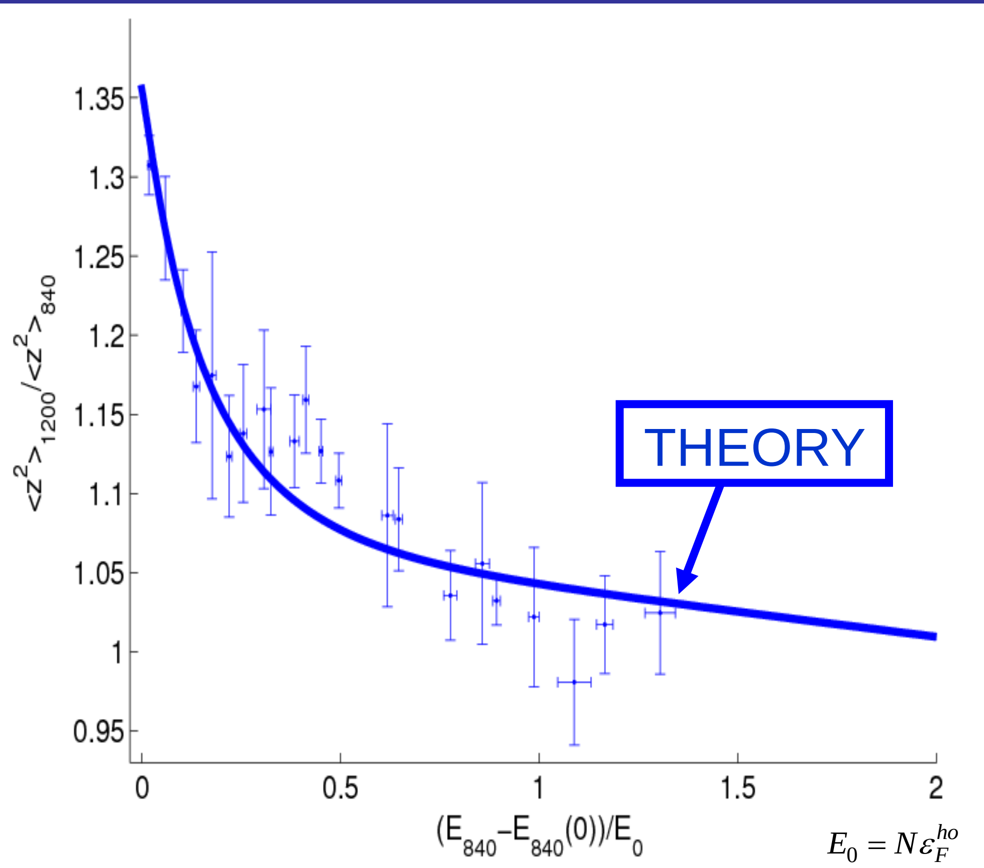
Using as an input the Monte Carlo results for the uniform system and experimental data (trapping potential, number of particles), we determine the density profiles.

Comparison with experiment

John Thomas' group at Duke University,

L.Luo, et al. Phys. Rev. Lett. 98, 080402, (2007)





Entropy as a function of energy (relative to the ground state) for the unitary Fermi gas in the harmonic trap.

Ratio of the mean square cloud size at B=1200G to its value at unitarity (B=840G) as a function of the energy. Experimental data are denoted by point with error bars.

$B = 1200G \Rightarrow 1/k_F a \approx -0.75$

Theory: **Bulgac, Drut, and Magierski PRL 99, 120401 (2007)**

Static and dynamic responses from QMC

Static:

$$\hat{H}(t) = \hat{H} - \hat{A}X$$

$$\delta B = \Phi X$$

$$\Phi = \int_0^{1/T} d\tau \left\langle \Delta\hat{A}(\tau)\Delta\hat{B} \right\rangle_0 ; \Delta\hat{A} = \hat{A} - \left\langle \hat{A} \right\rangle_0$$

Dynamic:

$$\hat{H}(t) = \hat{H} - \hat{A}X(t)$$

$$\delta B(t) = i \int_{-\infty}^{+\infty} dt \Phi(t - t')X(t')$$

$$\Phi(t) = i\theta(t) \left\langle [\hat{A}(t), \hat{B}] \right\rangle_0$$

$$\Phi(\omega) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \frac{A^{(+)}(\omega')}{\omega - \omega' + i0^+} \left(1 - e^{-\omega'/T}\right) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \frac{A^{(-)}(\omega')}{\omega - \omega' + i0^+} \left(e^{\omega'/T} - 1\right)$$

Examples od useful spectral function::

Pairing gap

Spectral weight function: $A(\vec{p},\omega)$

$$G^{ret/adv}(\vec{p},\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega' \frac{A(\vec{p},\omega')}{\omega-\omega'\pm i0^{+}}$$

$$G(\vec{p},\tau) = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, A(\vec{p},\omega)\frac{e^{-\omega\tau}}{1+e^{-\beta\omega}}$$

From Monte Carlo calcs.

$$G(\vec{p},\tau) = \frac{1}{Z} Tr\{e^{-(\beta-\tau)(\hat{H}-\mu\hat{N})}\hat{\psi}_{\uparrow}(\vec{p})e^{-\tau(\hat{H}-\mu\hat{N})}\hat{\psi}_{\uparrow}^{+}(\vec{p})\}$$

Constraints

$$\int_{-\infty}^{+\infty} A(\vec{p},\omega)\frac{d\omega}{2\pi} = 1$$

$$\int_{-\infty}^{+\infty} A(\vec{p},\omega)(1+e^{\beta\omega})^{-1}\frac{d\omega}{2\pi} = n(\vec{p})$$

In the limit of independent quasiparticles: $A(\vec{p},\omega) = 2\pi\delta(\omega - E(p))$

Shear viscosity

$$\eta(\omega) = \pi \frac{\rho_{xy,xy}(\boldsymbol{q}=0,\omega)}{\omega},$$

$$G_{ij,kl}(\boldsymbol{q},\tau) = \int_0^\infty \rho_{ij,kl}(\boldsymbol{q},\omega) \frac{\cosh\left[\omega(\tau-\beta/2)\right]}{\sinh\left(\omega\beta/2\right)} d\omega,$$

$$G_{ij,kl}(\boldsymbol{q},\tau) = \int d^3 r e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \langle \hat{\Pi}_{ij}(\boldsymbol{r},\tau)\hat{\Pi}_{kl}(\boldsymbol{0},0)\rangle,$$

$$i[\hat{j}_k(\boldsymbol{r}),\hat{H}] = \partial_l \hat{\Pi}_{kl}(\boldsymbol{r}),$$

Spin conductivity

$$\sigma_s(\omega) \;=\; \pi \rho_s^{(jj)}(\boldsymbol{q} \;=\; 0,\omega)/\omega$$

$$G_s^{(jj)}(\boldsymbol{q},\tau) = \frac{1}{V}\langle [\hat{j}^z_{\boldsymbol{q}\uparrow}(\tau) - \hat{j}^z_{\boldsymbol{q}\downarrow}(\tau)][\hat{j}^z_{-\boldsymbol{q}\uparrow}(0) - \hat{j}^z_{-\boldsymbol{q}\downarrow}(0)]\rangle,$$

$$G_s^{(jj)}(\boldsymbol{q},\tau) = \int_0^\infty \rho_s^{(jj)}(\boldsymbol{q},\omega) \frac{\cosh\left[\omega(\tau-\beta/2)\right]}{\sinh\left(\omega\beta/2\right)} d\omega,$$

Linear inverse problem

$$G(y) = \int_{-\infty}^{\infty} K(x,y)A(x)dx,$$

G is known from QMC with some error for a number of values of y, usually uniformly distributed within the interval: (0, 1/T)

Constraints:

$$\int_{-\infty}^{\infty} g_i(x)A(x)dx = c_i, \quad i = 1, 2, \ldots, L$$

and

$$A(x_j) \in [l_j, u_j], \quad j = 1, 2, \ldots, M,$$

SVD method:

$$\vec{G} = \mathcal{K}A.$$

$$\mathcal{K}u_i = \lambda_i \vec{v}_i, \qquad \mathcal{K}^\dagger \vec{v}_i = \lambda_i u_i. \tag{12}$$

The numbers λ_i are *singular values*, and u_i, $\vec{v}_i$ are *singular functions* and *singular vectors*, respectively. The definition of the conjugate operator $\mathcal{K}^\dagger$ and equations (12) allow to express the singular functions in the form:

$$u_i(x) = \frac{1}{\lambda_i} \sum_{k=1}^{N_\tau} K_k(x)(\vec{v}_i)_k, \tag{13}$$

where $(\vec{v}_i)_k$ denotes k-th element of vector $(\vec{v}_i)$.

