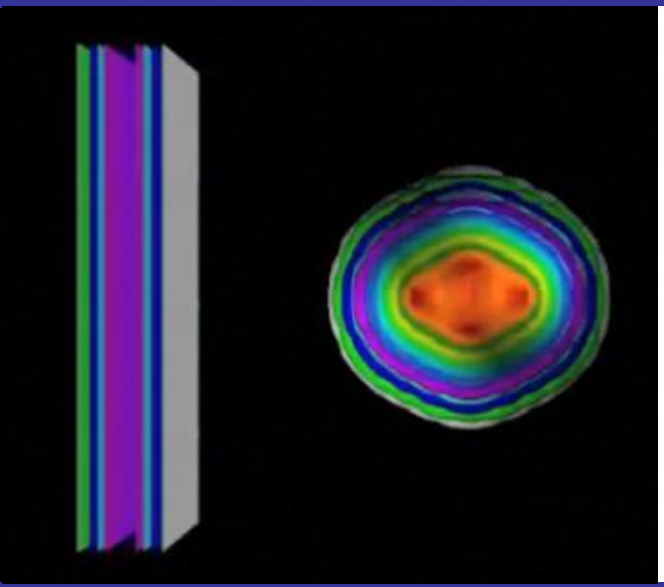
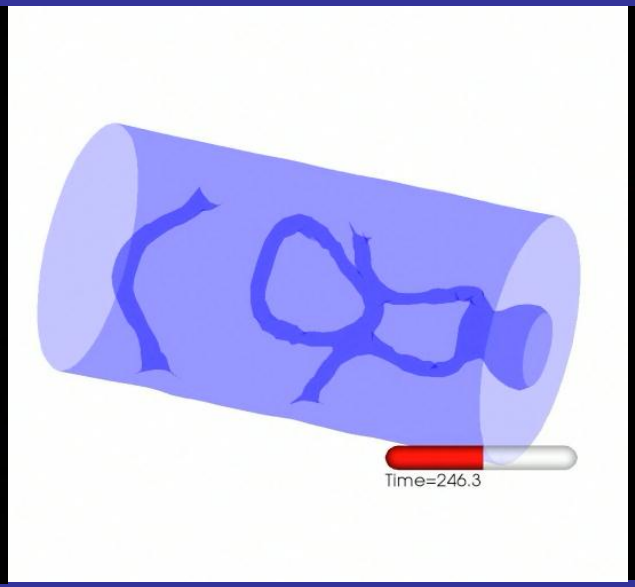
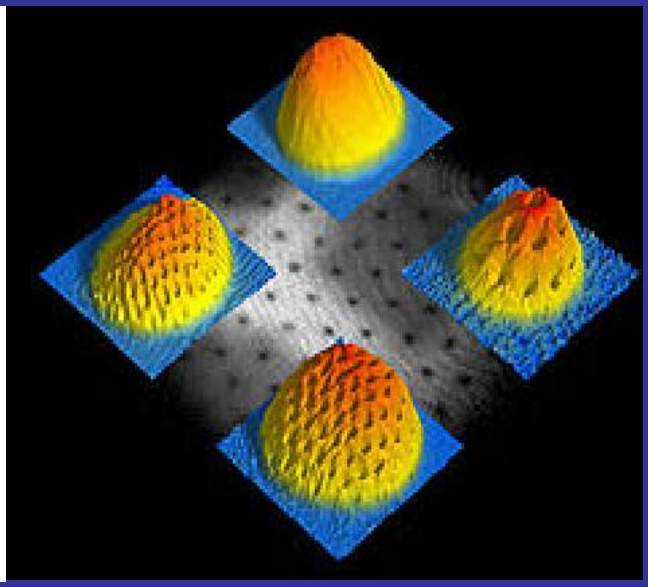
Dynamika układów nadciekłych: jąder atomowych i gazów kwantowych w ramach teorii funkcjonału gęstości







Piotr Magierski
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Yongle Yu
(Wuhan)

GOAL:

Description of fermionic superfluids (nuclei and quantum gases) far from equilibrium.

From quantum mechanics:

$$i\hbar \frac{\partial}{\partial t}\psi = \hat{H}\psi$$

However even if we know the Hamiltonian

can we solve in practice the above equation?

Consider a nucleus comprising of *A* nucleons

Its radius is of the order of: $R = r_0 A^{1/3}, \quad r_0 \approx 1.2\ fm$

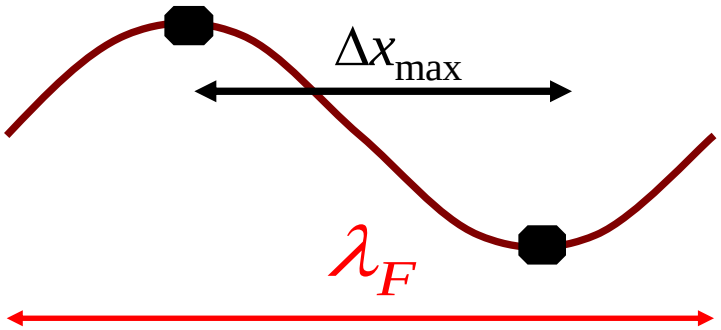
In order to make a reliable calculations we need to consider volumes at least of the order of $V \geq (2R)^3$

From the Fermi gas model we can estimate the momentum of the nucleon at the Fermi level:

$$p_F / \hbar = k_F = \left(\frac{3}{2}\pi^2 \rho\right)^{1/3}, \rho \approx 0.16\ fm^{-3} - \textit{nuclear saturation density}$$

$$\lambda = 2\pi / k \quad \textit{de Broglie}$$

$$\Delta x_{max} = \frac{\lambda_F}{2}$$ *Maximum distance between points*



It implies that the nuclear wave function has to be known at least in a number of points equal to:

$$\frac{V}{(\Delta x_{max})^3} = \frac{8(k_F r_0)^3}{\pi^3} A \approx A$$

But the wave function depends on A variables (disregarding spin):

$$\Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A, t)$$

and to store the wave function (at fixed *t*) we need to store A^A **complex numbers.**

For $A \approx 100$ **it means** 10^{200} **complex numbers!**

Not possible now and
most likely will never be!!!

But the situation is even worse because we need to evolve this wave function in time…

Time evolution

If we are interested in nuclear processes involving excitations up to several hundreds MeV we need to adjust the time step accordingly:

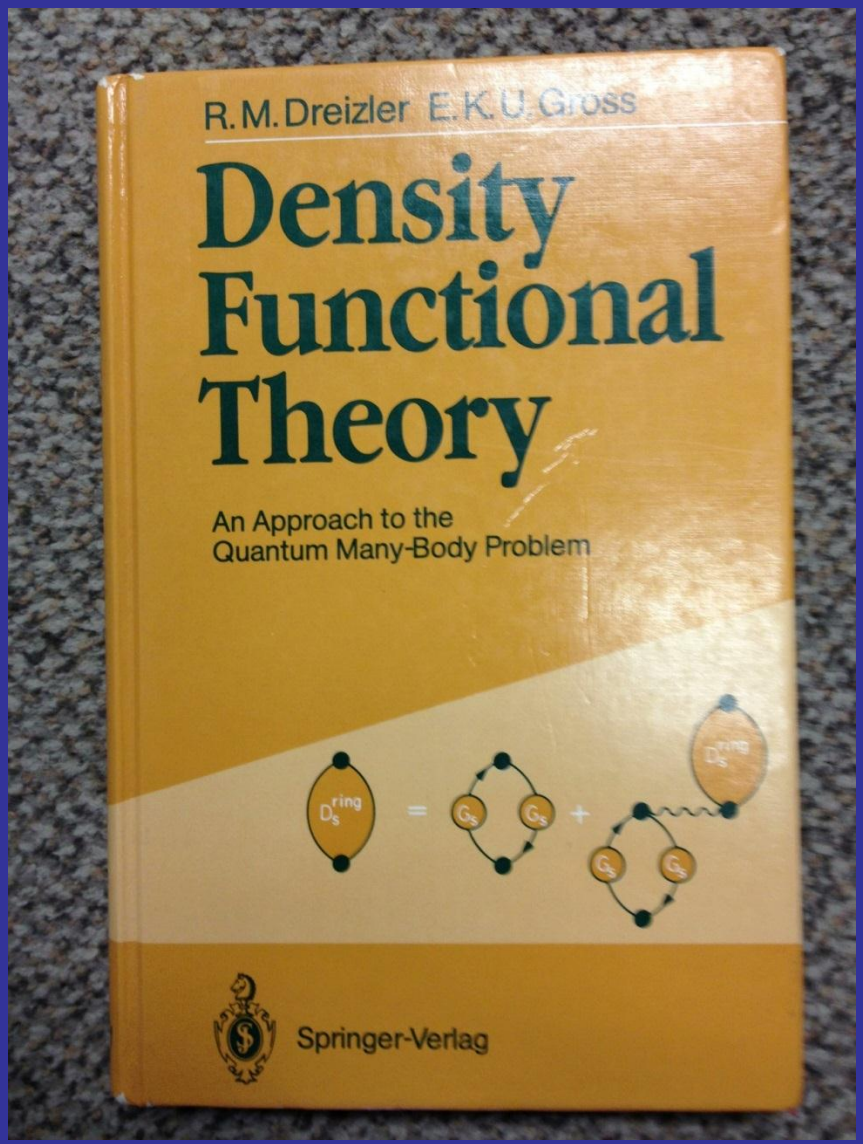
$$\Delta t = \frac{\hbar}{E_{exc}} \approx 0.1 \; fm/c$$

And in order to get the energy resolution of the order of *keVs* the length of the time evolution should be:

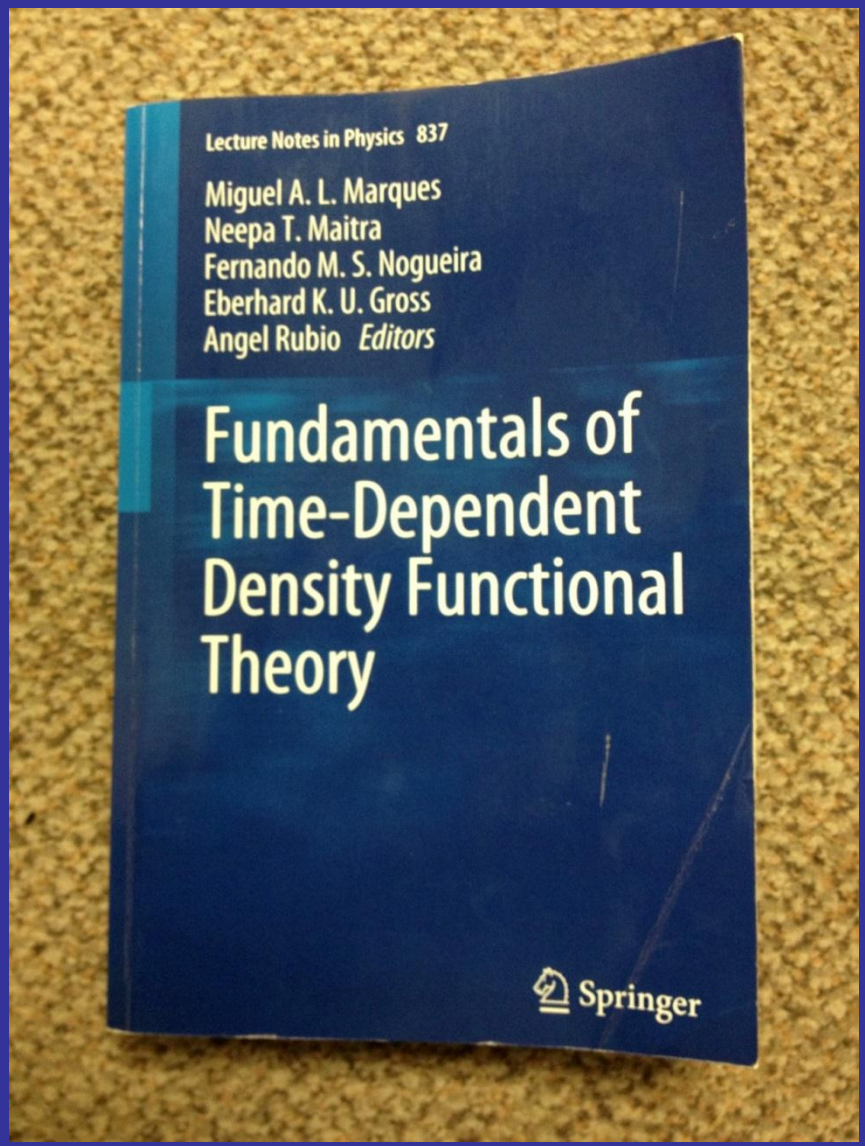
$$T = \frac{\hbar}{\Delta E} \approx 10^5 \; fm/c$$

Summarizing: we need to evolve the wave function through 1 million time steps without loosing numerical precision!

