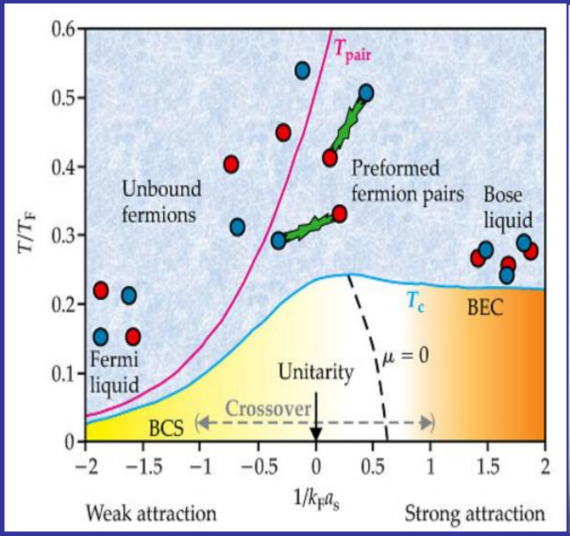
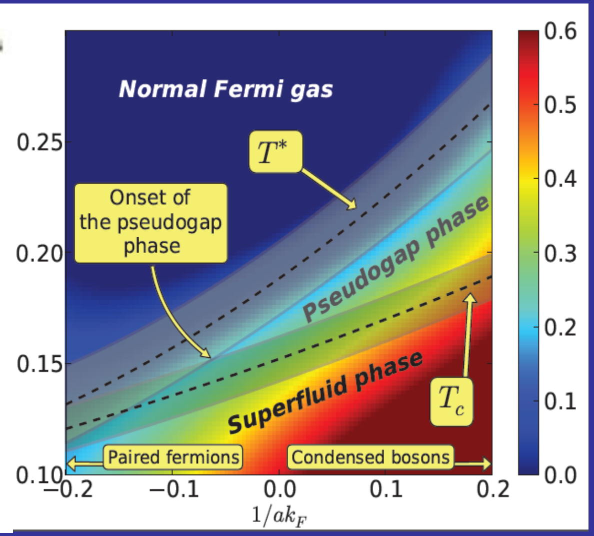
Transport properties of unitary Fermi gas from Quantum Monte Carlo





Piotr Magierski
Warsaw University of Technology

Unitary gas:

$n\, r_0^3 \ll 1$ $\quad$ $n\, |a|^3 \gg 1$

n - particle density
a - scattering length
r_0 - effective range

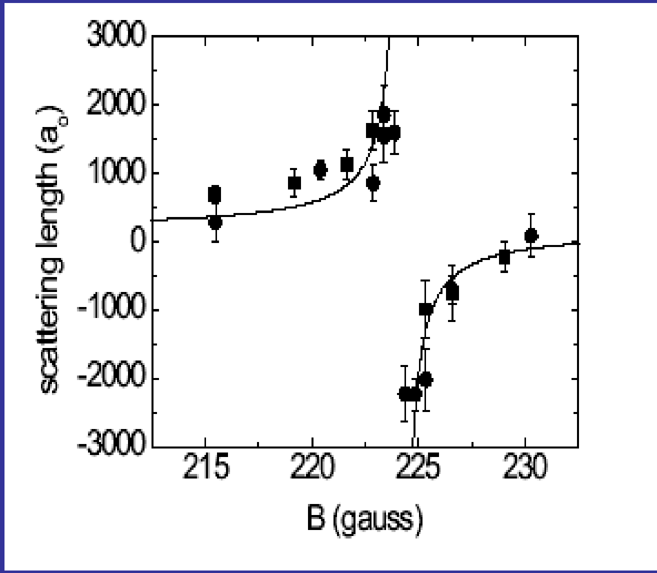
$$\text{i.e. } r_0 \to 0, a \to \pm\infty$$