The singular system forms a suitable basis for expansion of the unknown object A_P [1]:

$$A_P(x) = \sum_{i=1}^{M} b_i u_i(x), \tag{14}$$

where the expansion coefficients are given by

$$b_i = \frac{(\vec{v}_i, \vec{G})}{\lambda_i}. \tag{15}$$

Effects of noise:

$$\lambda_i \to 0 \quad \Rightarrow \quad \Delta b_i = \frac{(\vec{v}_i, \Delta\vec{G})}{\lambda_i} \to \infty.$$

Magierski, Wlazłowski, Comp. Phys. Comm. 183 (2012) 2264

<u>Maximum entropy method (MEM):</u>

Bayes' theorem:

$$p(\vec{A}|\vec{\tilde{G}}) = \frac{p(\vec{\tilde{G}}|\vec{G})p(\vec{A})}{p(\vec{\tilde{G}})},$$

Maximization of conditional probability:

$$\frac{\partial}{\partial A_j} p(\vec{A}|\vec{\tilde{G}}) = 0, \quad \text{j=1,..,N},$$

$$p(\vec{A}|\tilde{G}) \propto \exp\left(-\frac{1}{2}\sum_{i=1}^{N_\tau}\left(\frac{\tilde{G}_i - G_i}{\sigma_i}\right)^2 - \alpha\sum_{i=1}^{N} A_i \log\frac{A_i}{M_i}\right),$$

Relative entropy term

1. Create an SVD solution and apply it to construct the class of default models $\mathcal{M}(x;\vec{f})$;
2. Use the "self-consistent" MEM with constructed class of models $\mathcal{M}(x;\vec{f})$ to produce final solution.

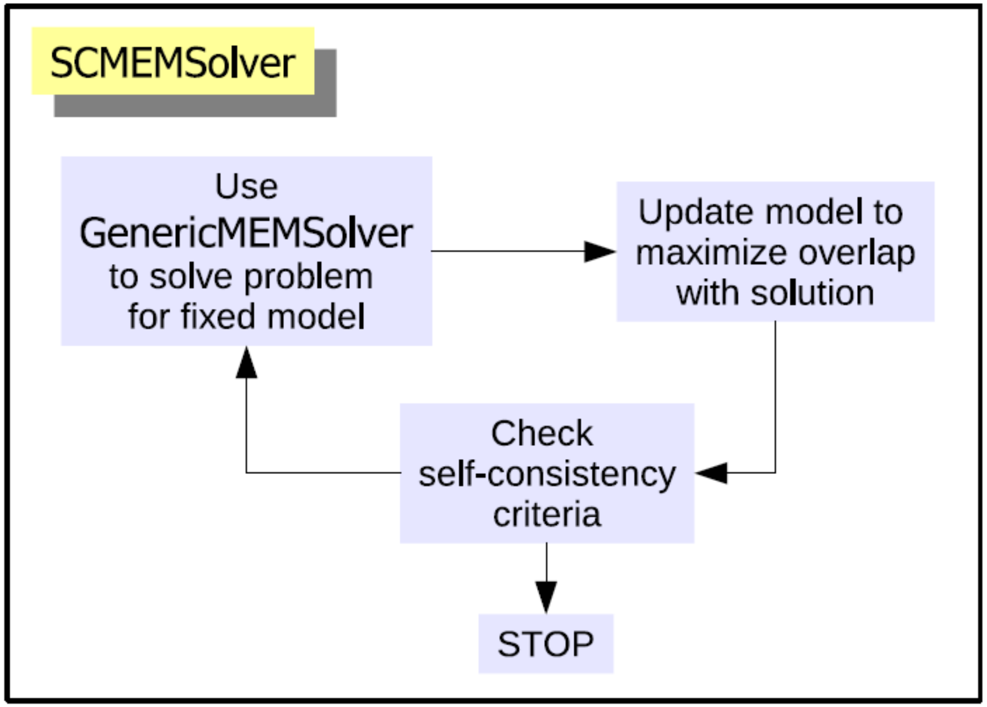


Figure 2: (Color online) The algorithm scheme for SCMEMSolver.

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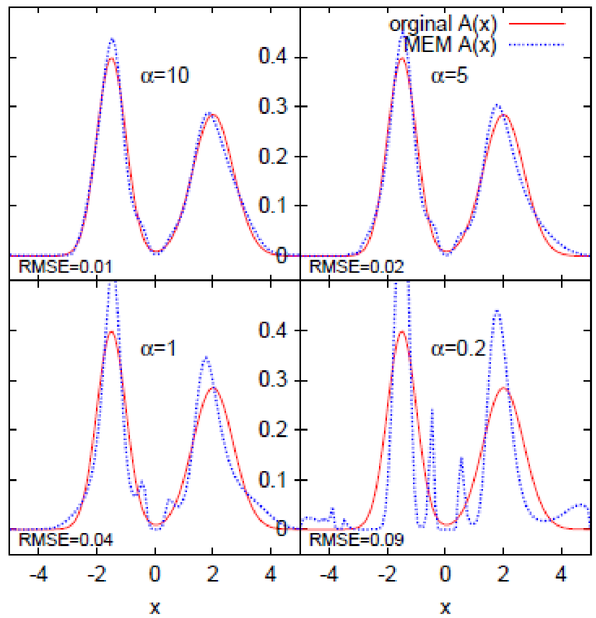

Figure 5: (Color online) Reconstruction of the artificial object function $A(x)$ by self-consistent MEM for different values of parameter α.

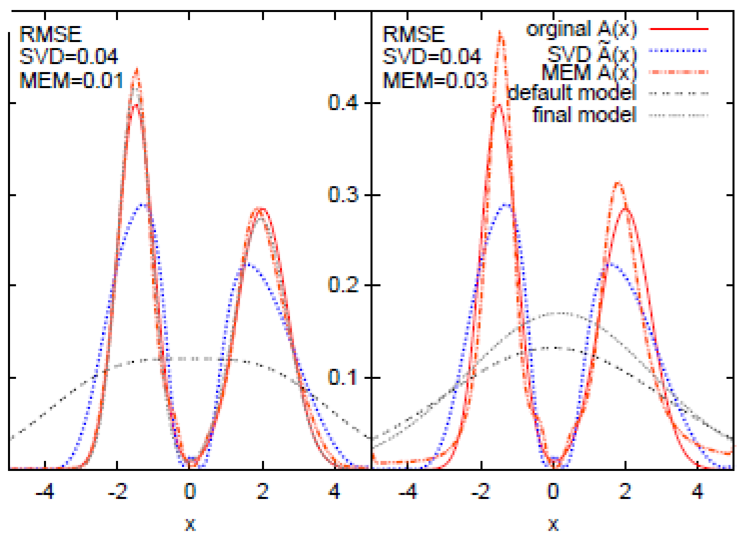

Figure 6: (Color online) The reconstruction ability of the spectral function for the full problem (data with noise + external constraints) of the SVD and MEM methods. The left panel shows the solution of the self-consistent MEM with a combination of two Gaussians functions as a default model class. The right panel shows the solution of the self-consistent MEM with Gaussian functions as a default model class.

