Quantum vortices in fermionic superfluids: from ultracold atoms to neutron stars

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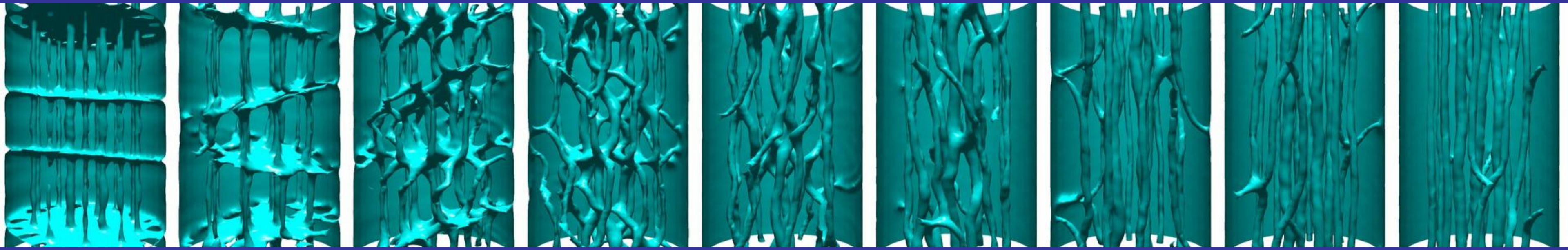

Generation and decay of fermionic turbulence

Collaborators:

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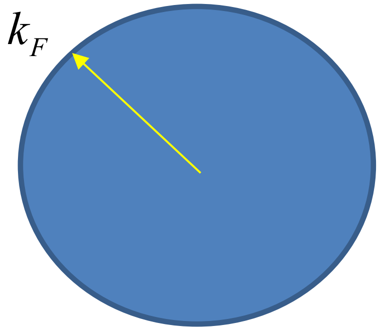
Gabriel Wlazłowski (WUT)
Klejdja Xhani (Univ. Of Torino)
Jie Yang(WUT -> Liaoning Normal Univ.)
Tomasz Zawiślak (WUT -> Univ. Trento)
Agata Zdanowicz (WUT)
and LENS exp. group - Giacomo Roati et al.

Cooper pairs

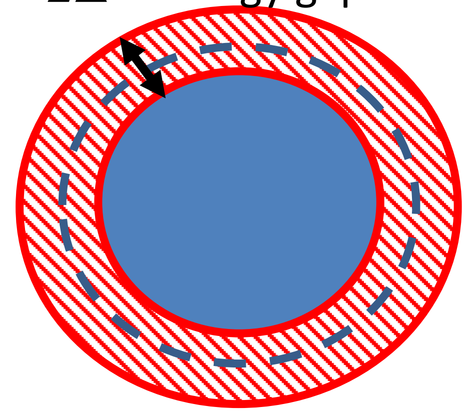
$$\varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