Density Functional Theory (DFT)



1990



2012

Density Functional Theory

Formally rigorous way of approaching *any* interacting problem by mapping it *exactly* to a much easier-to-solve noninteracting system.

Static case:

It can be shown that instead of wave function one may use a density distribution:

$$\rho(\vec{r}) = \int d^3 r_2 ... d^3 r_A \, | \, \Psi(\vec{r}, \vec{r}_2, ..., \vec{r}_A) \, |^2$$

Theorem (Hohenberg & Kohn):
The energy of the nondegenerate ground state of the Fermi system is uniquely determined by its density distribution.

$$\Psi \xleftrightarrow{1\text{–}1} \rho \quad \Rightarrow \quad E[\rho] = \left\langle \Psi[\rho] \middle| \hat{H} \middle| \Psi[\rho] \right\rangle$$

P. Hohenberg, W. Kohn, Phys. Rev. 136, B864 (1964)
W. Kohn, L.J. Sham, Phys. Rev. 140, A1133 (1965)

Time dependent case:

If $i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi, \quad \psi_0 = \psi(t_0)$ and $\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$

then (Runge & Gross theorem):

$$\left.\begin{array}{l}\rho(\vec{r},t)\\ \psi(\ldots,t_0)\end{array}\right\} \leftrightarrow e^{i\alpha(t)}\psi(\ldots,t)$$

Up to an arbitrary function α(t)

and consequently the functional exists:

$$F[\psi_0,\rho] = \int_{t_0}^{t_1} \langle \psi[\rho] | \left(i\hbar\frac{\partial}{\partial t} - \hat{H} \right) | \psi[\rho] \rangle \, dt$$

E. Runge, E.K.U Gross, PRL 52, 997 (1984)
B.-X. Xu, A.K. Rajagopal, PRA 31, 2682 (1985)
G. Vignale, PRA77, 062511 (2008)

Formalism for Time Dependent Phenomena: TDSLDA

A.K. Rajagopal and J. Callaway, Phys. Rev. B 7, 1912 (1973)
V. Peuckert, J. Phys. C 11, 4945 (1978)
E. Runge and E.K.U. Gross, Phys. Rev. Lett. 52, 997 (1984)

Local density approximation (no memory effects – adiabatic TDDFT)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k\uparrow}(\mathbf{r},t) \\ u_{k\downarrow}(\mathbf{r},t) \\ v_{k\uparrow}(\mathbf{r},t) \\ v_{k\downarrow}(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow,\uparrow}(\mathbf{r},t) & h_{\uparrow,\downarrow}(\mathbf{r},t) & 0 & \Delta(\mathbf{r},t) \\ h_{\downarrow,\uparrow}(\mathbf{r},t) & h_{\downarrow,\downarrow}(\mathbf{r},t) & -\Delta(\mathbf{r},t) & 0 \\ 0 & -\Delta^*(\mathbf{r},t) & -h^*_{\uparrow,\uparrow}(\mathbf{r},t) & -h^*_{\uparrow,\downarrow}(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & 0 & -h^*_{\uparrow,\downarrow}(\mathbf{r},t) & -h^*_{\downarrow,\downarrow}(\mathbf{r},t) \end{pmatrix} \begin{pmatrix} u_{k\uparrow}(\mathbf{r},t) \\ u_{k\downarrow}(\mathbf{r},t) \\ v_{k\uparrow}(\mathbf{r},t) \\ v_{k\downarrow}(\mathbf{r},t) \end{pmatrix}$$

Density functional contains normal densities, anomalous density (pairing) and currents:

$$E(t) = \int d^3r \left[\varepsilon(n(\vec{r},t),\tau(\vec{r},t),\nu(\vec{r},t),\vec{j}(\vec{r},t)) + V_{ext}(\vec{r},t)n(\vec{r},t) + \ldots \right]$$

Density functional for unitary Fermi gas

Nuclear energy functional: SLy4, SkP, SkM*,…

TDSLDA is formulated on the 3D lattice without any symmetry restrictions.
Initial conditions for TDSLDA are generated through adiabatic switching and quantum friction.

Why TDDFT?

- Unlike ATDHFB, TDSLDA does not require introduction of hard to define collective degrees of freedom and there are no ambiguities arising from defining potential energy surfaces (PES) and inertias
- Interaction with basically any external probe (weak or strong) easy to implement
- Accuracy of description in terms quantized trajectories with Maslov index corrections is at the same level of theoretical accuracy of a re-quantization of collective Hamiltonian
- One-body dissipation is automatically included in the formalism and this is a quantum approach (unlike diffusion models)
- TD equations are a consequence of the action minimum principle and thus the perils of using minimum of energy trajectories on PES and inertia are not encountered
- Overall computational effort in TDDSLDA is significantly less that in a ATHFB in a large collective space (five or more dimensions, which requires 5-9 million of configuration presently) and more accurate numerically
- There are ways to include collective surface hopping and thus the effects of two-body collisions (surface hopping already studied for more than two decades in chemistry and now entering the condensed matter filed for normal systems)

Limitations:

- Only one body observables can be described accurately (can be overcome by Balian-Veneroni method)
- Requires large computer resources
- The results depend on the quality of the density functional

Selected capabilities of the SLDA/TDSLDA codes:

- ✓ full 3D simulations with no symmetry restrictions
- ✓ number of evolved quasiparticle wave functions is of the order of the lattice size: $O(10^4)$- $O(10^6)$
- ✓ high numerical accuracy for spatial derivatives using FFTW
- ✓ for TD high-accuracy and numerically stable Adams–Bashforth–Milne 5th order predictor-corrector-modifier algorithm with only 2 evaluations of the rhs per time step and with no matrix operations
- ✓ Eg. we evolve 4x136626=546504 coupled eigenvectors for ^{238}U on the lattice: 50x50x80 fm (mesh size: 1.25fm) with energy cutoff 100MeV to an accuracy 10^{-8}
- ✓ volumes of the order of ($L = 80^3$) capable of simulating time evolution of 42000 neutrons at saturation density (possible application: neutron stars)
- ✓ capable of simulating up to times of the order of 10^{-19} s (a few million time steps)
- ✓ CPU vs GPU on Titan ≈ 15 speed-up (likely an additional factor of 4 possible)

Eg. for 137062 two component wave functions:

CPU version (4096 nodes x 16 PEs) - 27.90 sec for 10 time steps

GPU version (4096 PEs + 4096GPU) - 1.84 sec for 10 time steps

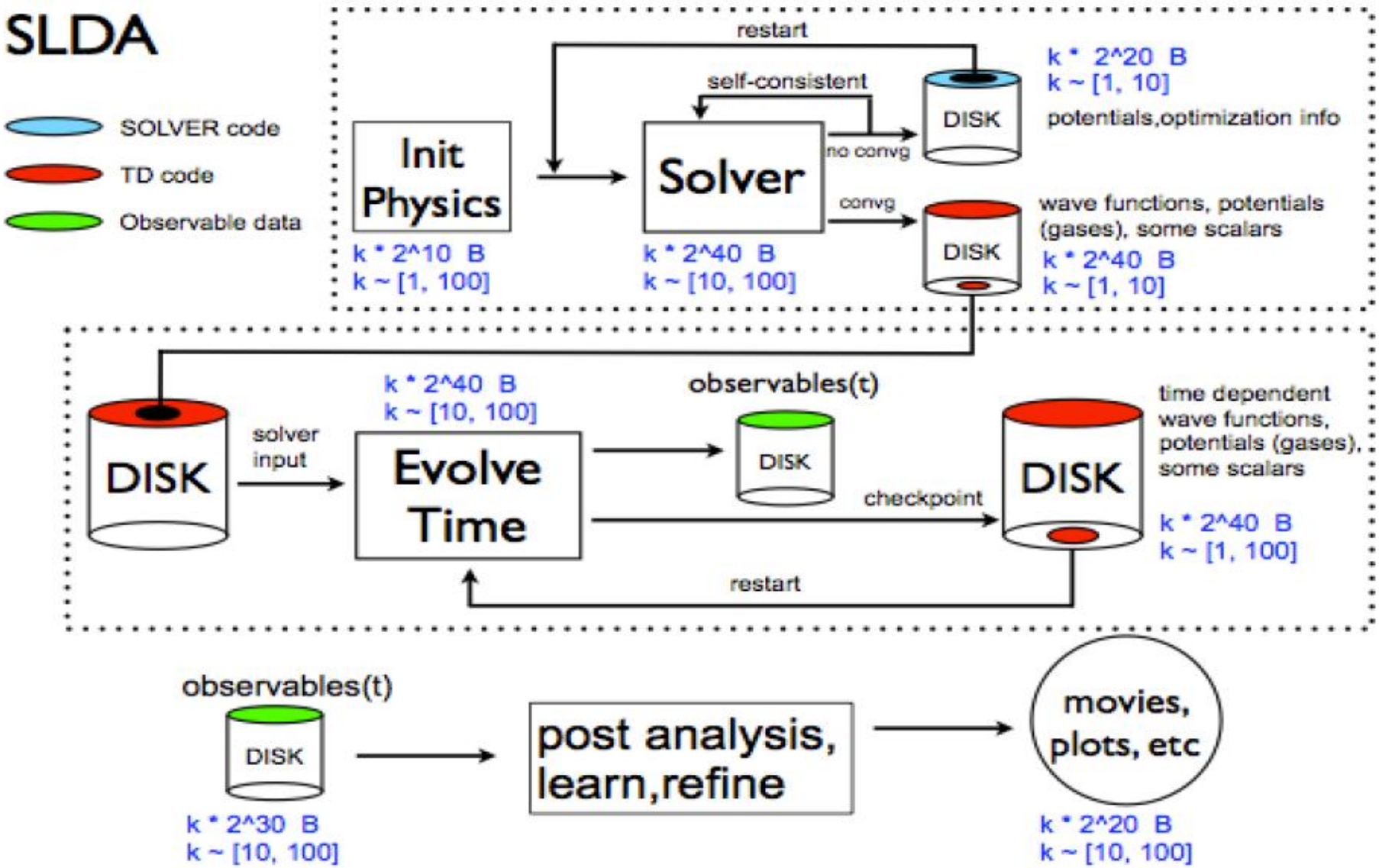


Figure 4: Structure of the (TD)SLDA production software and workflow. Data data magnitudes per analysis stage are printed blue and range from kilobytes to describe a problem, to hundreds of terabytes in memory, to tens of terabytes for checkpoint / restart, to up to nearly a hundred gigabytes data for post analysis.