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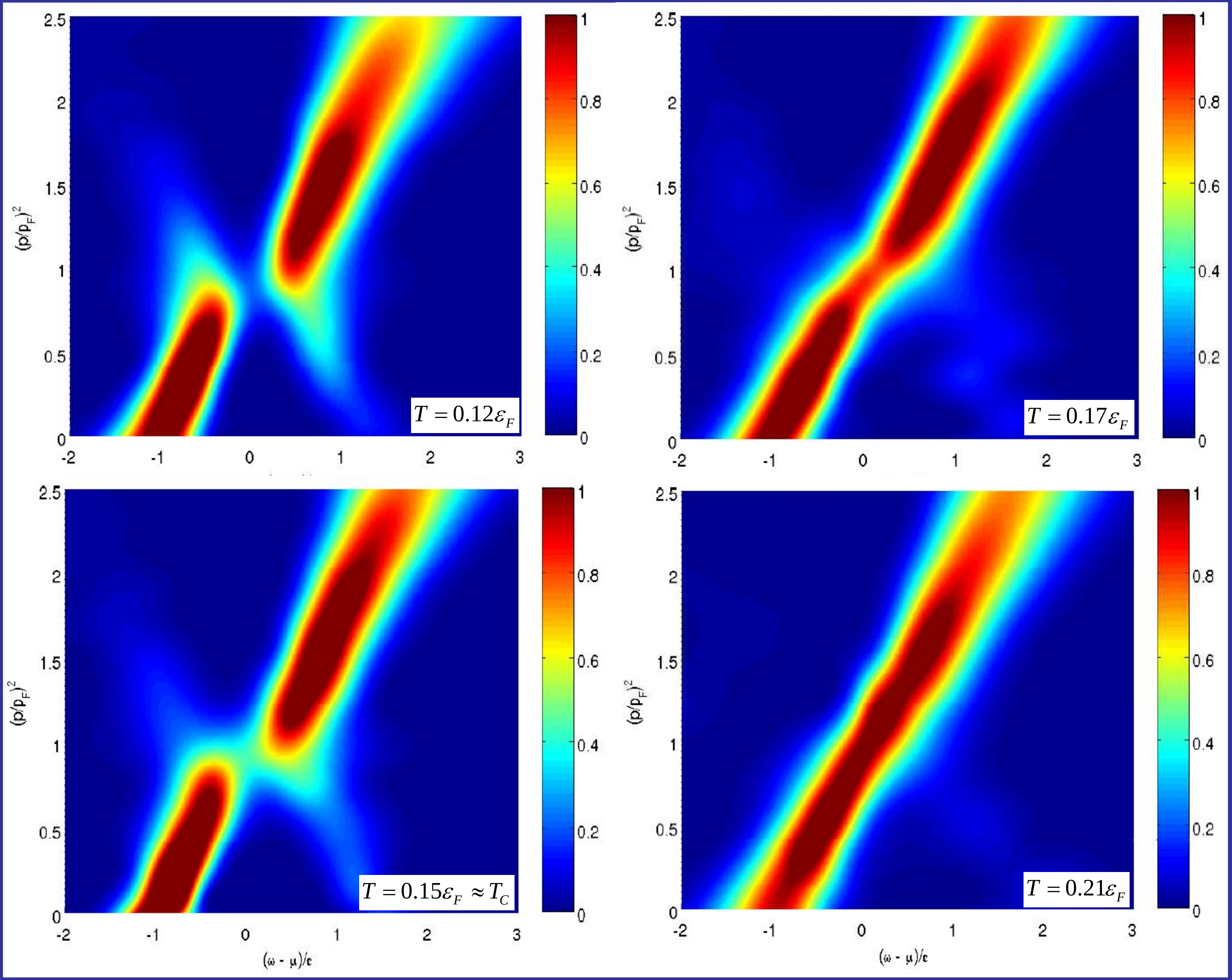
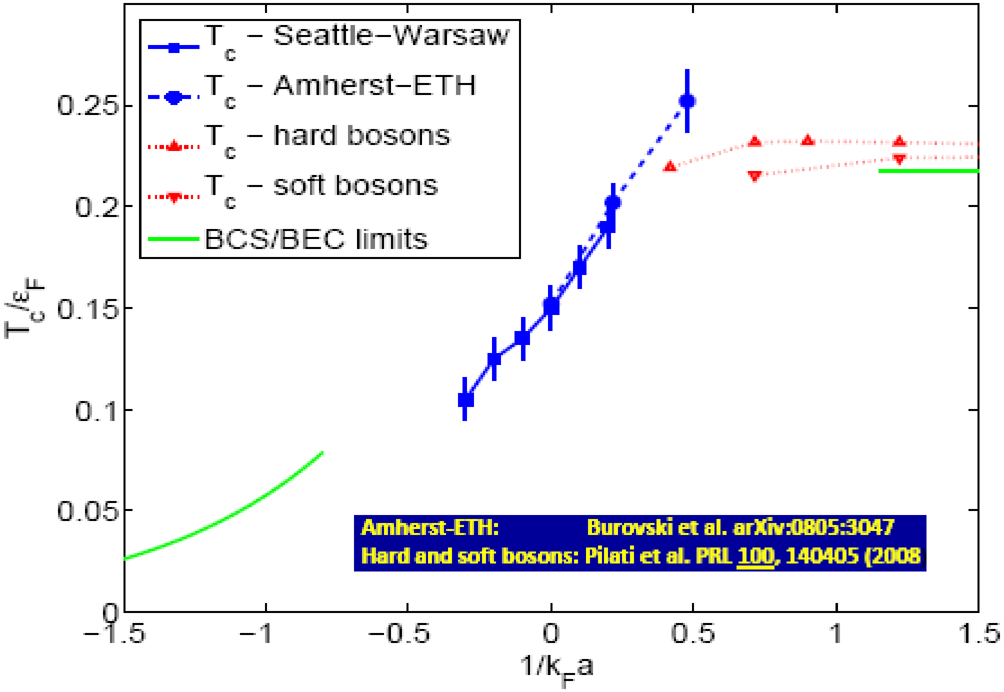
Spectral weight function at unitarity: $(k_F a)^{-1} = 0$