NONPERTURBATIVE REGIME

Universality: $E(x) = \xi(x) E_{FG}$; $x = T/\varepsilon_F$

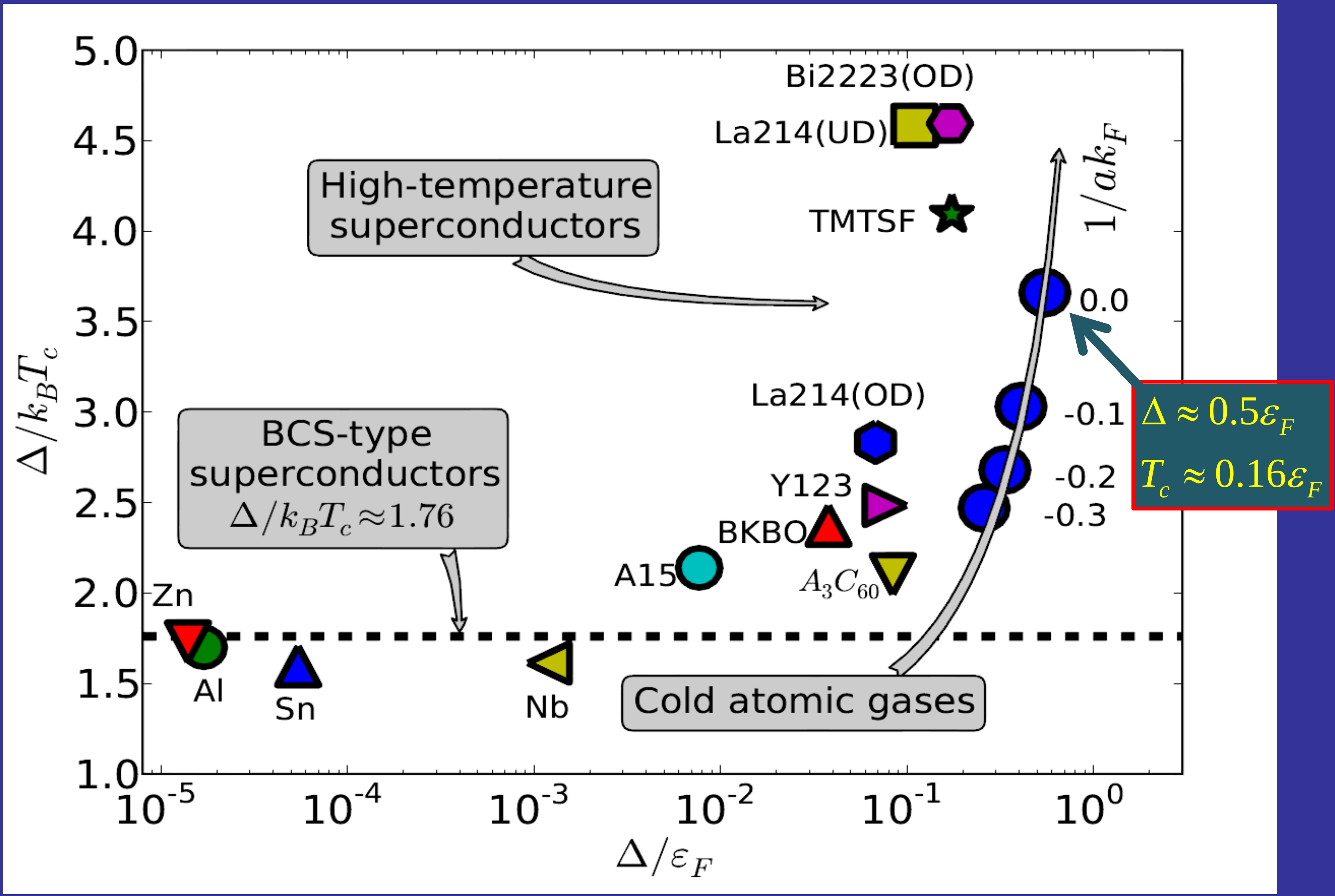
$\xi(0) = 0.37(1)$ - **Exp. estimate**

E_{FG} - **Energy of noninteracting Fermi gas**



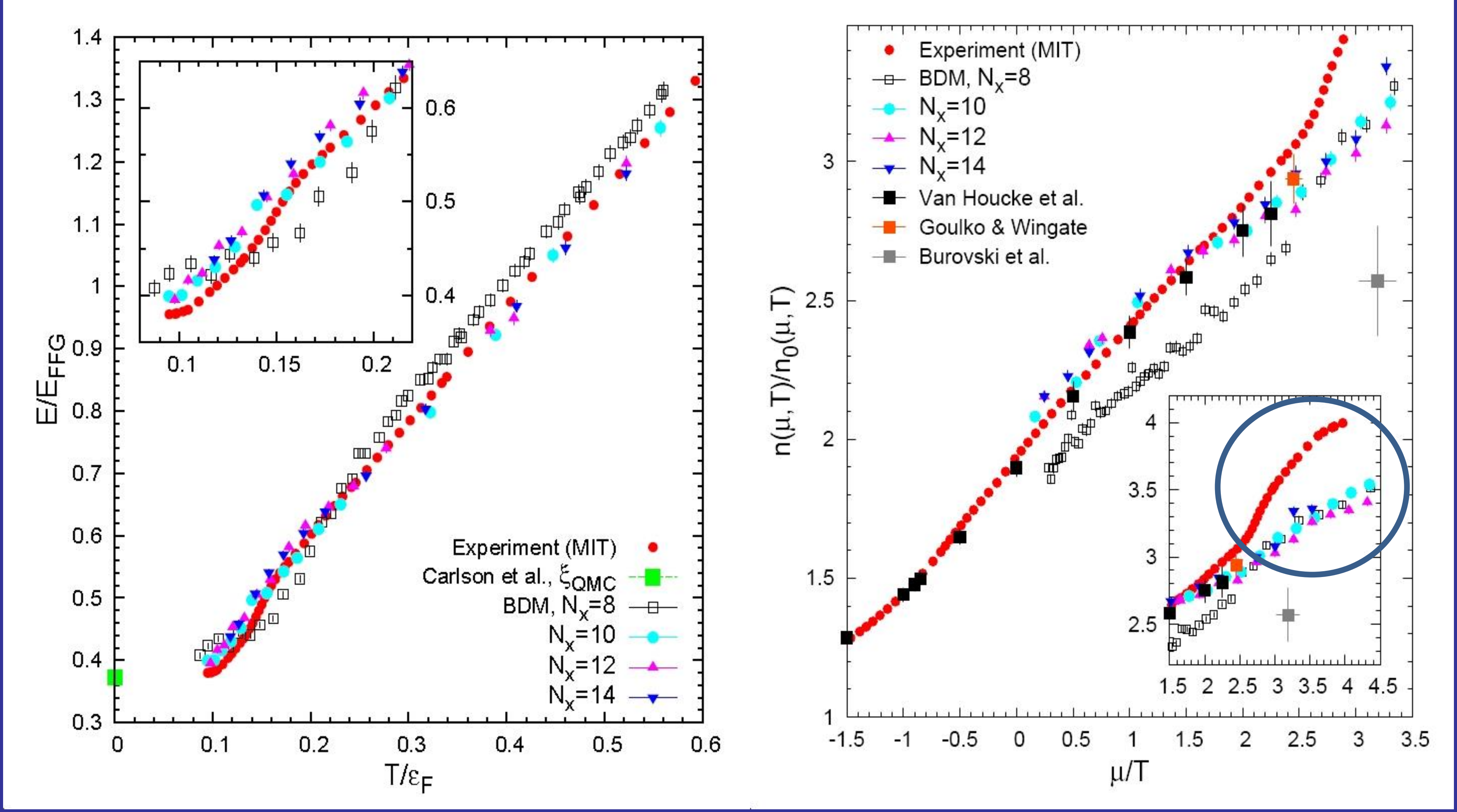
Regal and Jin, PRL 90, 230404 (2003)

Cold atomic gases and high Tc superconductors



From Fischer et al., Rev. Mod. Phys. 79, 353 (2007) & P. Magierski, G. Wlazłowski, A. Bulgac, Phys. Rev. Lett. 107, 145304 (2011)

Equation of state of the unitary Fermi gas - current status



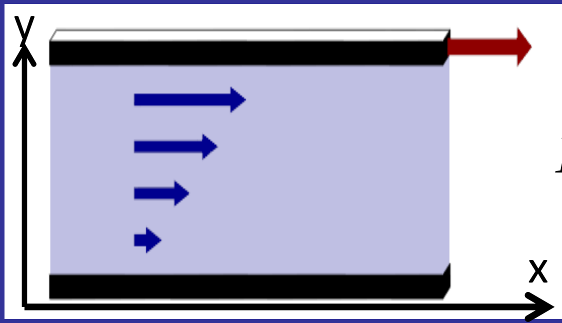
Experiment: M.J.H. Ku, A.T. Sommer, L.W. Cheuk, M.W. Zwierlein , Science 335, 563 (2012)

QMC (PIMC + Hybrid Monte Carlo): J.E.Drut, T.Lähde, G.Wlazłowski, P.Magierski, Phys. Rev. A 85, 051601 (2012)

Hydrodynamics at unitarity

No intrinsic length scale ⇒ **Uniform expansion keeps the unitary gas in equilibrium**

Consequence:
uniform expansion does not produce entropy = bulk viscosity is zero!



$$F = A\eta \frac{\partial v_x}{\partial y}$$

Shear viscosity:

For any physical fluid:

$$\frac{\eta}{S} \geq \frac{\hbar}{4\pi k_B}$$

KSS conjecture
Kovtun, Son, Starinets, Phys.Rev.Lett. 94, 111601, (2005)
from AdS/CFT correspondence