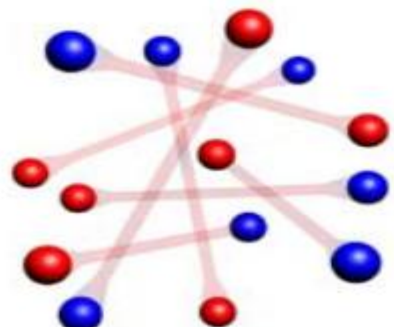


Fermi sphere

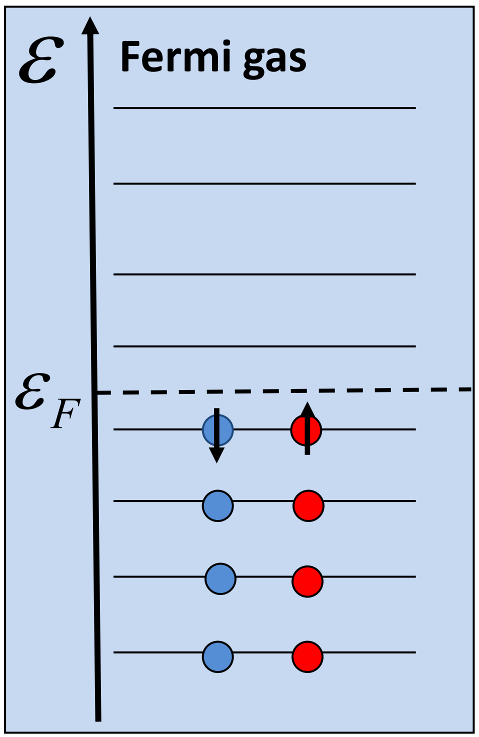
Attractive interaction

2Δ - Energy gap





Correlations between pairs (Cooper) of particles of opposite spins.



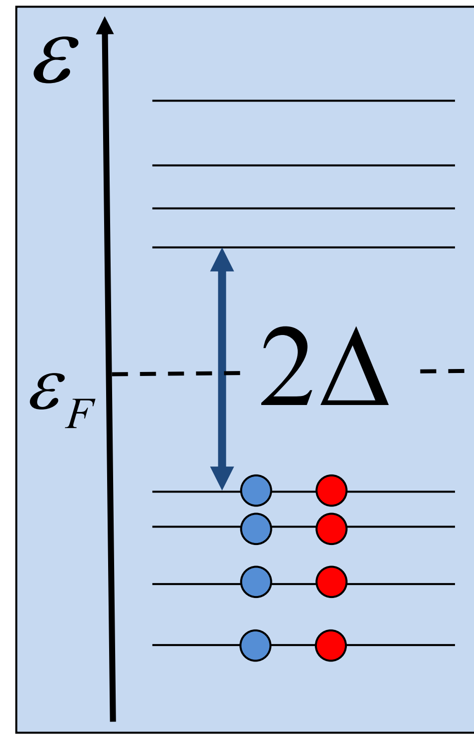
Energy of elementary excitation (quasiparticle):

$$E_{qp} = \sqrt{(\varepsilon - \varepsilon_F)^2 + |\Delta|^2} > |\Delta|$$

BCS theory:

$$|BCS\rangle = \prod_k \left(u_k + v_k a^{\dagger}_{k\uparrow} a^{\dagger}_{-k\downarrow}\right)|vacuum\rangle$$

$$\Delta = f(u_k, v_k)$$



Critical temperatures for superconductivity and superfluidity

- ✓ **Ultracold atomic gases:** $T_c \approx 10^{-12} - 10^{-9}$ eV
- ✓ **Liquid ^{3}He:** $T_c \approx 10^{-7}$ eV
- ✓ **Metals and alloys:** $T_c \approx 10^{-3} - 10^{-2}$ eV
- ✓ **Atomic nuclei and neutron stars:** $T_c \approx 10^{5} - 10^{6}$ eV
- **Color superconductivity (quarks) :** $T_c \approx 10^{7} - 10^{8}$ eV

(1 eV $\approx 10^4$ K)

Superfluidity and superconductivity

- **Requirement**: Bose-Einstein (BEC) condensation of interacting *bosons*.
- **Result**: linear dispersion relation
- **Consequence**: no viscosity (below certain flow velocity)
- **Theoretical description**: „Condensate wave function”

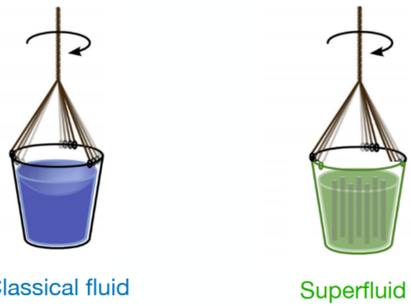
$$\Psi(\vec{r}) = \left|\Psi\left(\vec{r}\right)\right| e^{i\phi(\vec{r})}$$

- **Requirement**: arbitrary weak attraction between *fermions.*
- **Result**: formation of Cooper pairs
- **Consequence**: no resistance
- **Theoretical description**: Field of Cooper pairs

$$\Delta(\vec{r}) = \left|\Delta\left(\vec{r}\right)\right| e^{i\phi(\vec{r})}$$

Both phenomena are actually like two sides of the same coin!