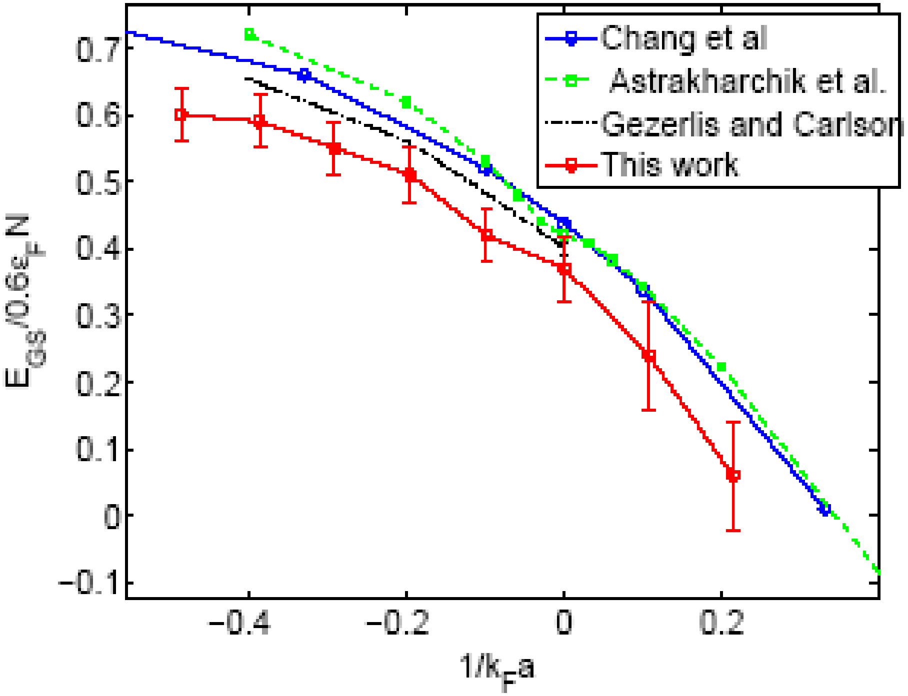


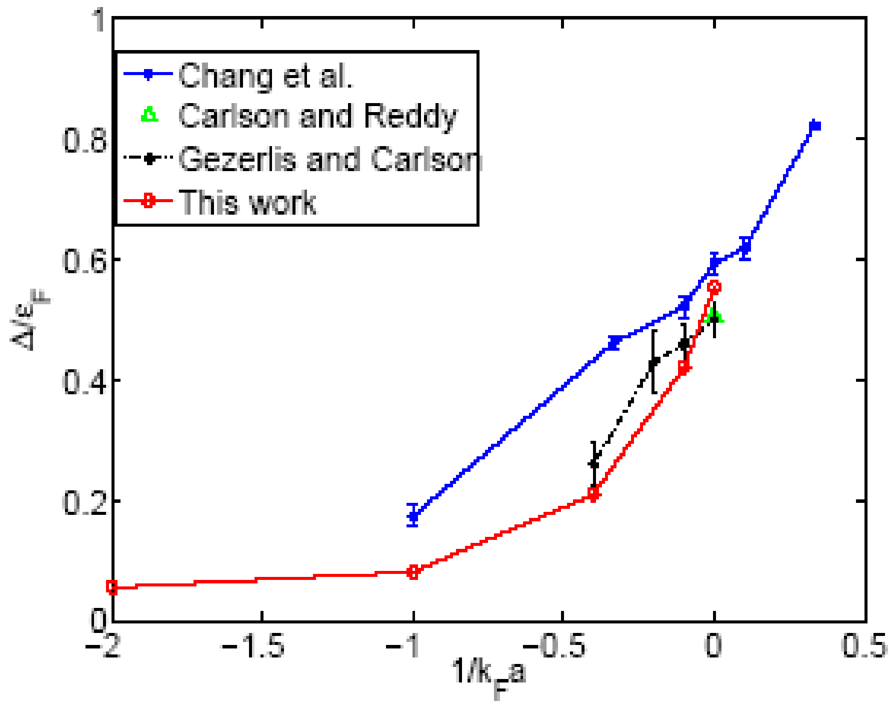


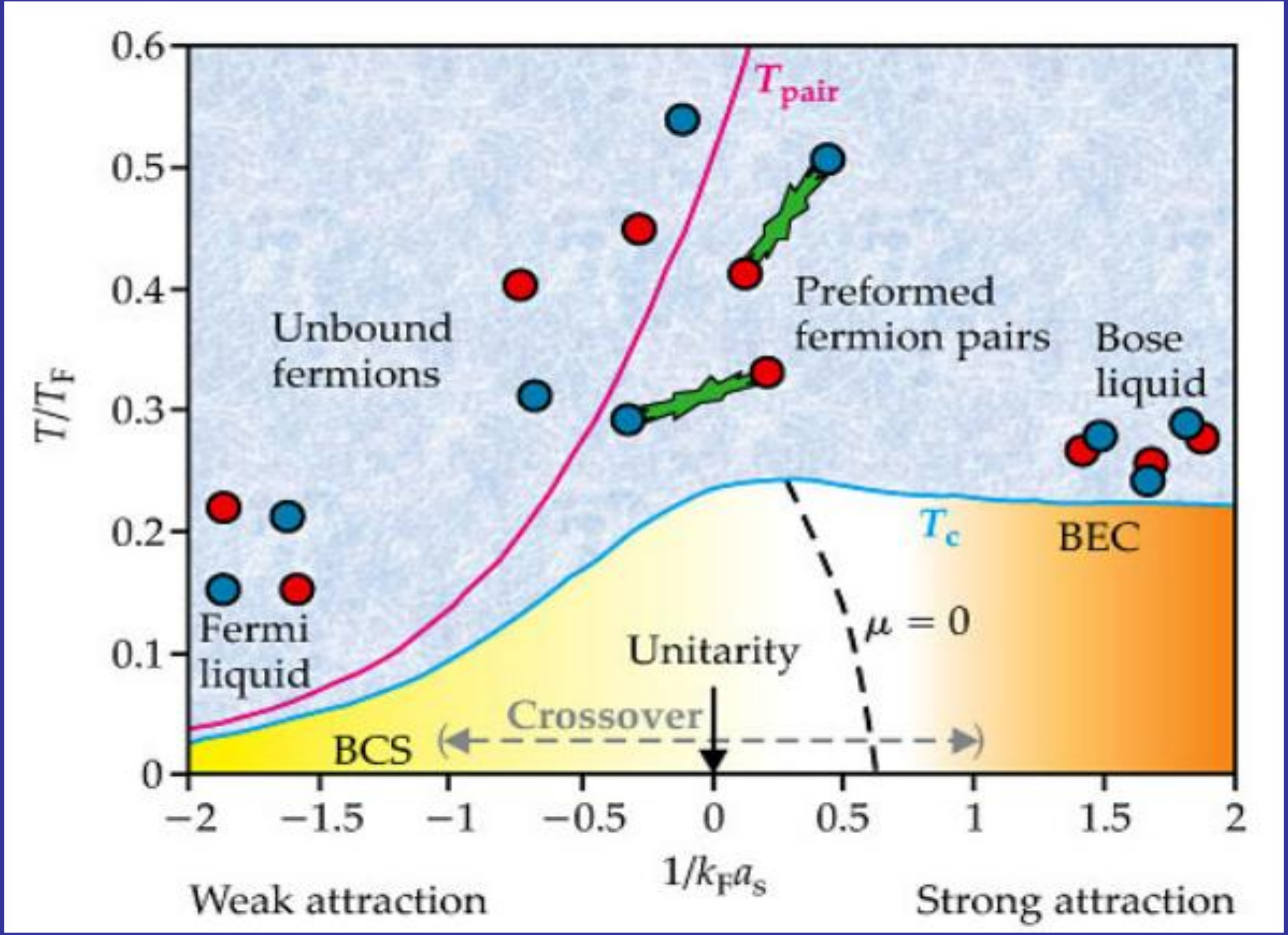
Results in the vicinity of the unitary limit:

- Critical temperature
- Ground state energy
- Pairing gap

Bulgac, Drut, Magierski, PRA78, 023625(2008)