Physical systems and processes we are interested in:

- ✓ Collective states in nuclei
- ✓ Nuclear large amplitude collective motion (LACM)
  (Induced) nuclear fission
- ✓ Excitation of nuclei with gamma rays and neutrons
- ✓ Coulomb excitation of nuclei with relativistic heavy-ions
- ✓ Nuclear reactions, fusion between colliding heavy-ions
- ✓ Neutron star crust and dynamics of vortices and their pinning mechanism

- ✓ Dynamics of vortices, Anderson-Higgs Mode
- ✓ Vortex crossing and reconnection and the onset of quantum turbulence
- ✓ Domain wall solitons and shock waves in collision of fermionic superfluid atomic clouds

Nuclear DFT

$$E = \int d^3r \mathcal{H}(\mathbf{r})$$

where

$$\begin{aligned}\mathcal{H}(\mathbf{r}) &= C^{\rho}\rho^2 + C^{s}\vec{s}\cdot\vec{s} + C^{\Delta\rho}\rho\nabla^2\rho + C^{\Delta s}\vec{s}\cdot\nabla^2\vec{s} + C^{\tau}(\rho\tau - \vec{j}\cdot\vec{j}) + \\ &+ C^{sT}(\vec{s}\cdot\vec{T} - \mathbf{J}^2) + C^{\nabla J}(\rho\vec{\nabla}\cdot\vec{J} + \vec{s}\cdot(\vec{\nabla}\times\vec{j})) + C^{\nabla s}(\vec{\nabla}\cdot\vec{s})^2 + C^{\gamma}\rho^{\gamma} - \Delta\chi^*\end{aligned}$$

where

$$\begin{aligned}J_i &= \sum_{k,l}\epsilon_{ikl}\mathbf{J}_{kl} \\ \mathbf{J}^2 &= \sum_{k,l}\mathbf{J}_{kl}^2\end{aligned}$$

- density: $\rho(\mathbf{r}) = \rho(\mathbf{r},\mathbf{r}')|_{r=r'}$
- spin density: $\vec{s}(\mathbf{r}) = \vec{s}(\mathbf{r},\mathbf{r}')|_{r=r'}$
- current: $\vec{j}(\mathbf{r}) = \frac{1}{2i}(\vec{\nabla} - \vec{\nabla}')\rho(\mathbf{r},\mathbf{r}')|_{r=r'}$
- spin current (2nd rank tensor): $\mathbf{J}(\mathbf{r}) = \frac{1}{2i}(\vec{\nabla} - \vec{\nabla}')\otimes\vec{s}(\mathbf{r},\mathbf{r}')|_{r=r'}$
- kinetic energy density: $\tau(\mathbf{r}) = \vec{\nabla}\cdot\vec{\nabla}'\rho(\mathbf{r},\mathbf{r}')|_{r=r'}$
- spin kinetic energy density: $\vec{T}(\mathbf{r}) = \vec{\nabla}\cdot\vec{\nabla}'\vec{s}(\mathbf{r},\mathbf{r}')|_{r=r'}$
- anomalous (pairing) density: $\chi(\mathbf{r}) = \chi(\mathbf{r},\mathbf{r}')|_{r=r'}$

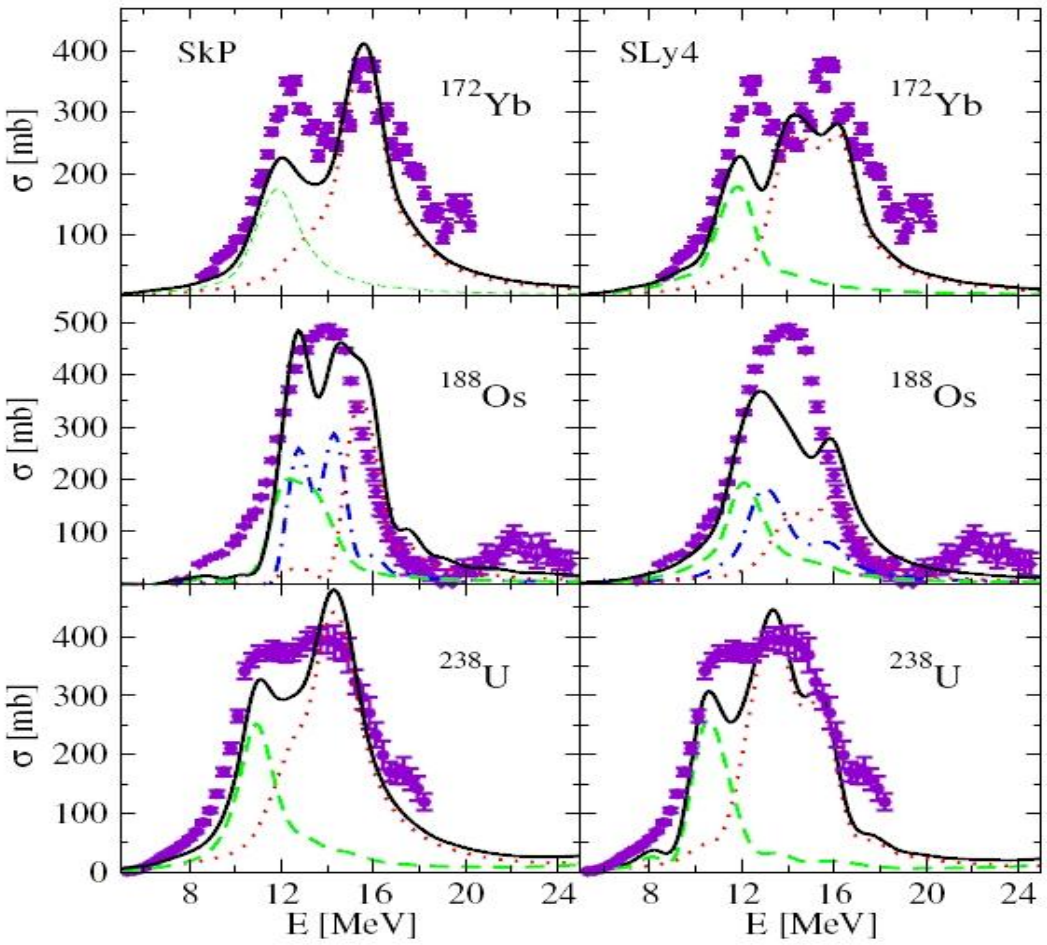
Nuclear dynamics from time dependent density functional theory

$$S(E) = \sum_\nu |\langle \nu | \hat{F} | 0 \rangle|^2 \delta(E - E_\nu)$$

$$S(\omega) = \mathrm{Im}\{\delta F(\omega)/[\pi f(\omega)]\}$$

Photoabsorption cross section for heavy, deformed nuclei.

(gamma,n) reaction through the excitation of GDR



I.Stetcu, A.Bulgac, P. Magierski, K.J. Roche, Phys. Rev. C84 051309 (2011)

Evolution of occupation probabilities

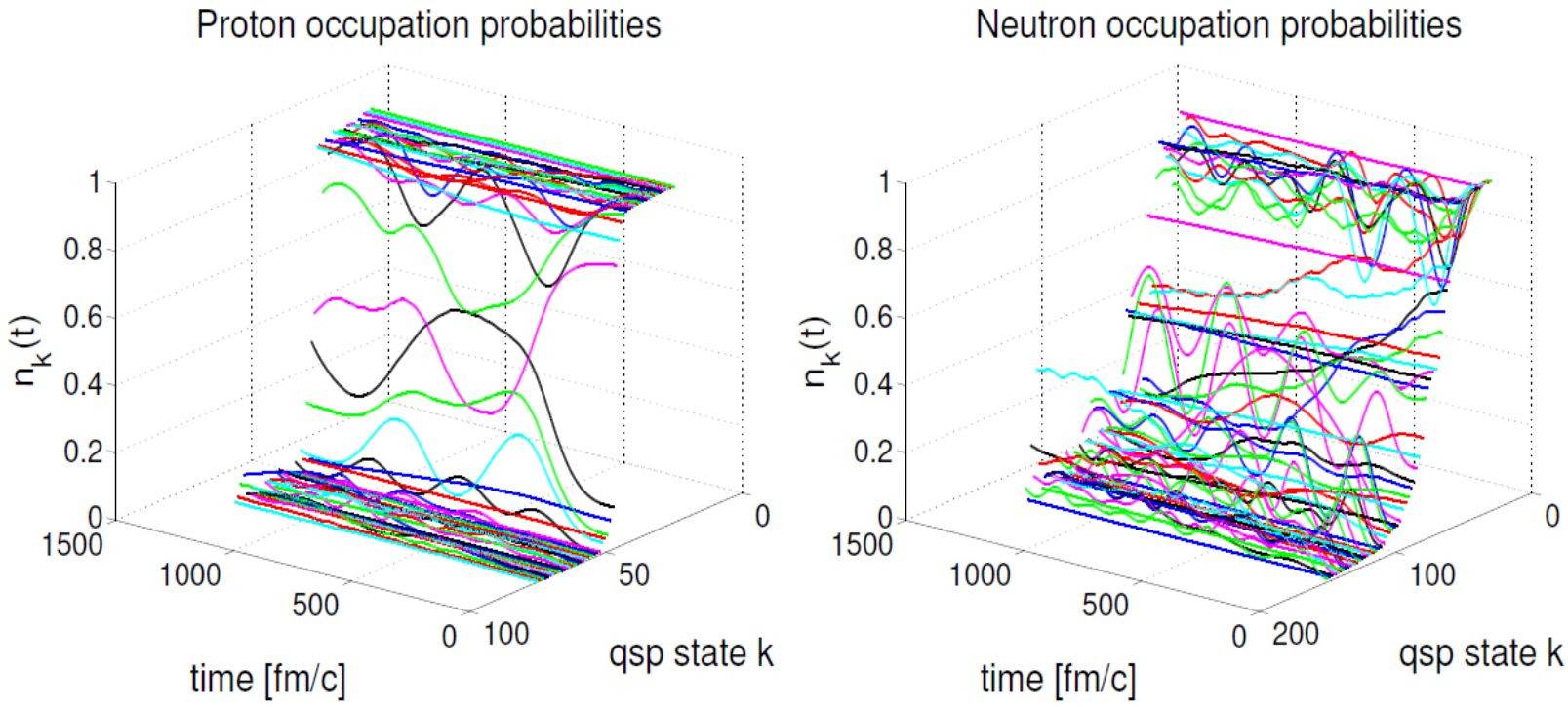


FIG. 1. (Color online) The time-dependent proton and neutron occupation probabilities of a a number of quasiparticle states around the Fermi level for ^{238}U calculated as described in the main text with SLy4.

Box size: 32.5fm (mesh size: 1.25fm)
Energy deposited into a nucleus: 45-50MeV
Adiabatic switching of external perturbation: C*exp[-(t-10)^2/2]
Time window for Fourier transform: 1600 fm/c
Time step: 0.12fm/c -> relative accuracy: 10^(-7)

Coupling to e.m. field:

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$
$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla}\psi \to \vec{\nabla}_A\psi = \left(\vec{\nabla} - i\frac{e}{\hbar c}\vec{A}\right)\psi$$
$$\vec{\nabla}\psi^* \to \vec{\nabla}_{-A}\psi^* = \left(\vec{\nabla} + i\frac{e}{\hbar c}\vec{A}\right)\psi^*$$
$$i\hbar\frac{\partial}{\partial t}\psi \to \left(i\hbar\frac{\partial}{\partial t} - e\phi\right)\psi$$

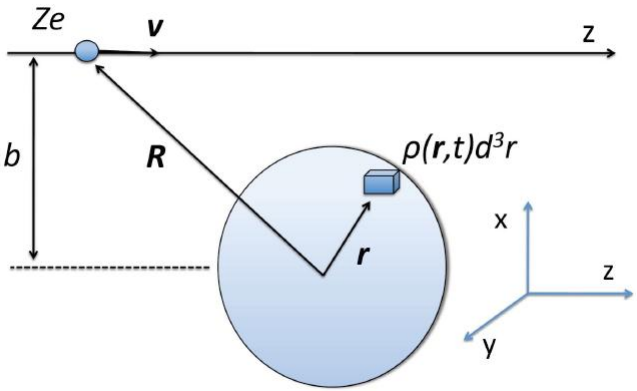
which implies that $\vec{\nabla}\psi\psi^* \to \vec{\nabla}\psi\psi^*$.