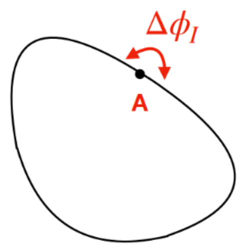
Quantum vortices – topological excitations in superfluids



The motion of an inviscid fluid is irrotational

$$\nabla \times v = 0$$

Quantized circulation



$$\Gamma = \oint v \cdot dl = \frac{\hbar}{m}\Delta\phi = \frac{\hbar}{m} 2\pi\, w$$

For a superfluid:

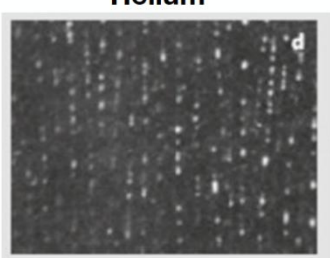
$$v = \frac{\hbar}{m}\nabla\phi$$

$\Delta\phi = 2\pi\, w \rightarrow$ single-valuedness of wavefunction

$w \rightarrow$ winding number

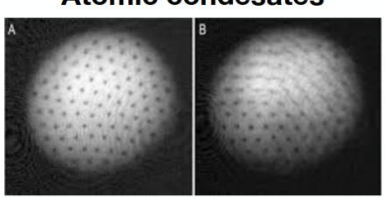
Topological excitations in a multiply-connected geometry
Vortices, (meta)stable currents in rings

Helium



From: G. Bewley, et al., *Nature* 441, 588 (2006).

Atomic condesates



From: J. R. Abo-Shaeer, et al., Science 292, Issue 5516, pp. 476-479 (2001)

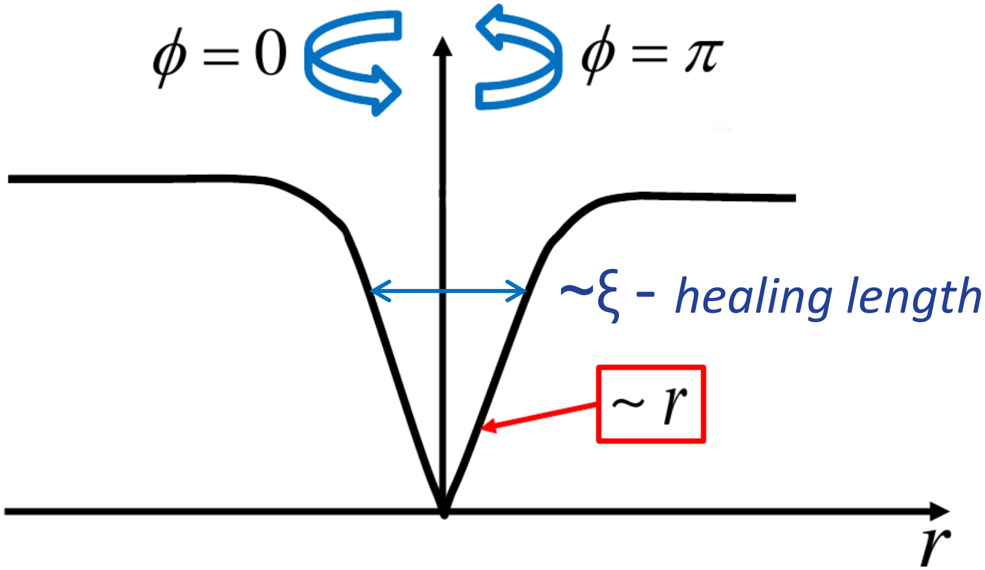
Topological excitations are long-lived as they are topologically protected:

- Fundamental physics and quantum simulation
- Quantum technologies

Anatomy of the vortex core

Bosonic vortex structure:
weakly interacting Bose gas at T=0 → Gross-Pitaevskii eq. (GPE)

$$\left[-\frac{1}{2m}\nabla^2 + g|\psi(\vec{r})|^2 + V_{ext}(\vec{r})\right]\psi(\vec{r}) = \mu\psi(\vec{r})$$



Order parameter:

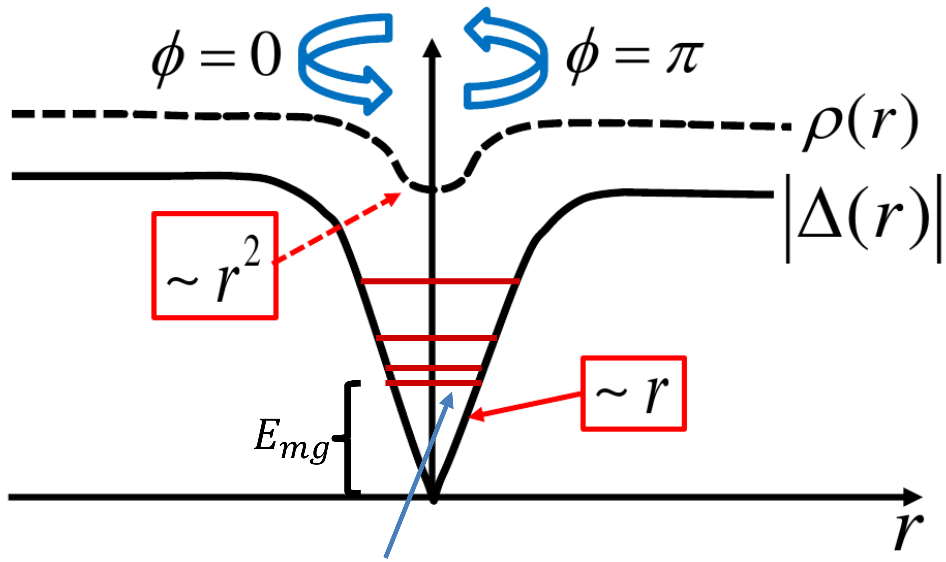
$$\psi(\vec{r}) = \sqrt{n(\vec{r})}e^{i\phi}$$

$$v_S = \frac{\hbar}{m}\nabla\phi$$

$$\kappa = \oint d\vec{l} \cdot \vec{v}_S = \frac{\hbar}{m}$$

Fermionic vortex structure:
Weakly interacting Fermi gas → Bogoliubov de Gennes (BdG) eqs.

$$\begin{pmatrix} h_\uparrow & \Delta \\ \Delta^* & -h_\downarrow^* \end{pmatrix} \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix} = \varepsilon_n \begin{pmatrix} u_{n,\uparrow} \\ v_{n,\downarrow} \end{pmatrix}$$



Form of the vortex-like solutions:

$$u_\eta(\mathbf{r}) = u_{nmk_z}(\rho) e^{im\varphi} e^{ik_z z}$$
$$v_\eta(\mathbf{r}) = v_{nmk_z}(\rho) e^{i(m+1)\varphi} e^{ik_z z}$$

CdGM (Andreev) states
C. Caroli, P. de Gennes, J. Matricon, Phys. Lett. 9, 307 (1964):

Minigap: $E_{mg} \sim \dfrac{|\Delta_\infty|^2}{\varepsilon_F}$ - energy scale for vortex core excitations.