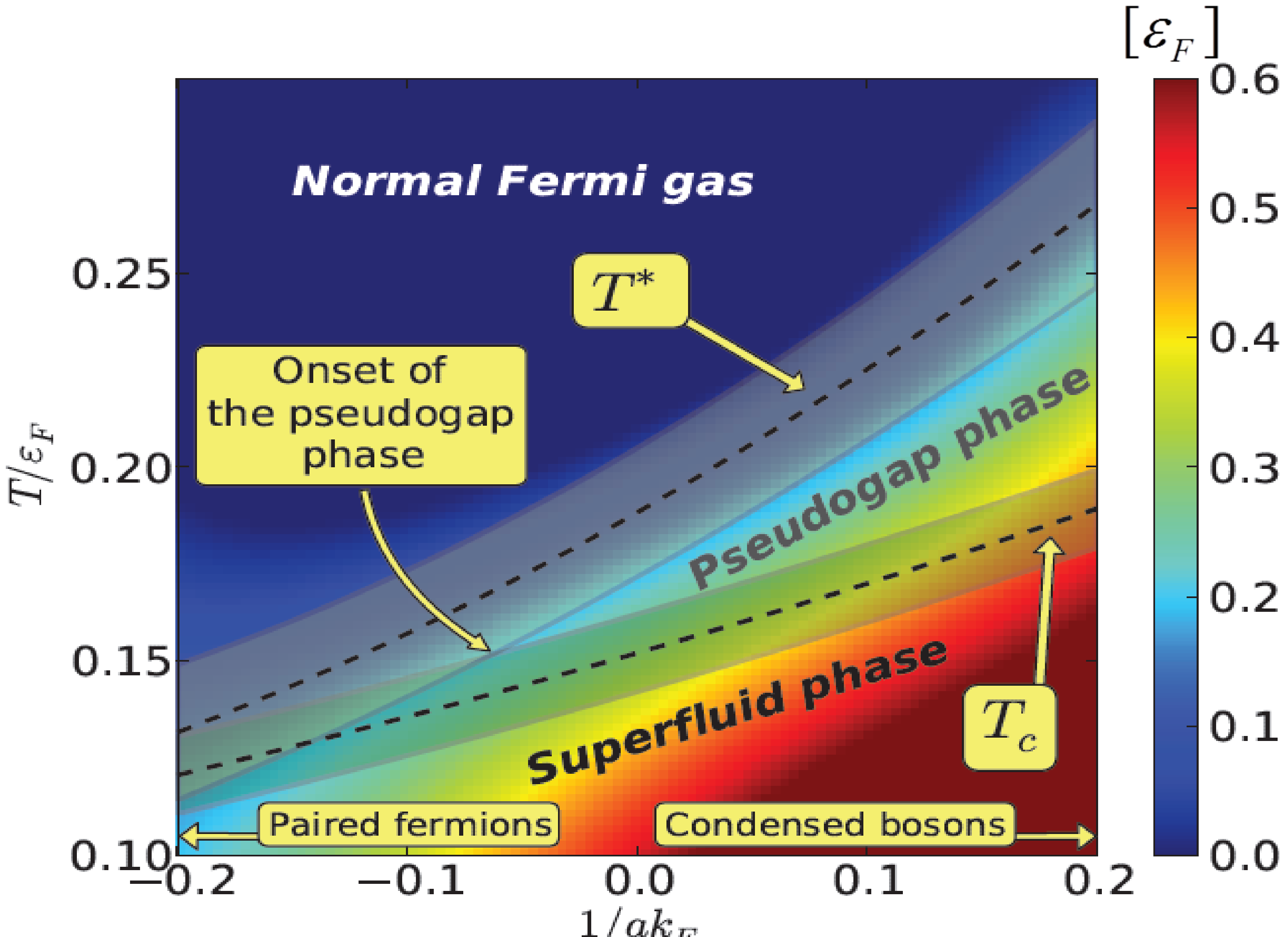
From Sa de Melo, Physics Today (2008)

<u>Pairing pseudogap:</u> suppression of low-energy spectral weight function due to incoherent pairing in the normal state ($T > T_c$)

<u>Important issue related to pairing pseudogap:</u>

- Are there sharp gapless quasiparticles in a normal Fermi liquid
  YES: Landau's Fermi liquid theory;
  NO: breakdown of Fermi liquid paradigm

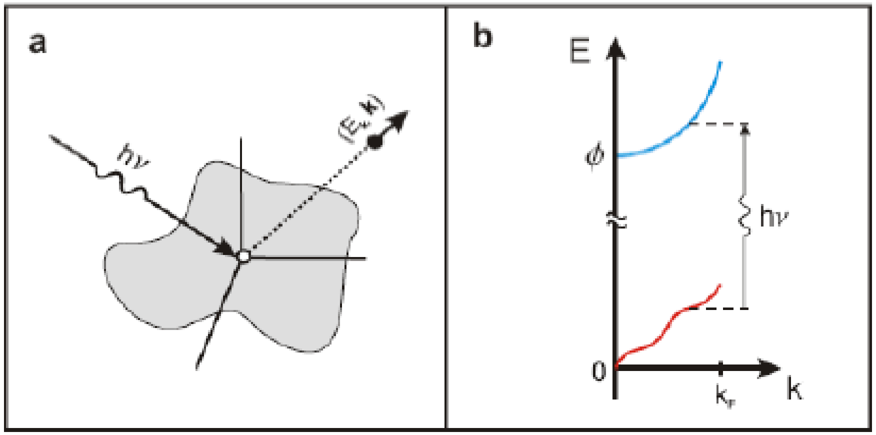
Gap in the single particle fermionic spectrum - theory



Ab initio **result: The onset of pseudogap phase at $1/ak_F \approx -0.05$.**

Magierski, Wlazłowski, Bulgac, *Phys. Rev. Lett.*107,145304(2011)
Magierski, Wlazłowski, Bulgac, Drut, *Phys. Rev. Lett.*103,210403(2009)

RF spectroscopy in ultracold atomic gases

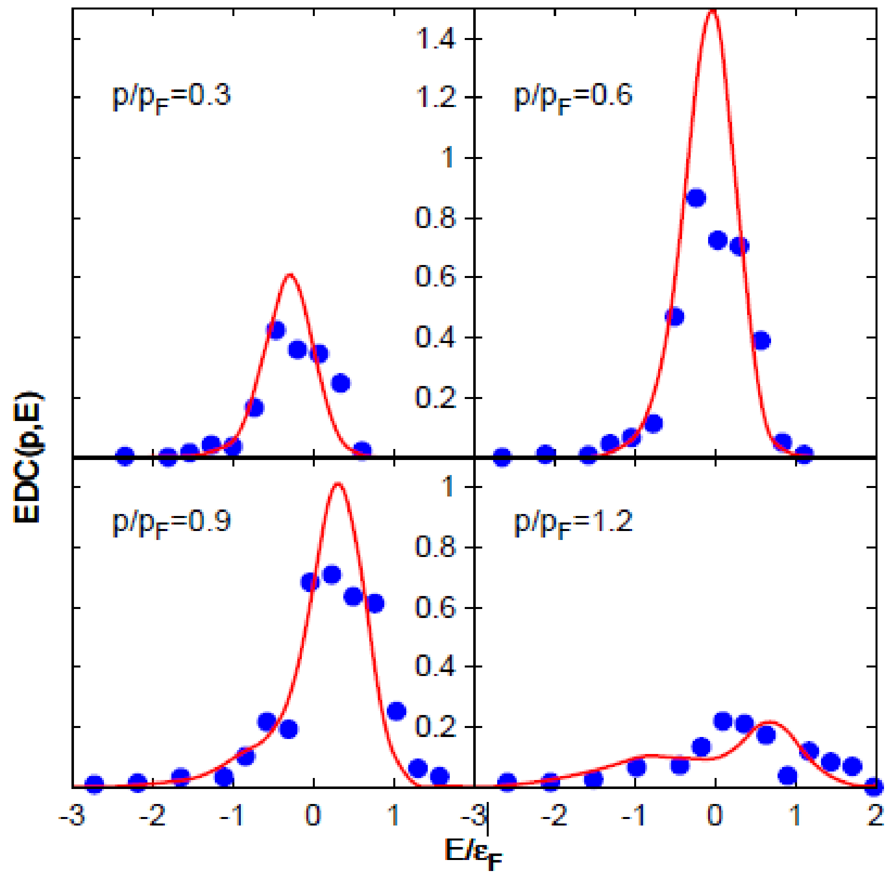


$$-E_s + h\nu = \frac{\hbar^2 k^2}{2m} + \phi$$

$$E(N) = E(N-1) + E_s$$

Stewart, Gaebler, Jin, Using photoemission spectroscopy to probe a strongly interacting Fermi gas, **Nature, 454, 744 (2008)**

$$\mathrm{EDC}(p,E,T) = \mathcal{C}p^2 \int_0^\infty dr\, r^2 \frac{1}{\varepsilon_F(r)} A\left[\frac{p}{p_F(r)}, \frac{E-\mu(r)}{\varepsilon_F(r)}, \frac{T}{\varepsilon_F(r)}\right] f(E-\mu(r)),$$



Experiment (blue dots): D. Jin's group
Gaebler et al. *Nature Physics 6, 569(2010)*
Theory (red line):
Magierski, Wlazłowski, Bulgac,
Phys.Rev.Lett.107,145304(2011)