Consequently the densities change according to:

- density: $\rho_A(\mathbf{r}) = \rho_A(\mathbf{r})$
- spin density: $\vec{s}_A(\mathbf{r}) = \vec{s}(\mathbf{r})$
- current: $\vec{j}_A(\mathbf{r}) = \vec{j}(\mathbf{r}) - \frac{e}{\hbar c}\vec{A}\rho(\mathbf{r})$
- spin current (2nd rank tensor): $\mathbf{J}_A(\mathbf{r}) = \mathbf{J}(\mathbf{r}) - \frac{e}{\hbar c}\vec{A} \otimes \vec{s}(\mathbf{r})$
- spin current (vector): $\vec{J}_A(\mathbf{r}) = \vec{J}(\mathbf{r}) - \frac{e}{\hbar c}\vec{A} \times \vec{s}(\mathbf{r})$
- kinetic energy density: $\tau_A(\mathbf{r}) = \left(\vec{\nabla} - i\frac{e}{\hbar c}\vec{A}\right) \cdot \left(\vec{\nabla}' + i\frac{e}{\hbar c}\vec{A}\right)\rho(\mathbf{r},\mathbf{r}')|_{r=r'}$
  $= \tau(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A}\cdot\vec{j}(\mathbf{r}) + \frac{e^2}{\hbar^2c^2}|\vec{A}|^2\rho(\mathbf{r}) = \tau(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A}\cdot\vec{j}_A(\mathbf{r}) - \frac{e^2}{\hbar^2c^2}|\vec{A}|^2\rho(\mathbf{r})$
- spin kinetic energy density: $\vec{T}_A(\mathbf{r}) = \left(\vec{\nabla} - i\frac{e}{\hbar c}\vec{A}\right) \cdot \left(\vec{\nabla}' + i\frac{e}{\hbar c}\vec{A}\right)\vec{s}(\mathbf{r},\mathbf{r}')|_{r=r'}$
  $= \vec{T}(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A}^T\cdot\mathbf{J}(\mathbf{r}) + \frac{e^2}{\hbar^2c^2}|\vec{A}|^2\vec{s}(\mathbf{r}) = \vec{T}(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A}^T\cdot\mathbf{J}_A(\mathbf{r}) - \frac{e^2}{\hbar^2c^2}|\vec{A}|^2\vec{s}(\mathbf{r})$

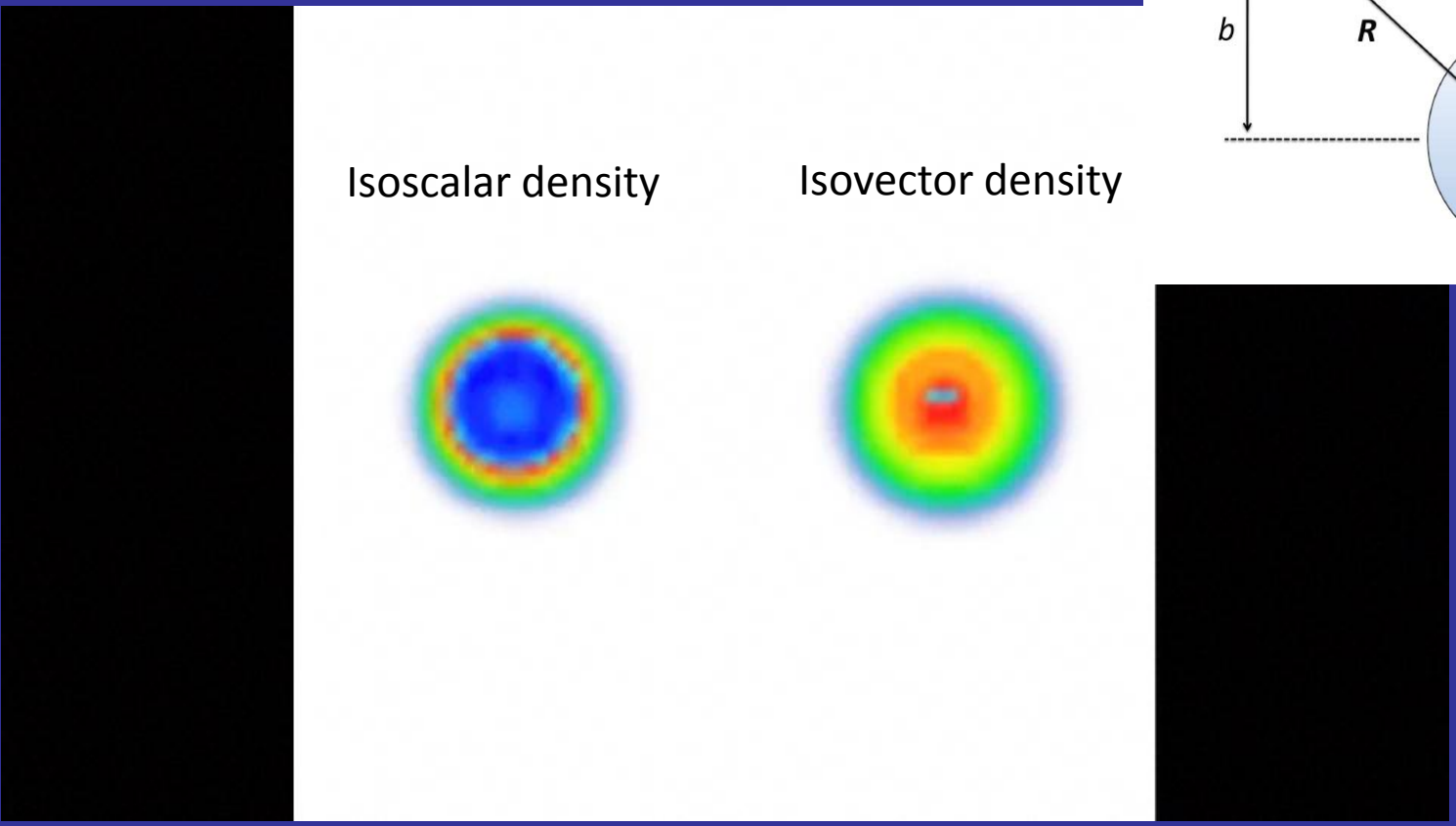
Relativistic Coulomb excitation

$$^{208}Pb + ^{238}U \rightarrow ^{238}U^{*} + \ldots \text{ at about 700 MeV/n}$$



Isoscalar density

Isovector density



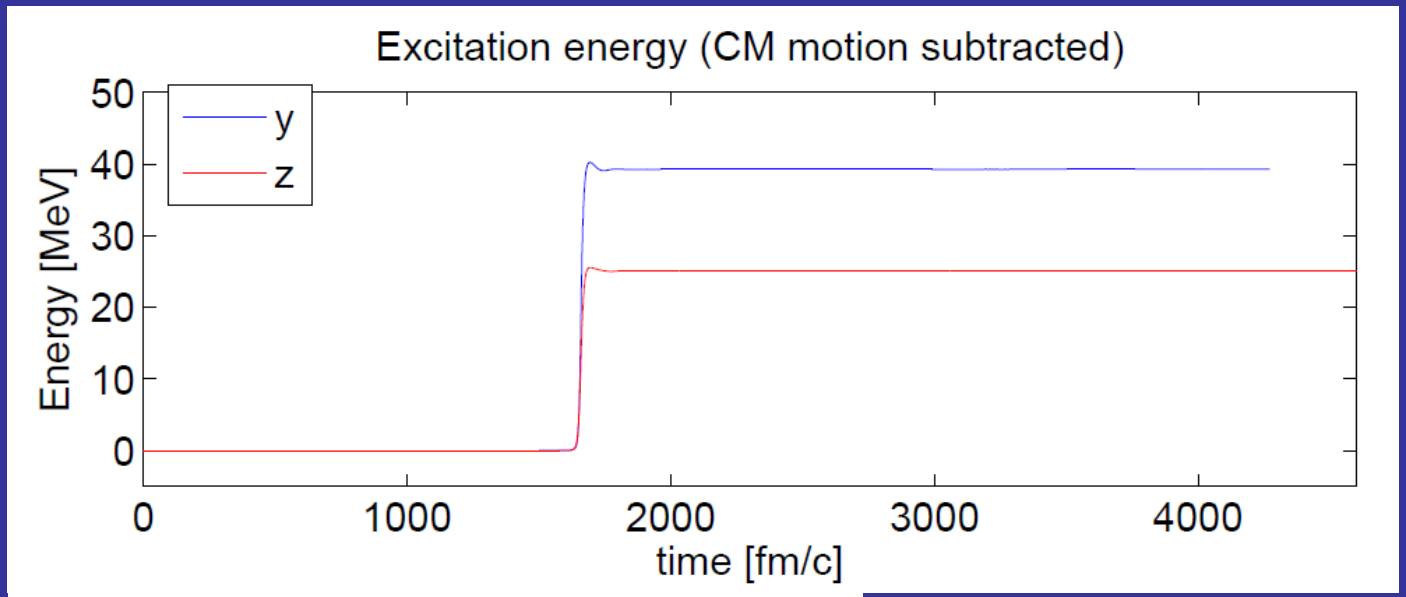
Coulomb excitation of GDR with relativistic heavy-ions computed in TDSLDA