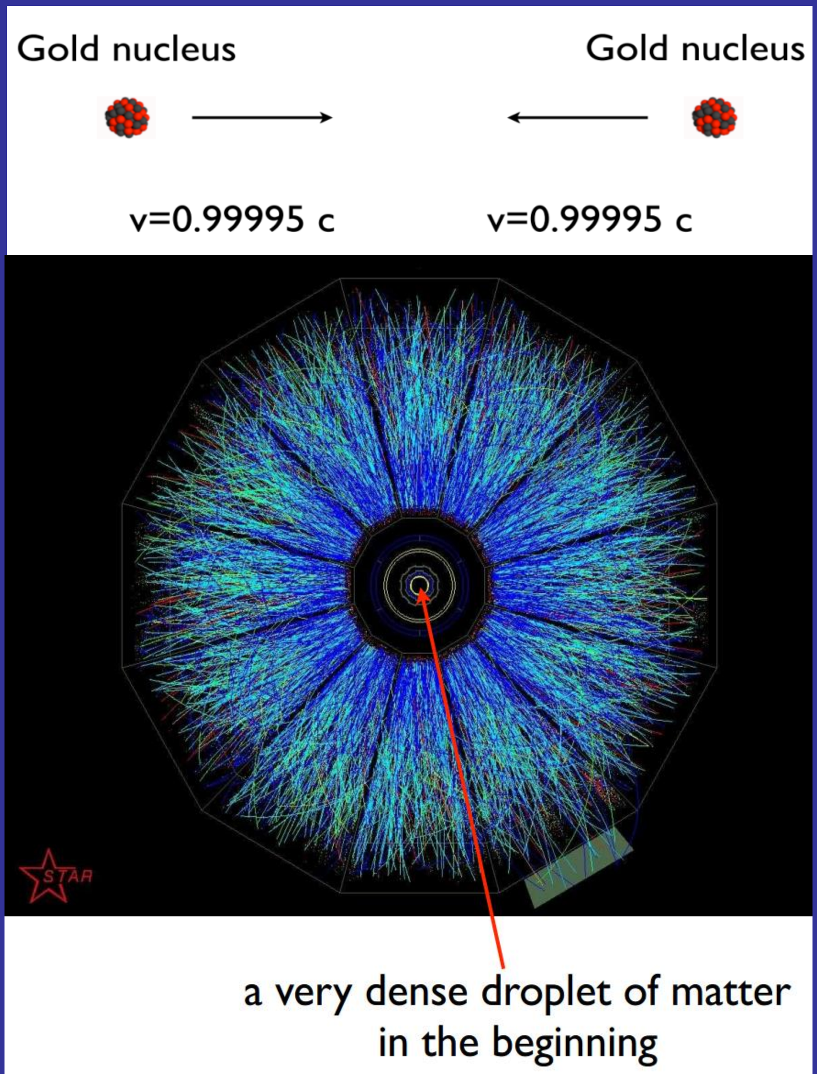
Maxwell classical estimate: $\eta \sim \text{mean free path}$

Perfect fluid $\frac{\eta}{S} = \frac{\hbar}{4\pi k_B}$ - strongly interacting quantum system = No well defined quasiparticles

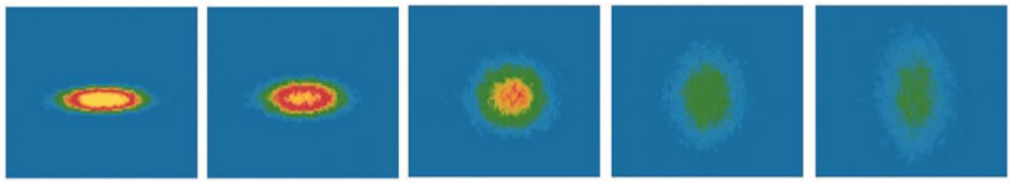
Candidates: unitary Fermi gas, quark-gluon plasma

Perfect fluid $\frac{\eta}{S} = \frac{\hbar}{4\pi k_B}$ - strongly interacting quantum system = No well defined quasiparticles

Candidates: quark gluon plasma, atomic gas



Expansion of a atomic gas cloud



(Cao et al, Science 2010)

Extremely low temperatures: 1 billionth of a degree

Shear viscosity

$$\eta(\omega) = \pi\rho_{xyxy}(q=0,\omega)/\omega$$

$$G_{xyxy}(q,\tau) = \int d^3r \left\langle \hat{\Pi}_{xy}(r,\tau)\hat{\Pi}_{xy}(0,0) \right\rangle e^{iqr}$$

$$G_{xyxy}(q,\tau) = \int_0^\infty \rho_{xyxy}(q,\omega) \frac{\cosh\left[\omega(\tau-\beta/2)\right]}{\sinh\left[\omega\beta/2\right]} d\omega$$

$$i\left[\hat{j}_k(r),\hat{H}\right] = \partial_l \hat{\Pi}_{kl}(r)$$

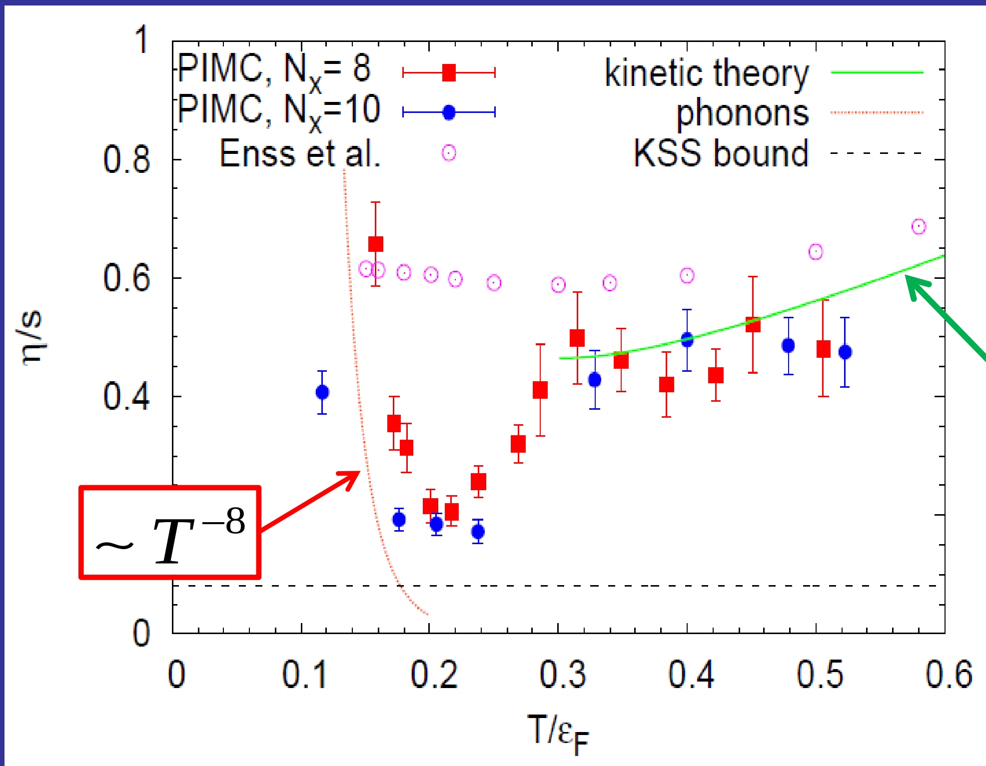
Additional symmetries and sum rules:

$$G(\tau) = G(\beta-\tau)$$

$$\frac{1}{\pi}\int_0^\infty d\omega \left[\eta(\omega) - \frac{C}{15\pi\sqrt{m\omega}}\right] = \frac{\varepsilon}{3},$$ *ε – energy density*

$$\eta(\omega\to\infty) \simeq \frac{C}{15\pi\sqrt{m\omega}}.$$

Shear viscosity to entropy density ratio

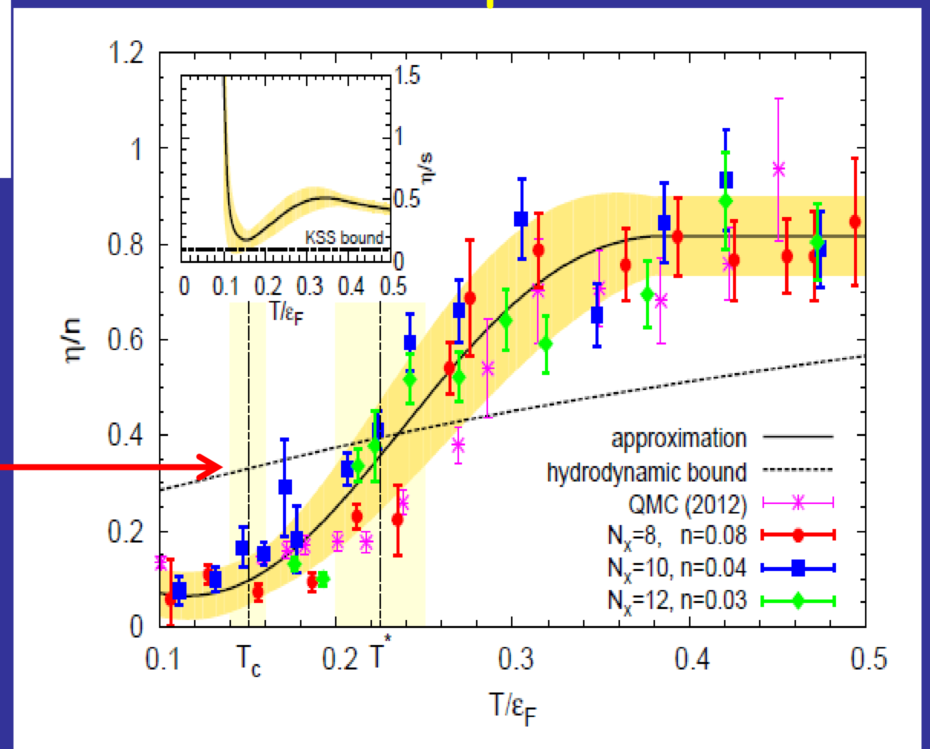


$\sim T^{3/2}$

G.Wlazłowski, P.Magierski,J.E.Drut,
Phys. Rev. Lett. 109, 020406 (2012)

Shear viscosity per unit density as a function of temperature

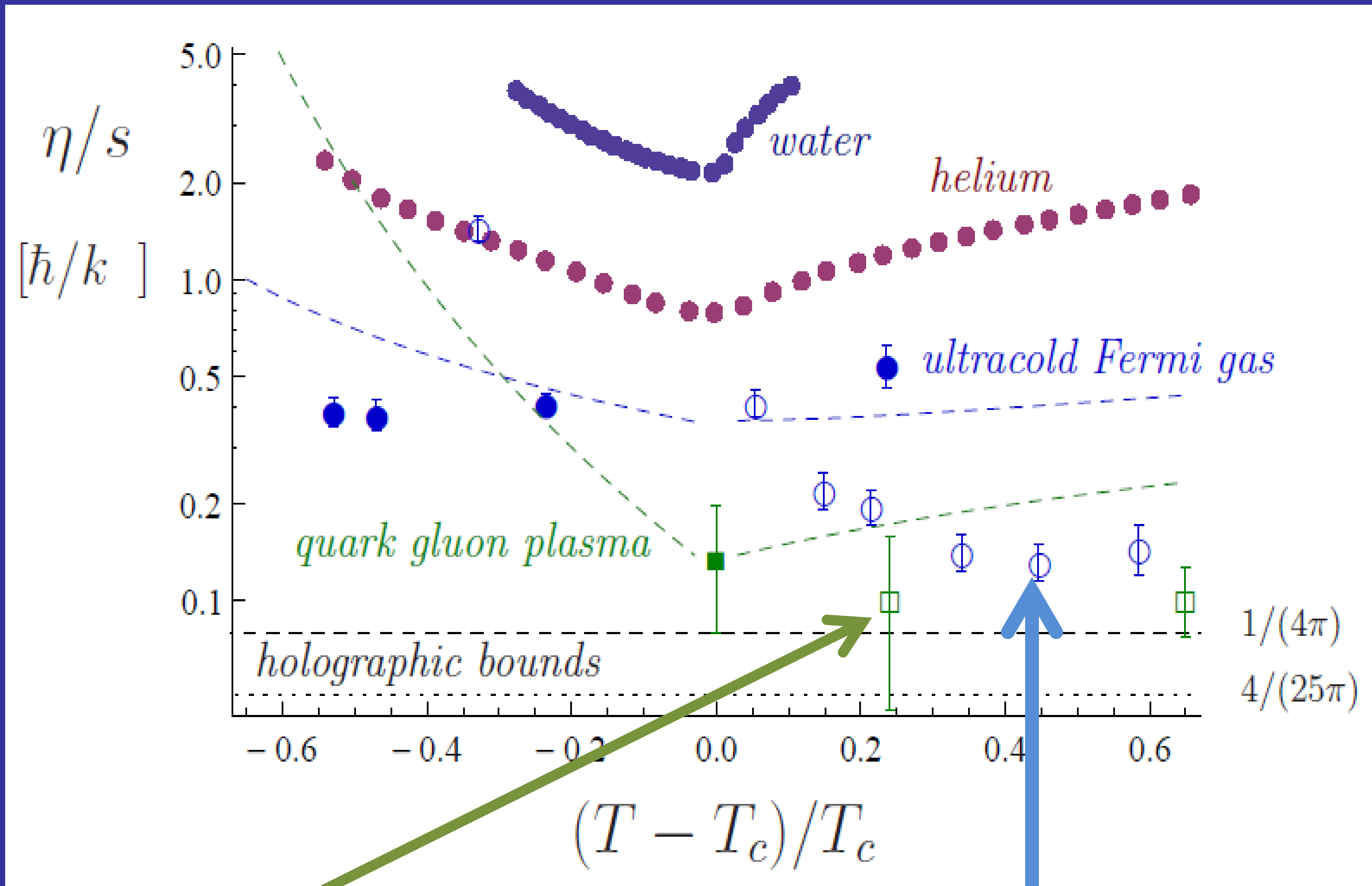
C. Chafin, T. Schafer,
PRA87,023629(2013)
P.Romatschke, R.E. Young,
PRA87,053606(2013)