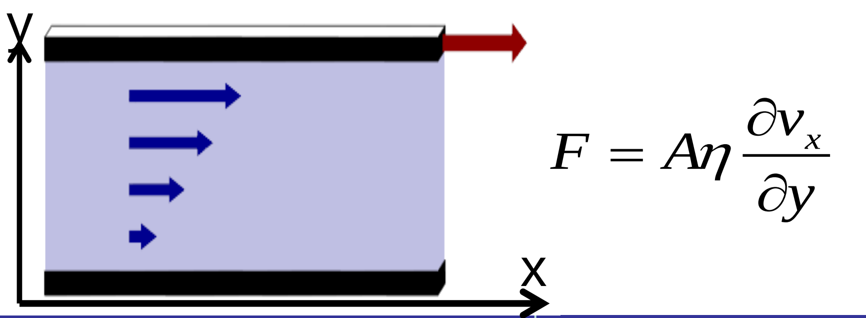
Hydrodynamics at unitarity

Scaling: $\psi_i(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) \rightarrow \frac{1}{\lambda^{3N/2}} \psi_i\left({}^{\vec{r}_1}\!/_{\lambda}, {}^{\vec{r}_2}\!/_{\lambda}, \ldots, {}^{\vec{r}_N}\!/_{\lambda} \right); E_i \rightarrow {}^{E_i}\!/_{\lambda^2}$

No intrinsic length scale ➡ **Uniform expansion keeps the unitary gas in equilibrium**

Consequence:
uniform expansion does not produce entropy = bulk viscosity is zero!

Shear viscosity:



For any physical fluid:

$$\frac{\eta}{S} \geq \frac{\hbar}{4\pi k_B}$$

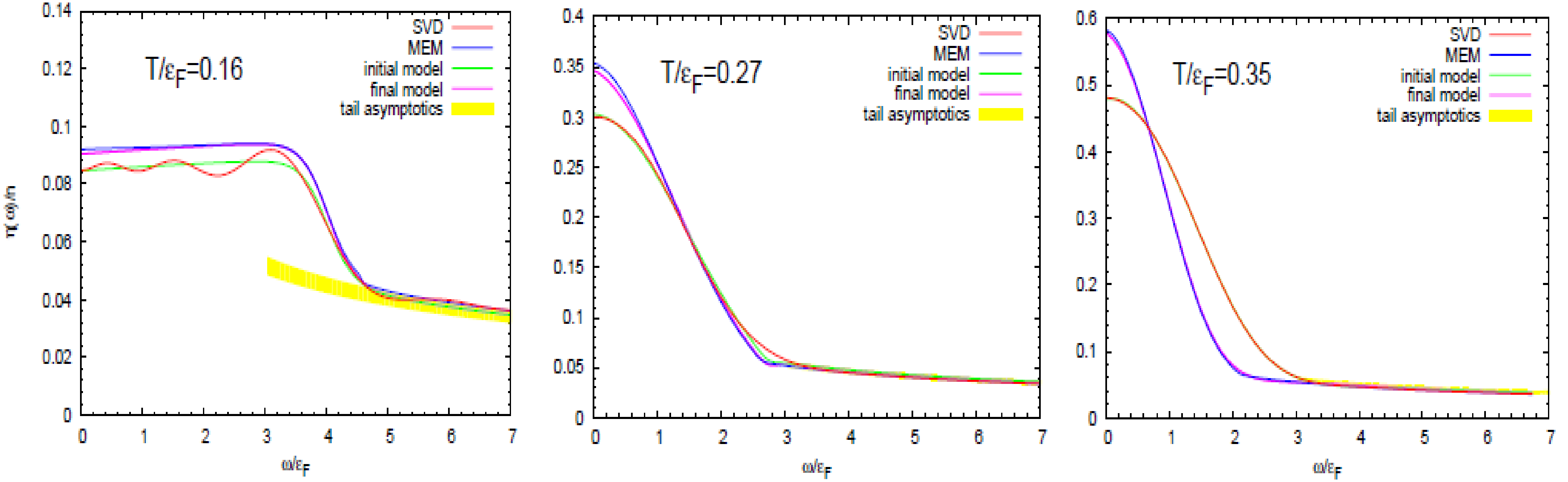
KSS conjecture
Kovtun,Son,Starinets, Phys.Rev.Lett. 94, 111601, (2005)
from AdS/CFT correspondence

Maxwell classical estimate: $\eta \sim$ mean free path

Perfect fluid $\frac{\eta}{S} = \frac{\hbar}{4\pi k_B}$ - strongly interacting quantum system = No well defined quasiparticles

Candidates: unitary Fermi gas, quark-gluon plasma

Combined Maximum Entropy and SVD methods were applied

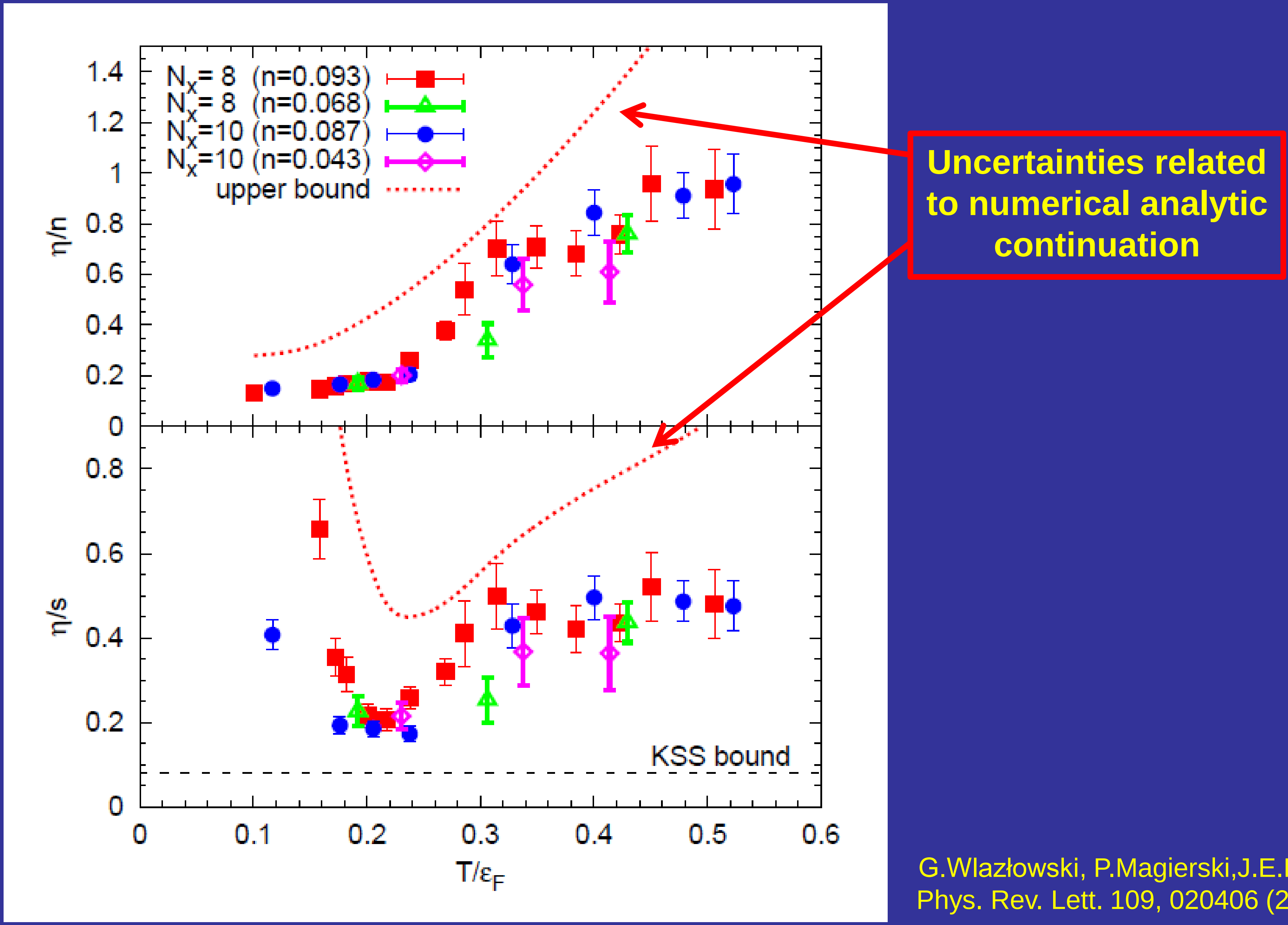


Additional symmetries and sum rules:

$$G(\tau) = G(\beta - \tau)$$

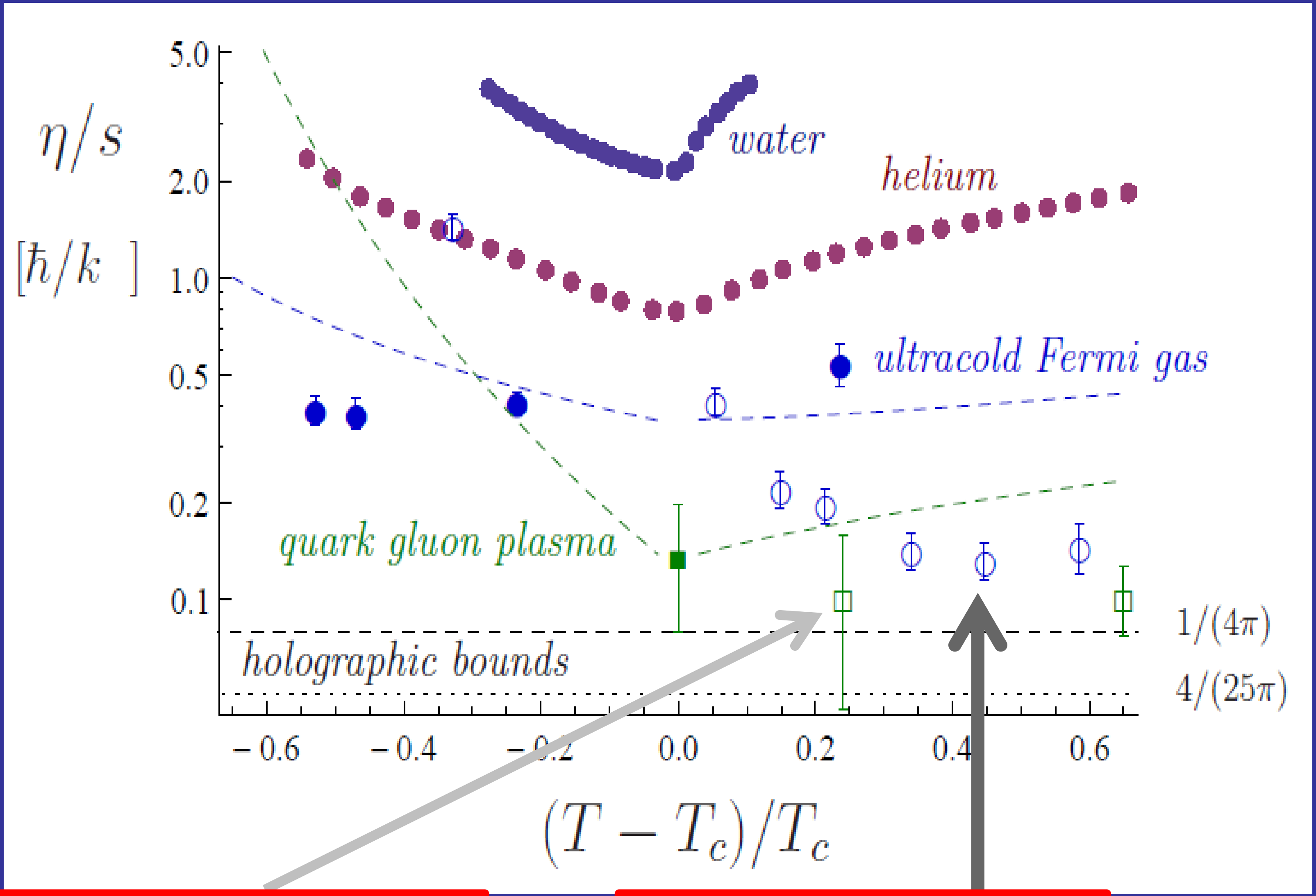
$$\frac{1}{\pi}\int_0^\infty d\omega \left[\eta(\omega) - \frac{C}{15\pi\sqrt{m\omega}}\right] = \frac{\varepsilon}{3}$$ ε – *energy density*

$$\eta(\omega \to \infty) \simeq \frac{C}{15\pi\sqrt{m\omega}}.$$



G.Wlazłowski, P.Magierski, J.E.Drut, Phys. Rev. Lett. 109, 020406 (2012)

Shear viscosity to entropy ratio – experiment vs. theory
(from A. Adams et al.1205.5180)



Lattice QCD (SU(3) gluodynamics):
H.B. Meyer, Phys. Rev. D 76, 101701 (2007)

QMC calculations for UFG:
G. Wlazłowski, P. Magierski, J.E. Drut,
Phys. Rev. Lett. 109, 020406 (2012)

Spin susceptibility, spin conductivity, spin diffusion

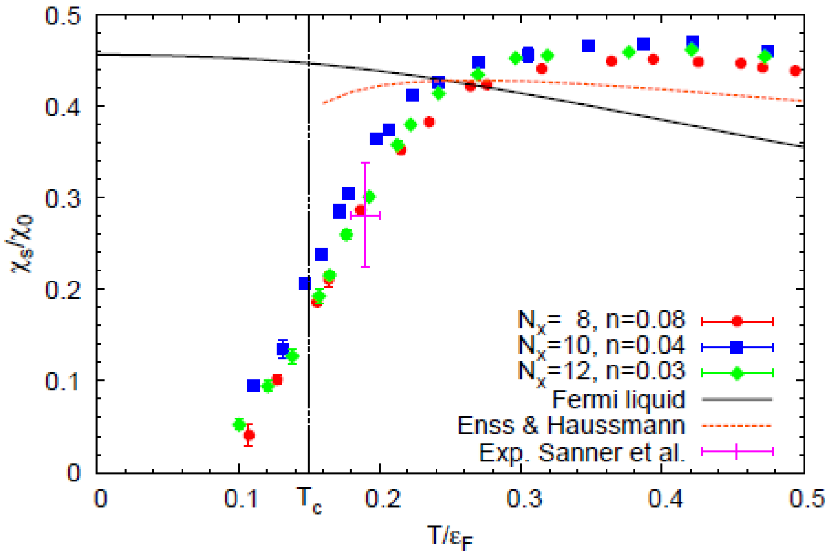


FIG. 2: (Color online) The static spin susceptibility as a function of temperature for an 8^3 lattice solid (red) circles, 10^3 lattice (blue) squares and 12^3 lattice (green) diamonds. Vertical black dotted line indicates the critical temperature of superfluid to normal phase transition $T_c = 0.15\,\varepsilon_F$. For comparison Fermi liquid theory prediction and recent results of the T-matrix theory produced by Enss and Haussmann [25] are plotted with solid and dashed (brown) lines, respectively. The experimental data point from Ref. [15] is also shown.