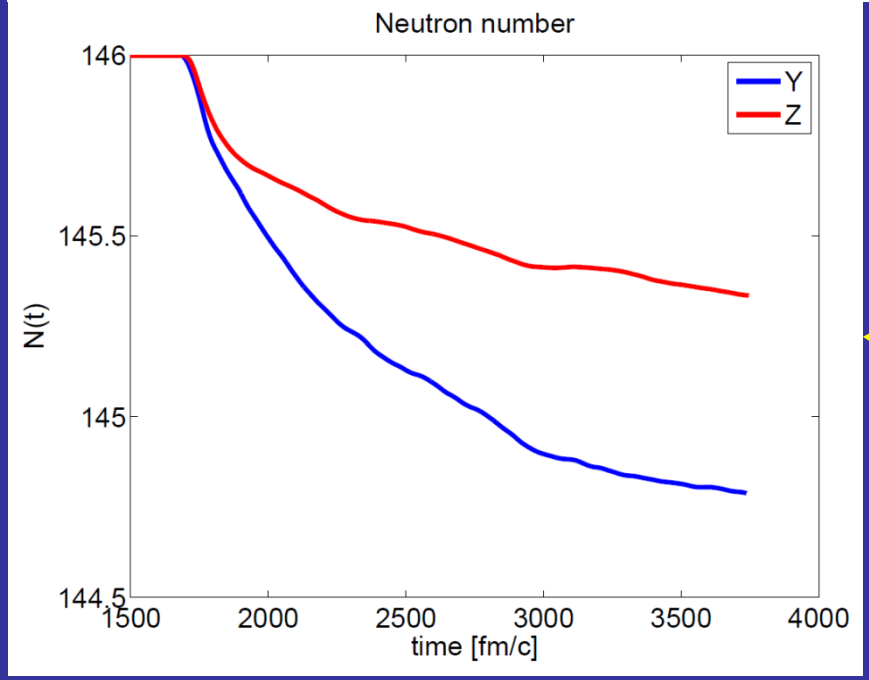
Movie

I. Stetcu *et al.*

Impact parameter b=12.2fm

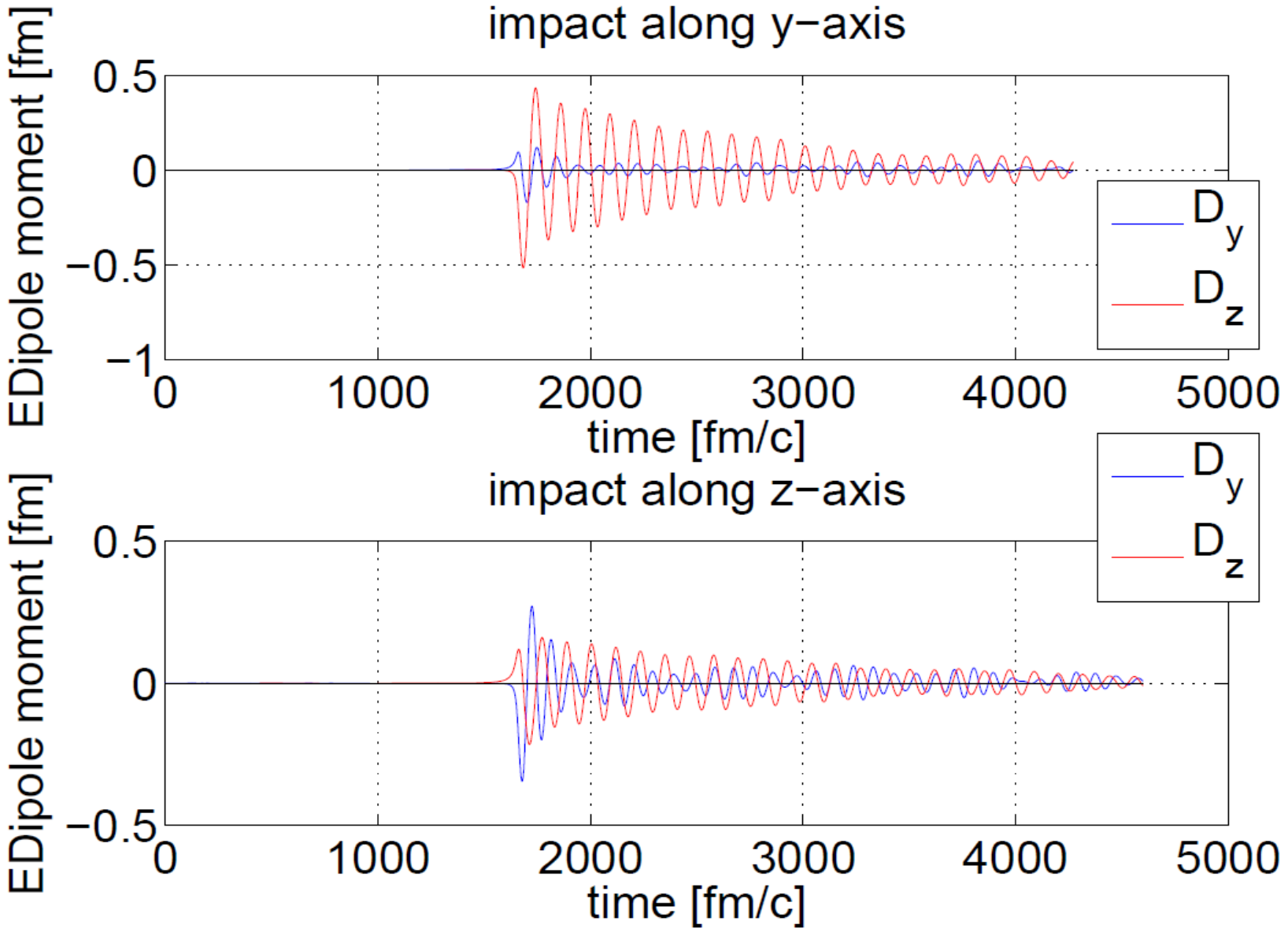
Energy deposited for two nuclear orientations (y – perpendicular, z – parallel)





Neutron emission

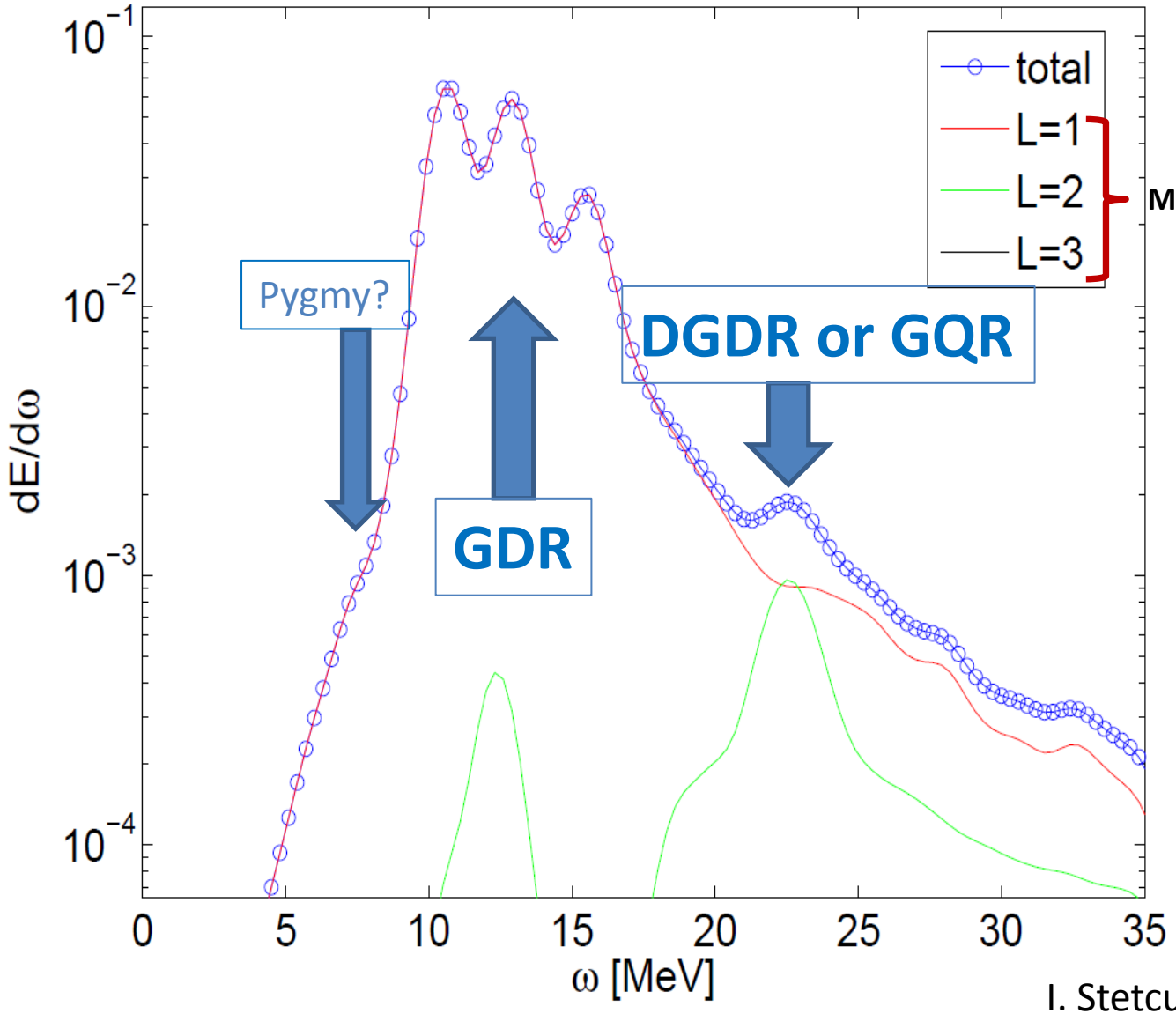
Electric dipole moment (along two axes: y, z) as a function of time



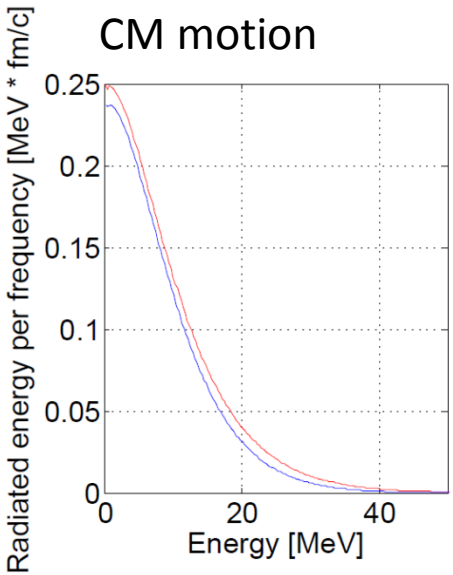
Oscillations are damped due to the one-body dissipation mechanism