Wlazłowski, Magierski, Bulgac, Roche,
Phys. Rev. A88, 013639 (2012)

Shear viscosity to entropy ratio – experiment vs. theory

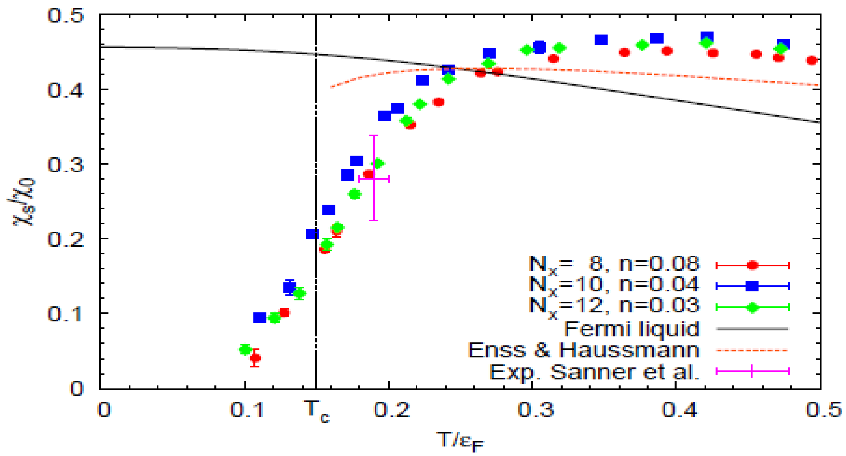
(from A. Adams et al. New Journal of Physics, "Focus on Strongly Correlated Quantum Fluids: from Ultracold Quantum Gases to QCD Plasmas,, arXive:***1205.5180)***



Lattice QCD (SU(3) gluodynamics):
H.B. Meyer, Phys. Rev. D 76, 101701 (2007)

QMC calculations for UFG:
G. Wlazłowski, P. Magierski, J.E. Drut,
Phys. Rev. Lett. 109, 020406 (2012)

Spin susceptibility and spin drag rate



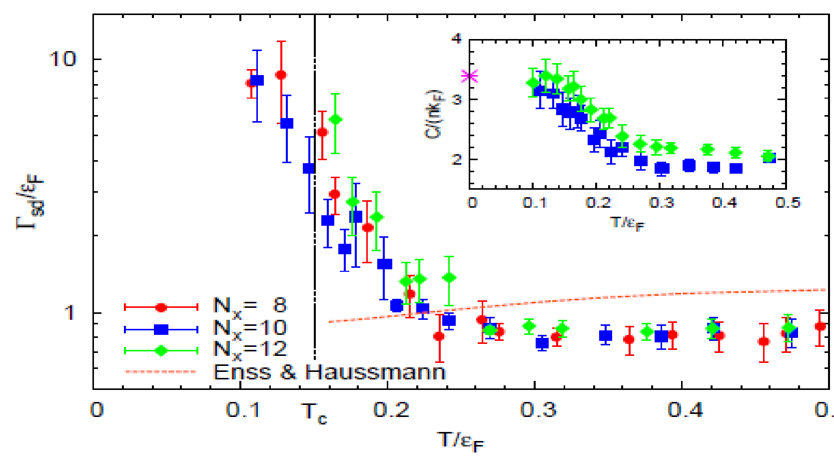


FIG. 2: (Color online) The static spin susceptibility as a function of temperature for an 8^3 lattice solid (red) circles, 10^3 lattice (blue) squares and 12^3 lattice (green) diamonds. Vertical black dotted line indicates the critical temperature of superfluid to normal phase transition $T_c = 0.15\,\varepsilon_F$. For comparison Fermi liquid theory prediction and recent results of the T-matrix theory produced by Enss and Haussmann [25] are plotted with solid and dashed (brown) lines, respectively. The experimental data point from Ref. [15] is also shown.

FIG. 3: (Color online) The spin drag rate $\Gamma_{sd} = n/\sigma_s$ in units of Fermi energy as a function of temperature for an 8^3 lattice solid (red) circles, 10^3 lattice (blue) squares and 12^3 lattice (green) diamonds. Vertical black dotted line locates the critical temperature of superfluid to normal phase transition. Results of the T-matrix theory are plotted by dashed (brown) line [25]. The inset shows extracted value of the contact density as function of the temperature. The (purple) asterisk shows the contact density from the QMC calculations of Ref. [29] at $T = 0$.

$$\Gamma = \frac{n}{\sigma_s}$$ - spin drag rate

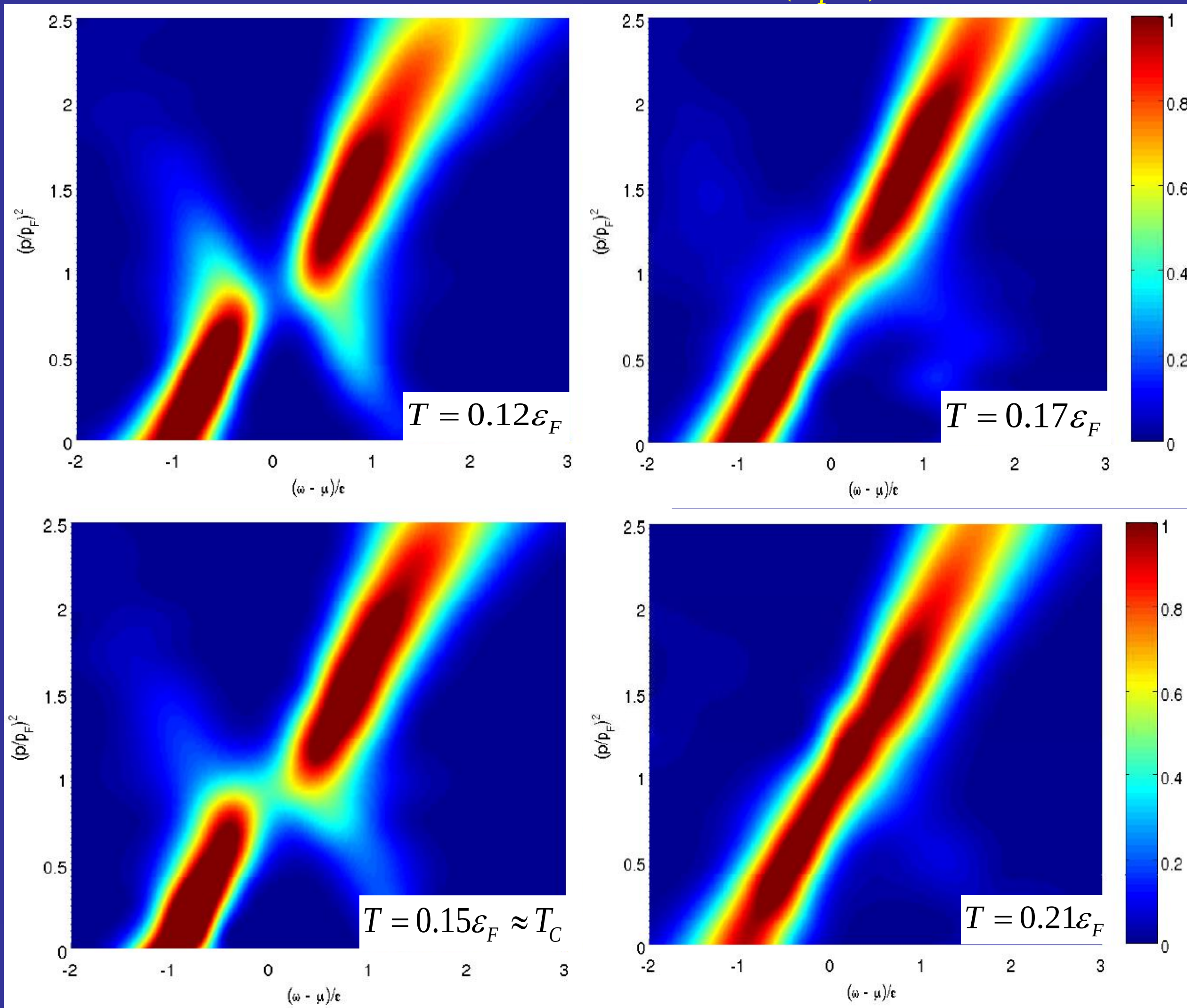
$$\sigma_s(\omega) = \pi\rho_s(q=0,\omega)/\omega$$ - spin conductivity