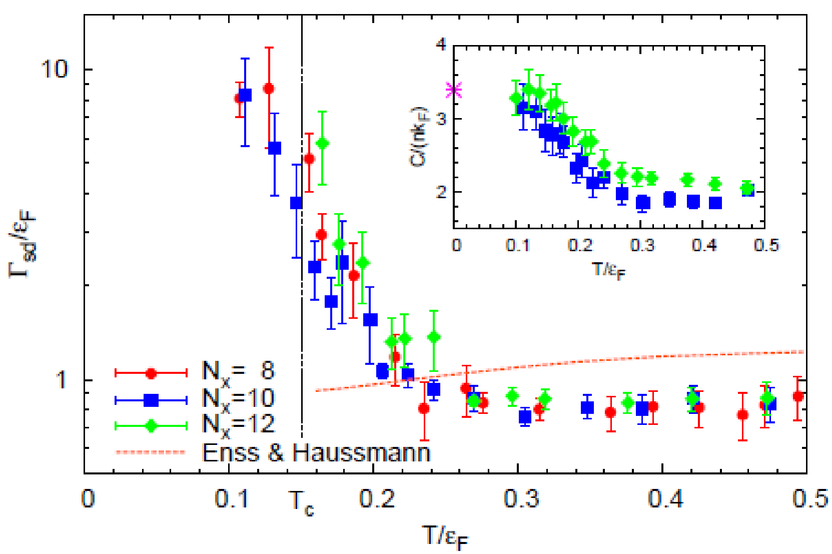


FIG. 3: (Color online) The spin drag rate $\Gamma_{sd} = n/\sigma_s$ in units of Fermi energy as a function of temperature for an 8^3 lattice solid (red) circles, 10^3 lattice (blue) squares and 12^3 lattice (green) diamonds. Vertical black dotted line locates the critical temperature of superfluid to normal phase transition. Results of the T-matrix theory are plotted by dashed (brown) line [25]. The inset shows extracted value of the contact density as function of the temperature. The (purple) asterisk shows the contact density from the QMC calculations of Ref. [29] at $T = 0$.

Wlazłowski, Magierski, Bulgac, Drut, Roche, arXive:1212.1503, to appear in Phys. Rev. Lett. (2013)

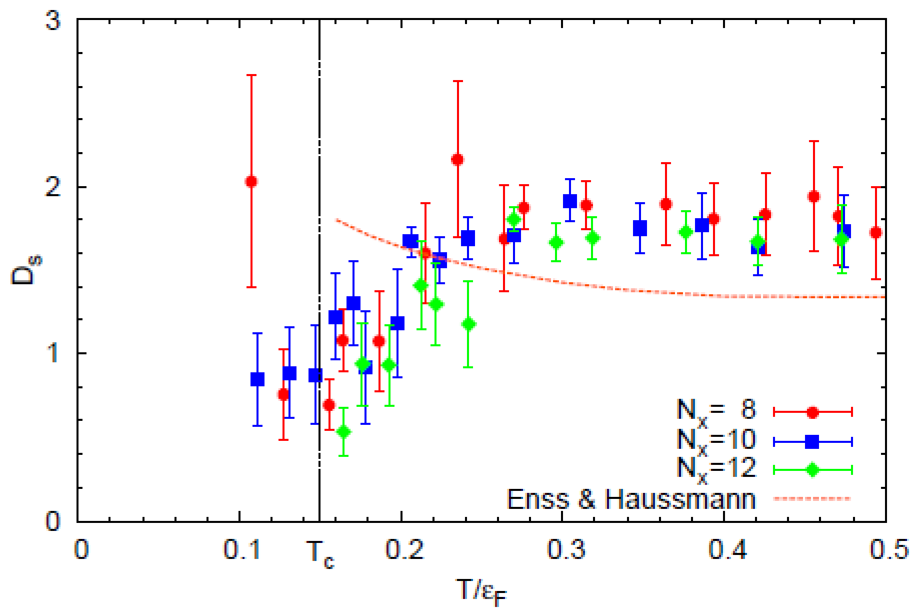
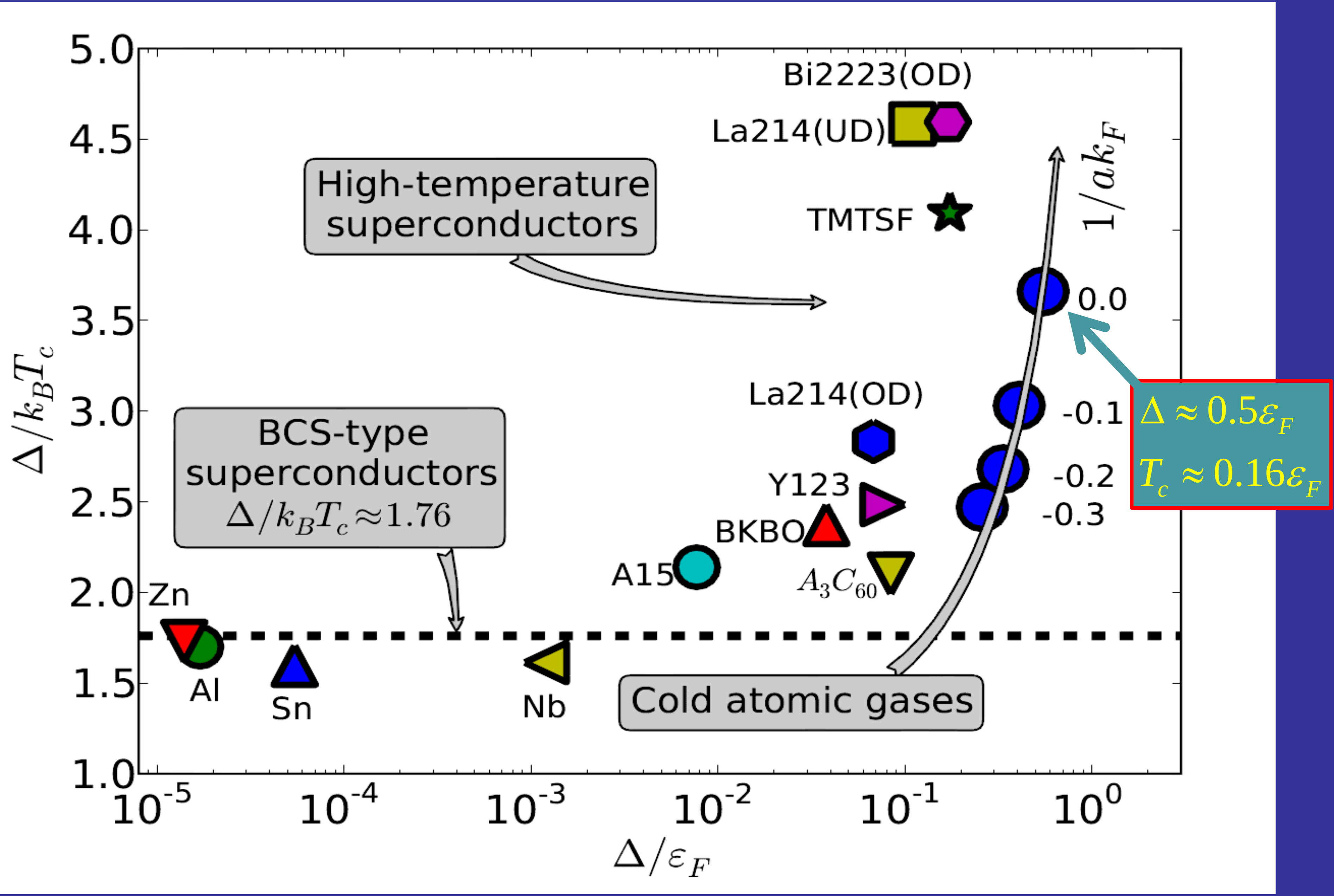


FIG. 4: (Color online) The spin diffusion coefficient obtained by the Einstein relation $D_s = \sigma_s/\chi_s$ as function of temperature. The notation is identical to Fig. 3.

Wlazłowski, Magierski, Bulgac, Drut, Roche, arXive:1212.1503, to appear in Phys. Rev. Lett. (2013)

Cold atomic gases and high Tc superconductors



From Fischer et al., Rev. Mod. Phys. 79, 353 (2007) & P. Magierski, G. Wlazłowski, A. Bulgac, Phys. Rev. Lett. 107, 145304 (2011)