Electromagnetic radiation due to the internal nuclear motion

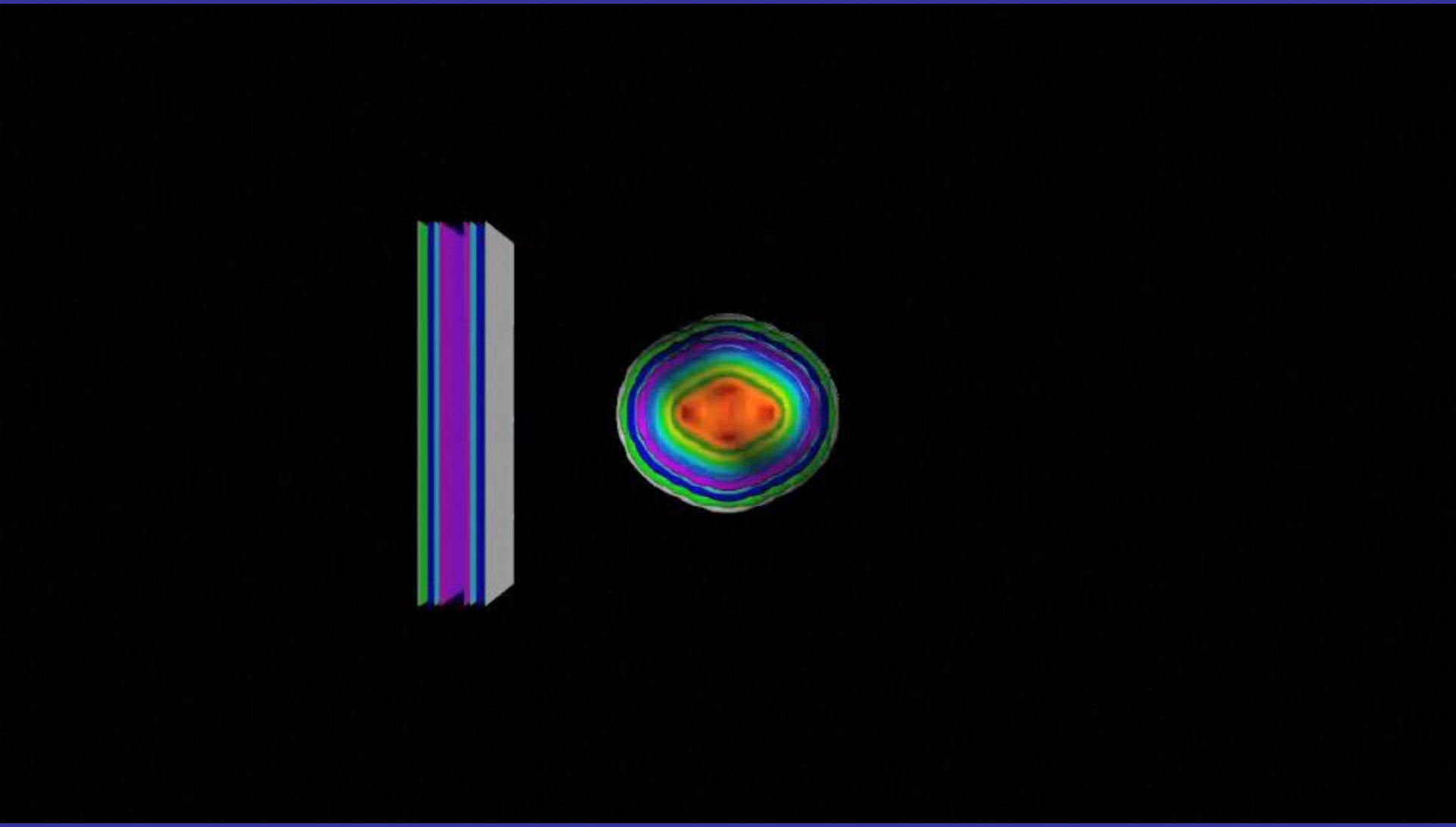
b (en, z)= 12.2 fm - impact parameter



Radiation due to CM motion



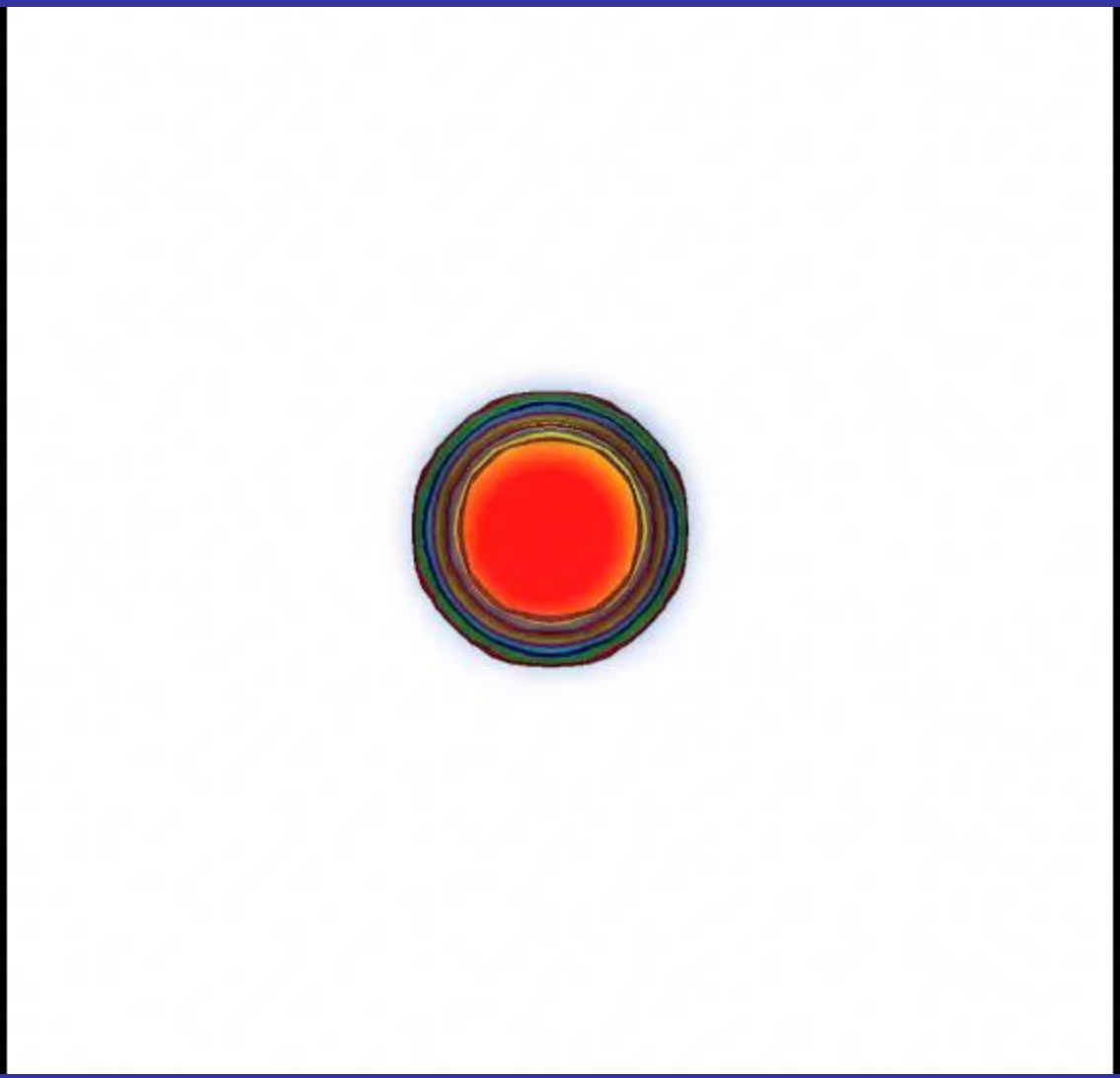
I. Stetcu et al.



Neutron scattering of ^{238}U computed in TDSLDA with absorbing boundary conditions

Movie

I. Stetcu *et al.*



Real-time induced fission of ^{280}Cf computed in TDSLDA

I. Stetcu *et al.*

Movie

Cold atomic gas in the unitary regime

$n\, r_0^3 \ll 1$ | $n\, |a|^3 \gg 1$

n - particle density
a - scattering length
r_0 - effective range

Universality: $E = \xi_0 E_{FG}$ for $T = 0$

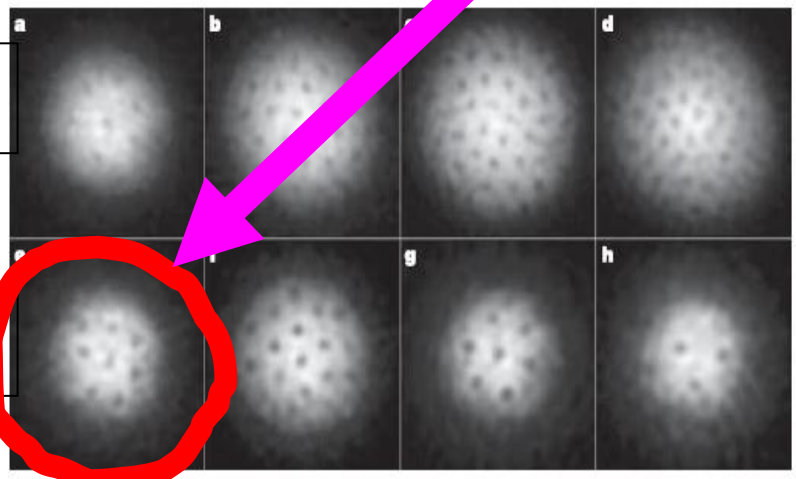
$\xi_0 = 0.376(5)$ - Bertsch parameter (Exp.estimate)

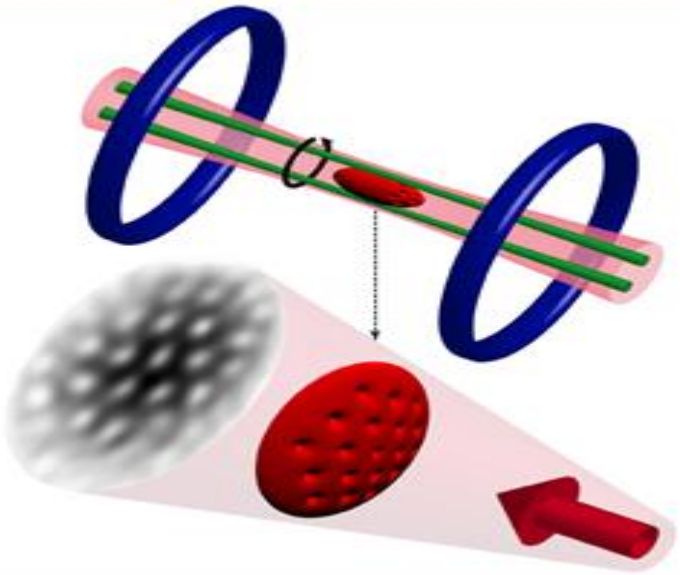
E_{FG} - Energy of noninteracting Fermi gas

UNITARY REGIME

BEC side: a>0

BCS side: a<0

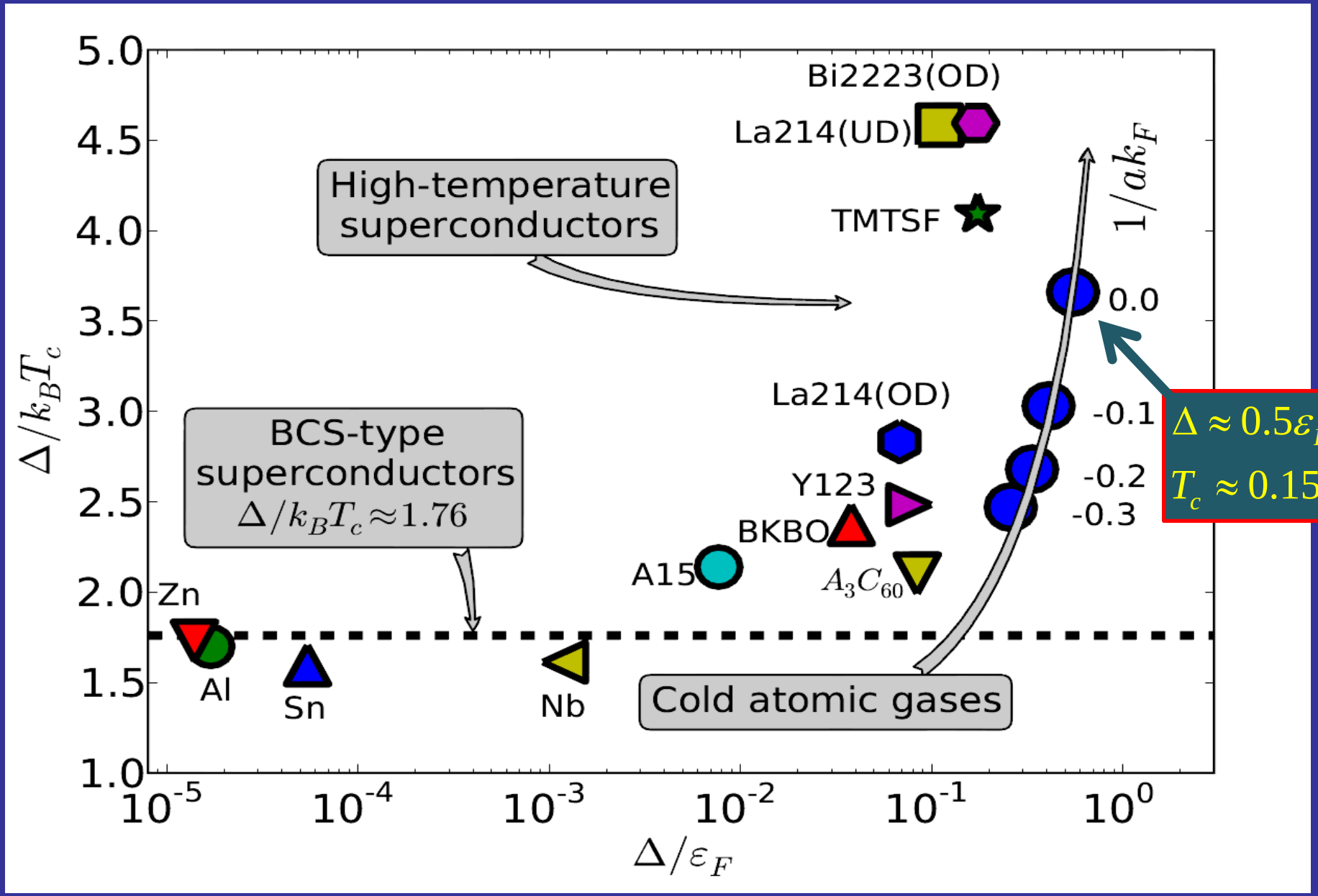




M.W. Zwierlein *et al.*, Nature, 435, 1047 (2005)

system of fermionic ^6Li atoms

Cold atomic gases and high Tc superconductors



From Fischer et al., Rev. Mod. Phys. 79, 353 (2007) &
P. Magierski, G. Wlazłowski, A. Bulgac, Phys. Rev. Lett. 107, 145304 (2011)