$$G_s(q,\tau) = \frac{1}{V}\left\langle\left(\hat{j}^z_{q\uparrow}(\tau) - \hat{j}^z_{q\downarrow}(\tau)\right)\left(\hat{j}^z_{-q\uparrow}(0) - \hat{j}^z_{-q\downarrow}(0)\right)\right\rangle$$

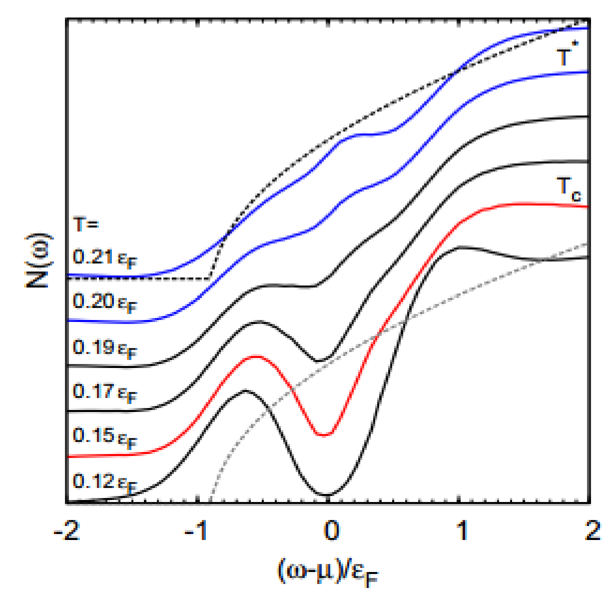
$$G_s(q,\tau) = \int_0^\infty \rho_s(q,\omega)\,\frac{\cosh[\omega(\tau-\beta/2)]}{\sinh[\omega\beta/2]}\,d\omega$$

Wlazłowski, Magierski, Bulgac, Drut, Roche,
Phys. Rev. Lett. 110, 090401,(2013)

Spectral weight function at unitarity: $(k_F a)^{-1} = 0$



Density of states profiles



Wlazłowski, Magierski, Drut, Bulgac
Phys. Rev. Lett. 110, 090401 (2013)

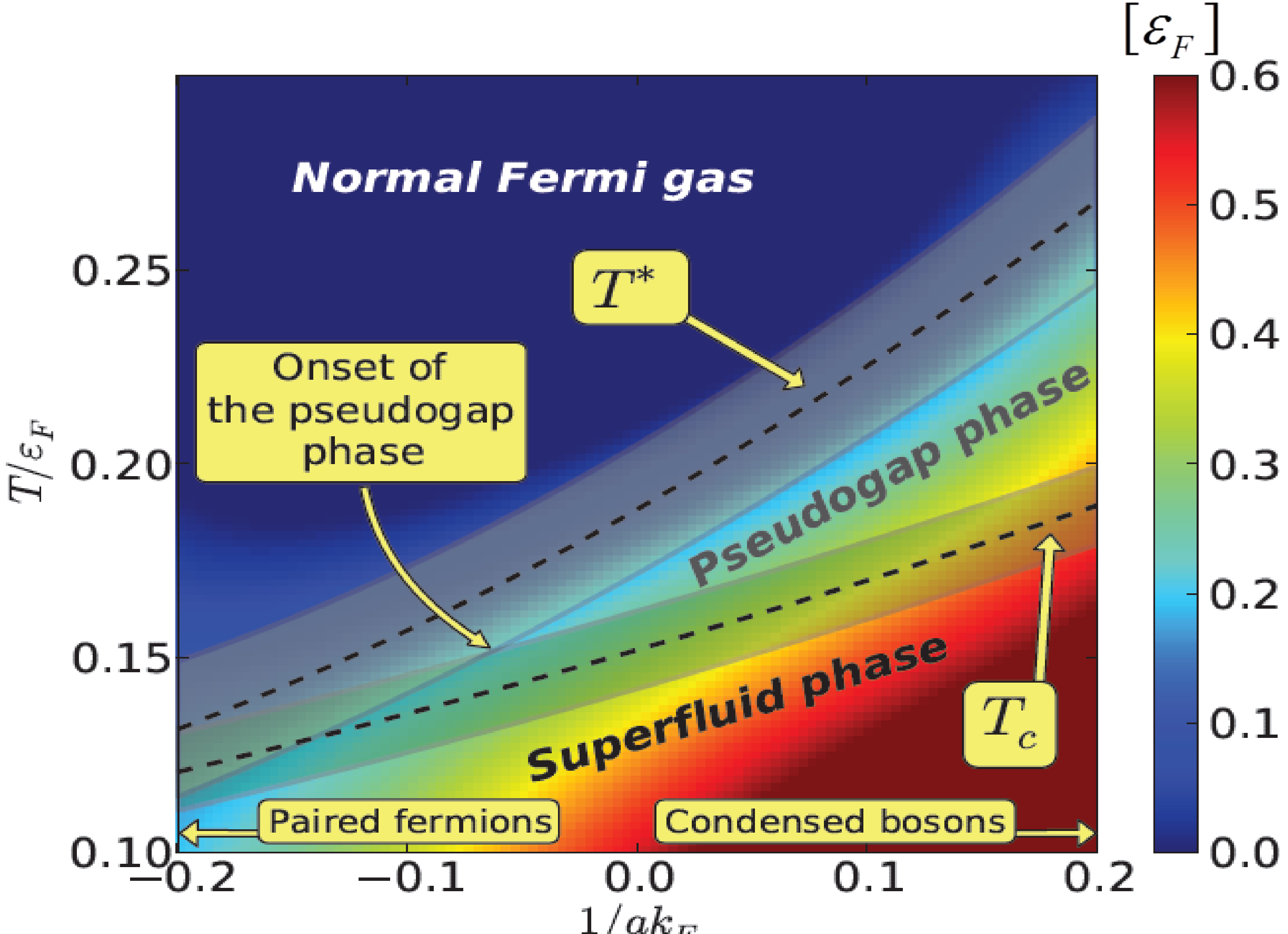
Spectral weight function: $A(\vec{p}, \omega)$

$$G^{ret/adv}(\vec{p},\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega' \frac{A(\vec{p},\omega')}{\omega - \omega' \pm i0^+}$$

$$G(\vec{p},\tau) = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, A(\vec{p},\omega)\frac{e^{-\omega\tau}}{1+e^{-\beta\omega}}$$

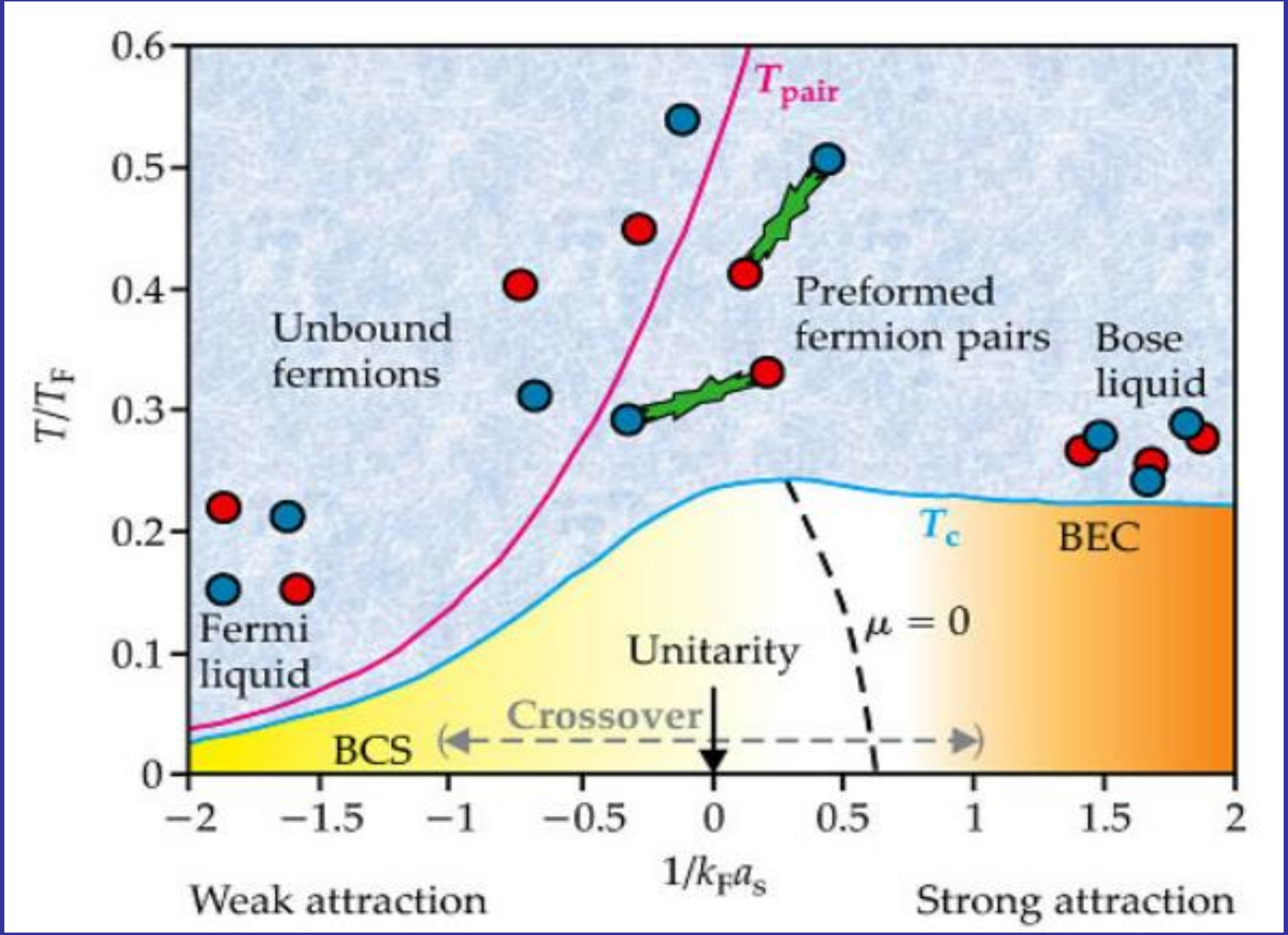
From Monte Carlo calcs.

Gap in the single particle fermionic spectrum - theory



Ab initio **result: The onset of pseudogap phase at $1/ak_F \approx -0.05$.**

Magierski, Wlazłowski, Bulgac, *Phys. Rev. Lett.* 107,145304(2011)
Magierski, Wlazłowski, Bulgac, Drut, *Phys. Rev. Lett.* 103,210403(2009)



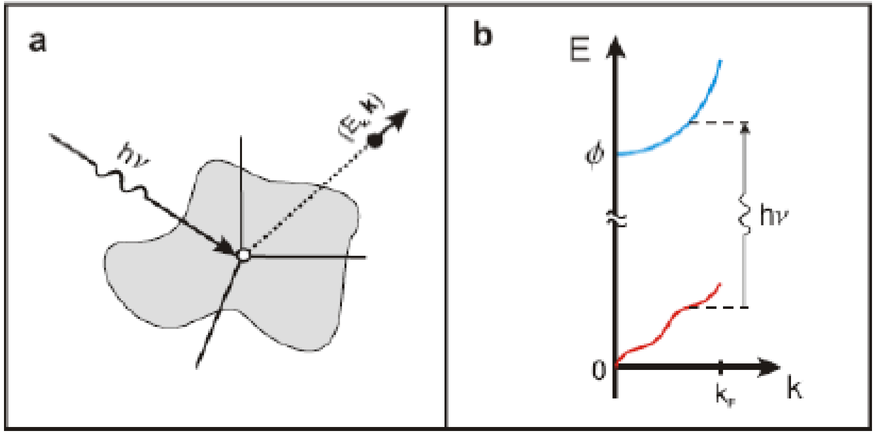
From Sa de Melo, Physics Today (2008)

Pairing pseudogap: suppression of low-energy spectral weight function due to incoherent pairing in the normal state ($T > T_c$)