SLDA for unitary Fermi gas

SLDA – Superfluid Local Density Approximation

SLDA energy density functional at unitarity

$$\varepsilon(\vec{r}) = \left[\alpha\frac{\tau_c(\vec{r})}{2} - \Delta(\vec{r})\nu_c(\vec{r})\right] + \beta\frac{3(3\pi^2)^{2/3}n^{5/3}(\vec{r})}{5}$$

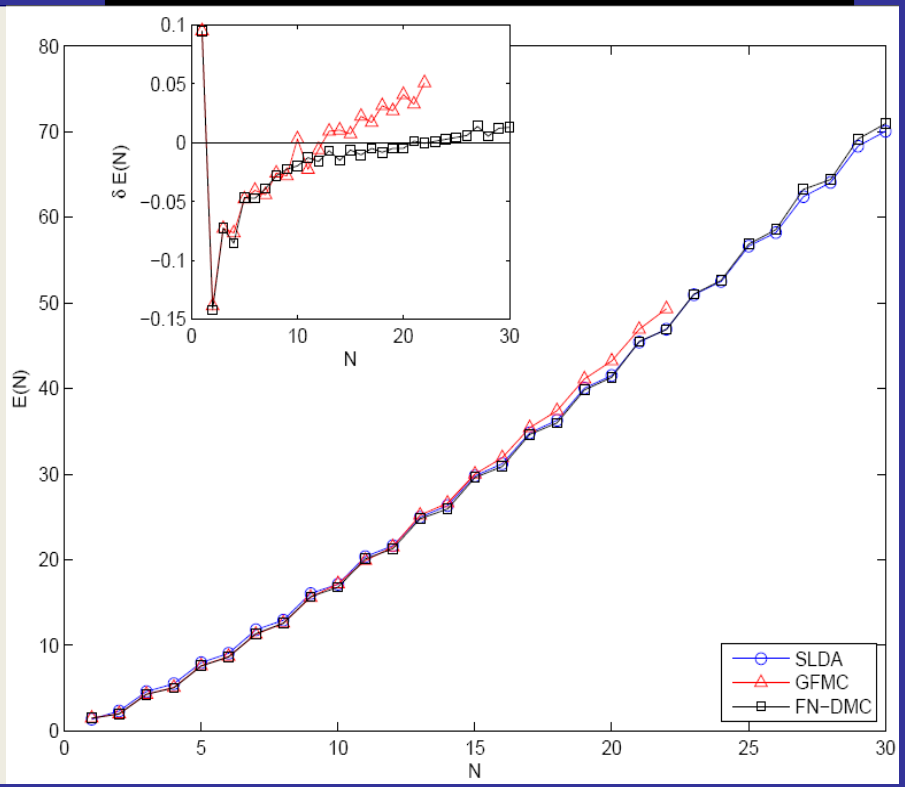
$$n(\vec{r}) = 2\sum_{0<E_k<E_c}\left|\mathrm{v}_k(\vec{r})\right|^2, \quad \tau_c(\vec{r}) = 2\sum_{0<E_k<E_c}\left|\vec{\nabla}\mathrm{v}_k(\vec{r})\right|^2,$$

$$\nu_c(\vec{r}) = \sum_{0<E<E_c}\mathrm{u}_k(\vec{r})\mathrm{v}_k^*(\vec{r})$$

$$U(\vec{r}) = \beta\frac{(3\pi^2)^{2/3}n^{2/3}(\vec{r})}{2} - \frac{\left|\Delta(\vec{r})\right|^2}{3\gamma n^{2/3}(\vec{r})} + V_{ext}(\vec{r})$$

$$\Delta(\vec{r}) = -g_{eff}(\vec{r})\nu_c(\vec{r})$$

Fermions at unitarity in a harmonic trap
Total energies E(N)



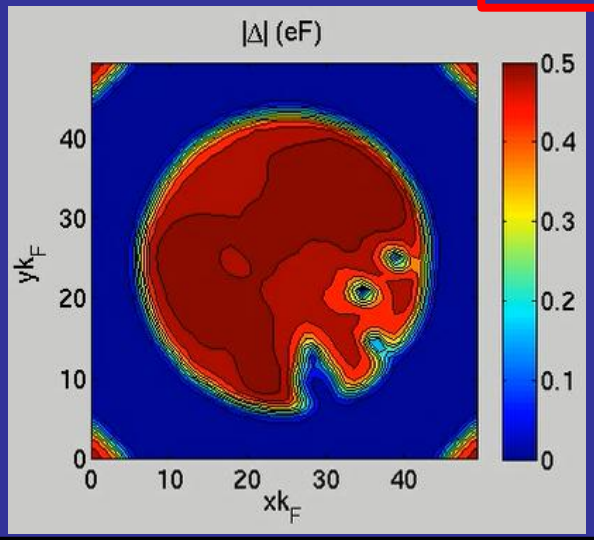
GFMC - Chang and Bertsch, Phys. Rev. A 76, 021603(R) (2007)
FN-DMC - von Stecher, Greene and Blume, PRL 99, 233201 (2007)
PRA 76, 053613 (2007)

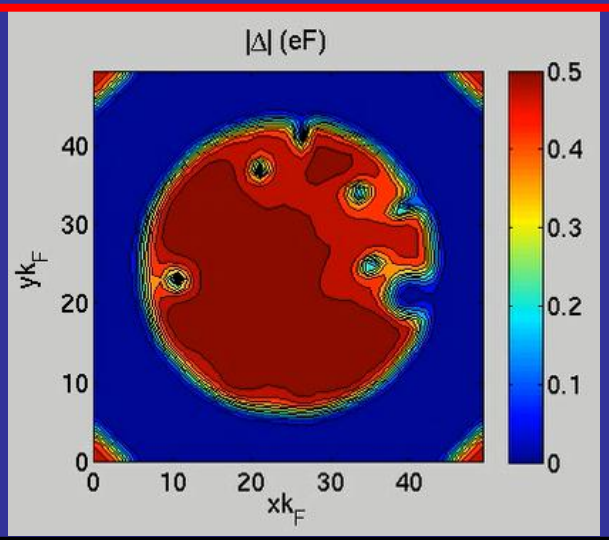
Bulgac, PRA 76, 040502(R) (2007)

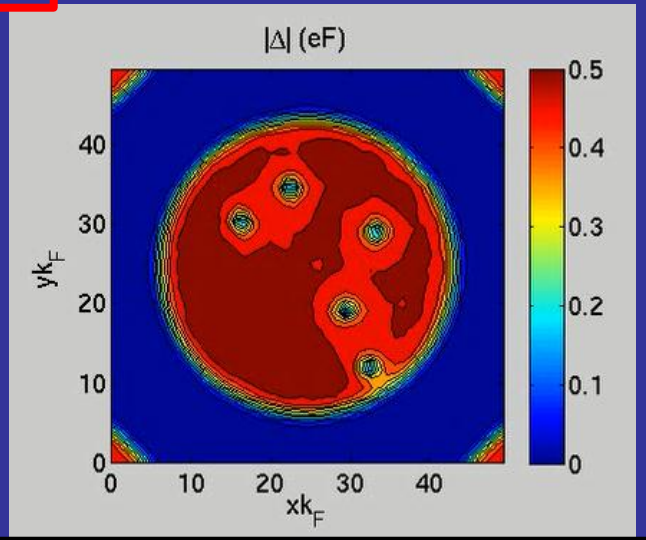
Normal State				Superfluid State			
(N_a, N_b)	E_{FNDMC}	E_{ASLDA}	(error)	(N_a, N_b)	E_{FNDMC}	E_{ASLDA}	(error)
(3,1)	6.6 ± 0.01	6.687	1.3%	(1,1)	2.002 ± 0	2.302	15%
(4,1)	8.93 ± 0.01	8.962	0.36%	(2,2)	5.051 ± 0.009	5.405	7%
(5,1)	12.1 ± 0.1	12.22	0.97%	(3,3)	8.639 ± 0.03	8.939	3.5%
(5,2)	13.3 ± 0.1	13.54	1.8%	(4,4)	12.573 ± 0.03	12.63	0.48%
(6,1)	15.8 ± 0.1	15.65	0.93%	(5,5)	16.806 ± 0.04	16.19	3.7%
(7,2)	19.9 ± 0.1	20.11	1.1%	(6,6)	21.278 ± 0.05	21.13	0.69%
(7,3)	20.8 ± 0.1	21.23	2.1%	(7,7)	25.923 ± 0.05	25.31	2.4%
(7,4)	21.9 ± 0.1	22.42	2.4%	(8,8)	30.876 ± 0.06	30.49	1.2%
(8,1)	22.5 ± 0.1	22.53	0.14%	(9,9)	35.971 ± 0.07	34.87	3.1%
(9,1)	25.9 ± 0.1	25.97	0.27%	(10,10)	41.302 ± 0.08	40.54	1.8%
(9,2)	26.6 ± 0.1	26.73	0.5%	(11,11)	46.889 ± 0.09	45	4%
(9,3)	27.2 ± 0.1	27.55	1.3%	(12,12)	52.624 ± 0.2	51.23	2.7%
(9,5)	30 ± 0.1	30.77	2.6%	(13,13)	58.545 ± 0.18	56.25	3.9%
(10,1)	29.4 ± 0.1	29.41	0.034%	(14,14)	64.388 ± 0.31	62.52	2.9%
(10,2)	29.9 ± 0.1	30.05	0.52%	(15,15)	70.927 ± 0.3	68.72	3.1%
(10,6)	35 ± 0.1	35.93	2.7%	(1,0)	1.5 ± 0.0	1.5	0%
(20,1)	73.78 ± 0.01	73.83	0.061%	(2,1)	4.281 ± 0.004	4.417	3.2%
(20,4)	73.79 ± 0.01	74.01	0.3%	(3,2)	7.61 ± 0.01	7.602	0.1%
(20,10)	81.7 ± 0.1	82.57	1.1%	(4,3)	11.362 ± 0.02	11.31	0.49%
(20,20)	109.7 ± 0.1	113.8	3.7%	(7,6)	24.787 ± 0.09	24.04	3%
(35,4)	154 ± 0.1	154.1	0.078%	(11,10)	45.474 ± 0.15	43.98	3.3%
(35,10)	158.2 ± 0.1	158.6	0.27%	(15,14)	69.126 ± 0.31	62.55	9.5%
(35,20)	178.6 ± 0.1	180.4	1%				

A. Bulgac, M.M. Forbes, P. Magierski, *in* BCS-BEC crossover and the Unitary Fermi gas „Lecture Notes in Physics" v.836, p. 305, ed. W. Zwerger (2012)