Important issue related to pairing pseudogap:

- Are there sharp gapless quasiparticles in a normal Fermi liquid
  YES: Landau's Fermi liquid theory;
  NO: breakdown of Fermi liquid paradigm

RF spectroscopy in ultracold atomic gases

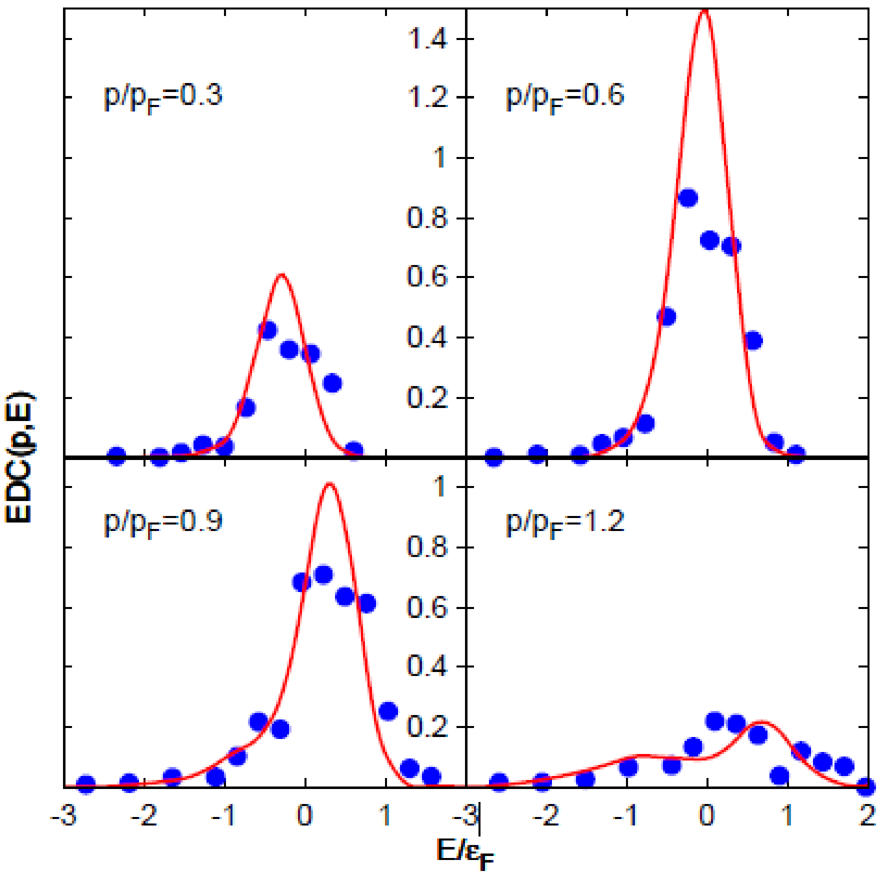


$$-E_s + h\nu = \frac{\hbar^2 k^2}{2m} + \phi$$

$$E(N) = E(N-1) + E_s$$

Stewart, Gaebler, Jin, **Nature, 454, 744 (2008)**

$$\mathrm{EDC}(p,E,T) = \mathcal{C}p^2 \int_0^\infty dr\, r^2 \frac{1}{\varepsilon_F(r)} A\left[\frac{p}{p_F(r)}, \frac{E-\mu(r)}{\varepsilon_F(r)}, \frac{T}{\varepsilon_F(r)}\right] f(E-\mu(r)),$$



Experiment (blue dots): D. Jin's group
Gaebler et al. *Nature Physics 6, 569(2010)*

Theory (red line):
Magierski, Wlazłowski, Bulgac,
Phys.Rev.Lett.107,145304(2011)

Summary:

- We have determined the shear viscosity for UFG from an ab-initio approach.

- The minimum of the shear viscosity-to-entropy density ratio appears slightly above the critical temperature and exceeds about twice the KSS bound.

- The shear viscosity-to-entropy density ratio is very close to the value estimated for quark-gluon plasma.

- Spin susceptibility (both static and dynamic) indicates the presence of pair correlations above Tc, which supports the existence of the pseudogap regime in UFG

Collaborators:

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(U. Washington)

Kenneth J. Roche
(PNNL)

Joaquin E. Drut
(U. North Carolina)

Gabriel Wlazłowski
(WUT/ U. Washington)

Postdoc and doctoral positions (either for physicists or computer scientists) available at the Faculty of Physics (WUT):

Field: Nonequilibrium processes in superfluid Fermi systems: ultracold atomic gases, atomic nuclei and neutron stars.

Tools: Time dependent DFT for superfluid systems and Quantum Monte Carlo

Computational issues: Parallel programming (MPI), programming for hybrid architectures (CUDA).

Interested persons should contact Piotr Magierski,
Faculty of Physics, (email: piotrm @ uw.edu, http://nuclear.fizyka.pw.edu.pl)

Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} = \int d^3r \sum_{s=\uparrow\downarrow} \hat{\psi}_s^{\dagger}(\vec{r}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \hat{\psi}_s(\vec{r}) \; - \; g \int d^3r \; \hat{n}_{\uparrow}(\vec{r}) \hat{n}_{\downarrow}(\vec{r})$$

$$\hat{N} = \int d^3r \left(\hat{n}_{\uparrow}(\vec{r}) + \hat{n}_{\downarrow}(\vec{r}) \right); \quad \hat{n}_s(\vec{r}) = \hat{\psi}_s^{\dagger}(\vec{r}) \hat{\psi}_s(\vec{r})$$

Path Integral Monte Carlo for fermions on 3D lattice

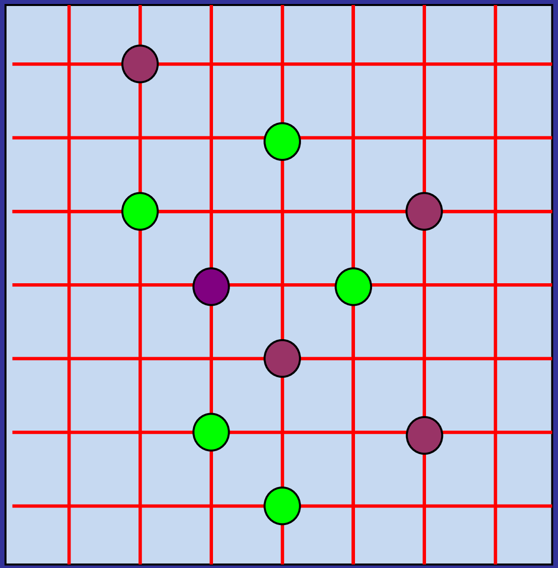
$Volume = L^3$

$lattice\ spacing = \Delta x$

Coordinate space

L – limit for the spatial correlations in the system

$k_{cut} = \frac{\pi}{\Delta x}$; Δx



- Spin up fermion:

- Spin down fermion:

External conditions:

T - temperature

μ - chemical potential

Periodic boundary conditions imposed