Excitation of vortices through stirring

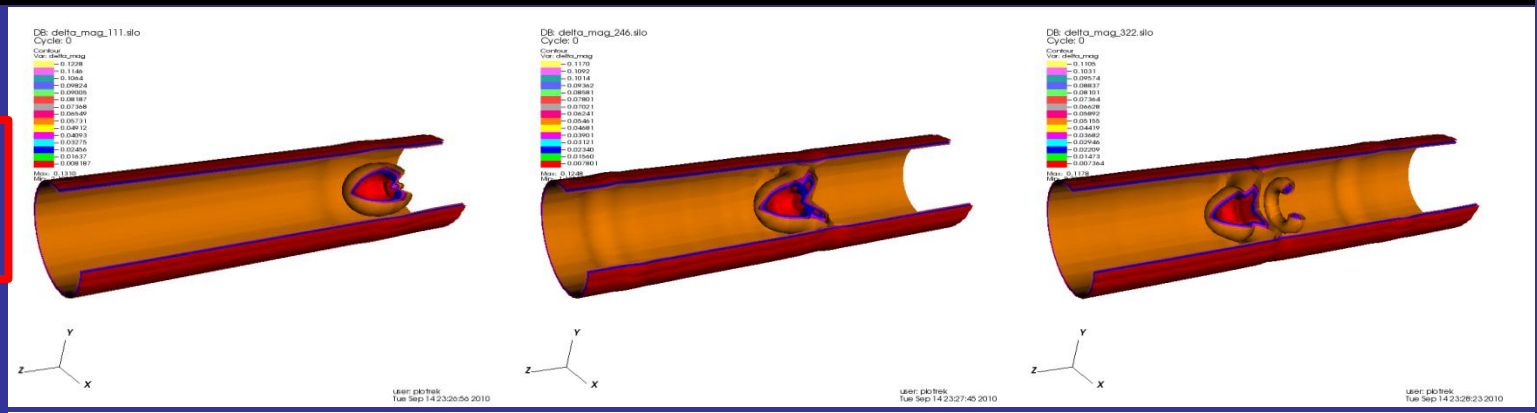


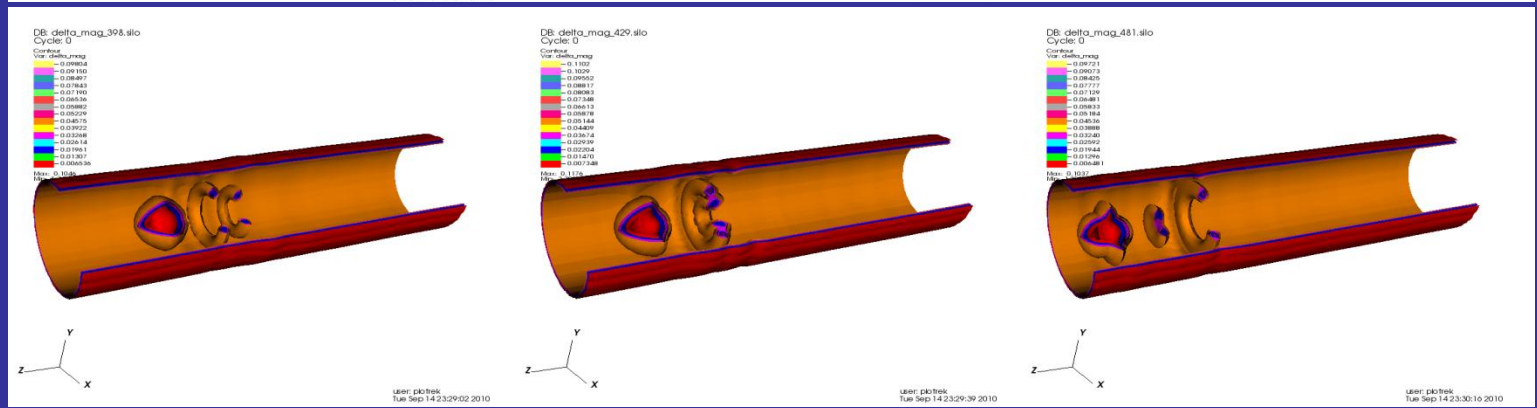




dynamics of vortex rings

Heavy spherical object moving through the superfluid unitary Fermi gas





Road to quantum turbulence

Classical turbulence: energy is transfered from large scales to small scales where it eventually dissipates.

Kolmogorov spectrum: $E(k)=C\,\varepsilon^{2/3}\,k^{-5/3}$

E – kinetic energy per unit mass associated with the scale 1/k
ε - energy rate (per unit mass) transfered to the system at large scales.
k - wave number (from Fourier transformation of the velocity field).
C – dimensionless constant.

Superfluid turbulence (quantum turbulence): disordered set of quantized vortices. The friction between the superfluid and normal part of the fluid serves as a source of energy dissipation.

Problem: how the energy is dissipated in the superfluid system at small scales at T=0? - „pure" quantum turbulence

Possibility: vortex reconnections → Kelvin waves → phonon radiation

Vortex reconnections

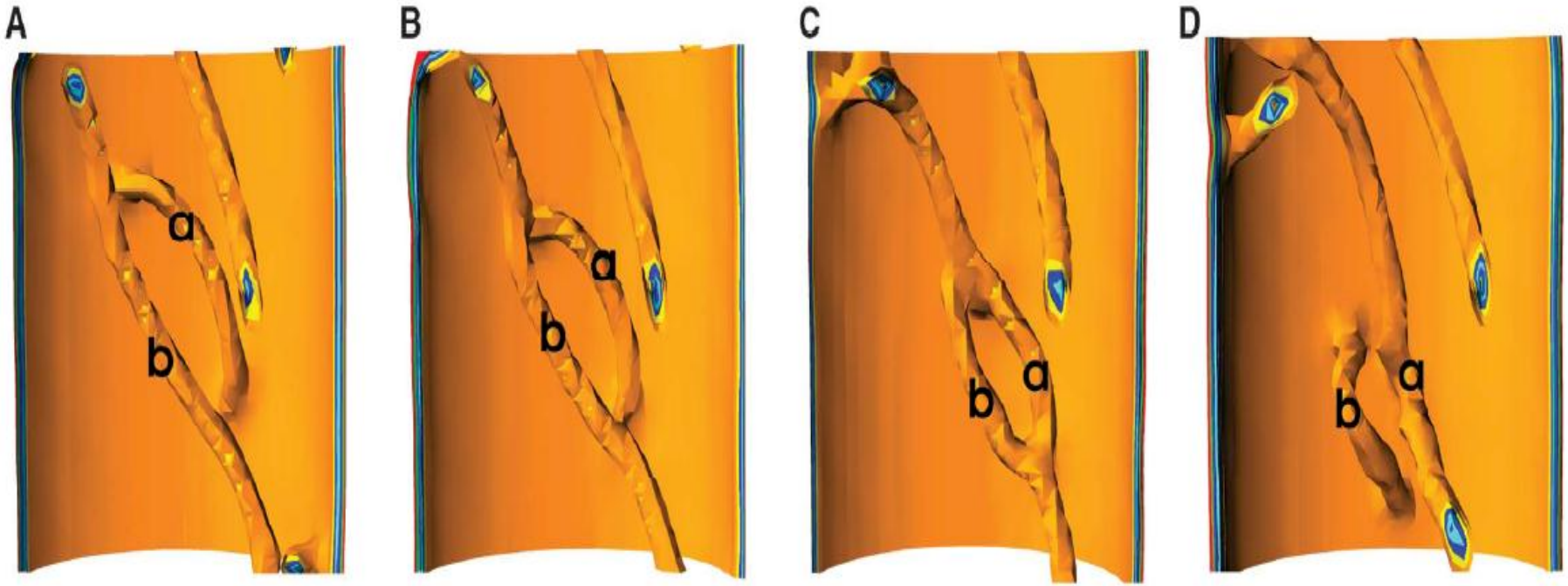


Fig. 3. (**A** to **D**) Two vortex lines approach each other, connect at two points, form a ring and exchange between them a portion of the vortex line, and subsequently separate. Segment (a), which initially belonged to the vortex line attached to the wall, is transferred to the long vortex line (b) after reconnection and vice versa.

Bulgac, Luo, Magierski, Roche, Yu, Science 332, 1288 (2011)

More movies here: www.phys.washington.edu/groups/qmbnt/UFG/

A new method to construct the ground state which eschews big matrix diagonalization:

adiabatic switching with quantum friction

$$i\hbar\dot{\Psi}(x,t) = \left[H(x,t) + U(x,t)\right]\Psi(x,t)$$

$$E = \langle\Psi|H|\Psi\rangle$$

$$\dot{E} = \langle\Psi|\dot{H}|\Psi\rangle + \frac{2}{\hbar}\mathrm{Im}\langle\Psi|HU|\Psi\rangle$$

$$\text{if } U \propto -\hbar\vec{\nabla}\cdot\vec{j} = \hbar\dot{\rho} \;\Rightarrow\; \dot{E} \le \langle\Psi|\dot{H}|\Psi\rangle$$

$$\text{We choose } U = -\beta\frac{\hbar\vec{\nabla}\cdot\vec{j}}{\rho}$$

$$\vec{j}(\vec{r}) = \frac{\hbar}{m}\mathrm{Im}\sum_n \psi_n^*(\vec{r},t)\vec{\nabla}\psi_n(\vec{r},t)$$

Main advantage:
Replace iterative procedure which requires $O(N^3)$ operations for diagonalization with time evolution which requires only $O(N^2 \ln(N))$ operations per time step.

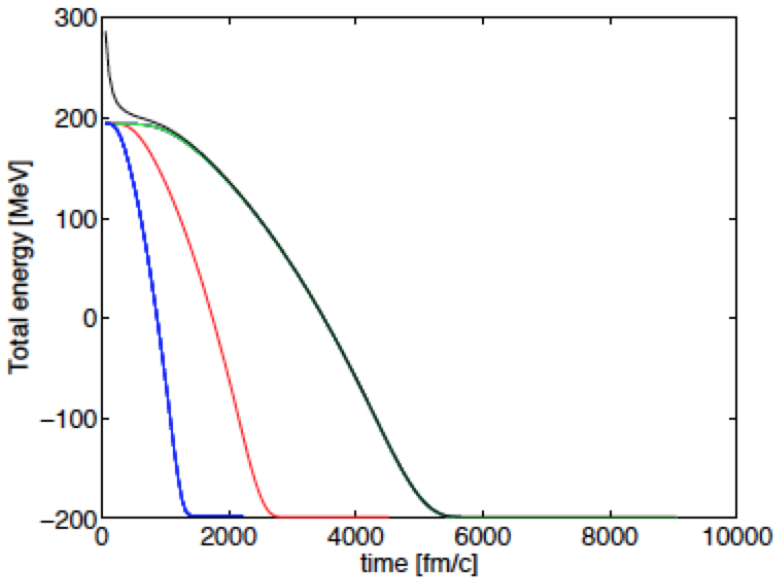


FIG. 2. (Color online) The total instantaneous energy of a system of twenty non-interacting neutrons evolving from an initial 3D harmonic oscillator potential to a final symmetrized Woods-Saxon potential. The curves correspond to quasi-adiabatic evolution with friction $(1-s_t)H_0 + s_t H_1 + U_t$ for various switching periods T (two-thirds of the simulation time) and just friction $H_1 + U_t$ for the remaining third of the simulation. That the energy is constant during this time demonstrates that the ground state has been reached. Note: there are three curves for the longest T corresponding to different simulations with $\{24^3, 32^3, 40^3\}$ lattices of 1 fm spacing: this demonstrates the infrared (IR) convergence.

TDSLDA applications:

1) Nuclear physics:

- **Electromagnetic response**
- **Pairing vibrations**
- **Heavy ion collisions**
- **Induced fission**
- **Neutron scattering/capture**

2) Neutron stars:

- **Dynamics of vortices**
- **Vortex pinning mechanism in the neutron star crust (glitches)**

3) Various applications in cold atom physics.

Papers we published so far on SLDA and TDSLDA
(stars indicate papers with significant nuclear physics content):

arXiv:1306.4266
* arXiv:1305.6891
* Phys. Rev. Lett. 110, 241102 (2013)
* Phys. Rev. C 87 051301(R) (2013)
* Ann. Rev. Nucl. Part. Phys. 63, 97 (2013)
* Phys. Rev. C 84, 051309(R) (2011)
Phys. Rev. Lett. 108, 150401 (2012)
Science, 332, 1288 (2011)
J. Phys. G: Nucl. Phys. 37, 064006 (2010)
Phys. Rev. Lett. 102, 085302 (2009)
Phys. Rev. Lett. 101, 215301 (2008)
* J.Phys. Conf. Ser. 125, 012064 (2008)
arXiv:1008.3933 chapter 9 in Lect. Notes Phys. vol. 836
Phys. Rev. A 76, 040502(R) (2007)
* Int. J. Mod. Phys. E 13, 147 (2004)
Phys. Rev. Lett. 91, 190404 (2003)
* Phys. Rev. Lett. 90, 222501 (2003)
* Phys. Rev. Lett. 90, 161101 (2003)
* Phys. Rev. C 65,051305(R) (2002)
* Phys. Rev. Lett. 88, 042504 (2002)

Plus a few other chapters in various books.