Density of states: $g(\varepsilon) \sim \dfrac{\varepsilon_F}{|\Delta_\infty|^2}$; $\varepsilon \ll |\Delta_\infty|$

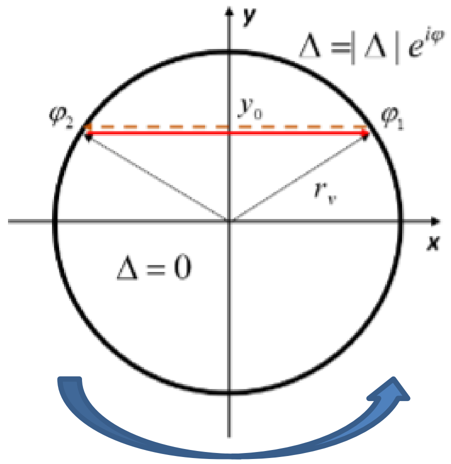
Vortex core structure in Andreev approximation:

$$\frac{E(0,L_z)}{\varepsilon_F}k_F r_V\sqrt{1-\left(\frac{L_z}{k_F r_V}\right)^2}+\arccos\left(\frac{-L_z}{k_F r_V}\right)-\arccos\left(\frac{E(0,L_z)}{|\Delta_\infty|}\right)=0$$

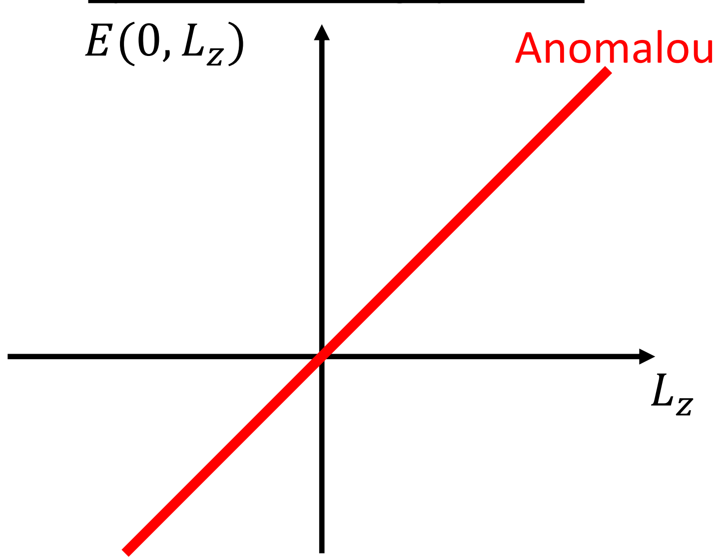
$$E(0,L_z)=E(0)L_z,\ E\ll|\Delta_\infty|$$

$$E(0,L_z)\approx\frac{|\Delta_\infty|^2}{\varepsilon_F\frac{r_V}{\xi}\left(\frac{r_V}{\xi}+1\right)}\frac{L_z}{\hbar},\qquad \xi=\frac{\varepsilon_F}{k_F|\Delta_\infty|}$$

Schematic section of the core



Spectrum of in-gap states



Works well in deep BCS limit: $\frac{1}{k_F a_s}\ll 0$

M. Stone, Phys. Rev. B 54, 13222 (1996)
P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

Quasiparticle mobility along the vortex line

$$E(k_z) = \frac{E(0)}{\sqrt{1-\left(\frac{k_z}{k_F}\right)^2}}\ ;\ k_z < k_F$$

C. Caroli, P. de Gennes, J. Matricon, Phys. Lett. 9, 307 (1964):

In Andreev approximation:

$$\sqrt{\varepsilon_F + E}\sin\alpha = \sqrt{\varepsilon_F - E}\sin\beta$$

$$k_h = \sqrt{2(\varepsilon_F - E)}$$

$$k_p = \sqrt{2(\varepsilon_F + E)}$$

$$v_z = k_z \frac{\sqrt{k_p^2 - k_z^2} - \sqrt{k_h^2 - k_z^2}}{\sqrt{k_p^2 - k_z^2} + \sqrt{k_h^2 - k_z^2}}$$

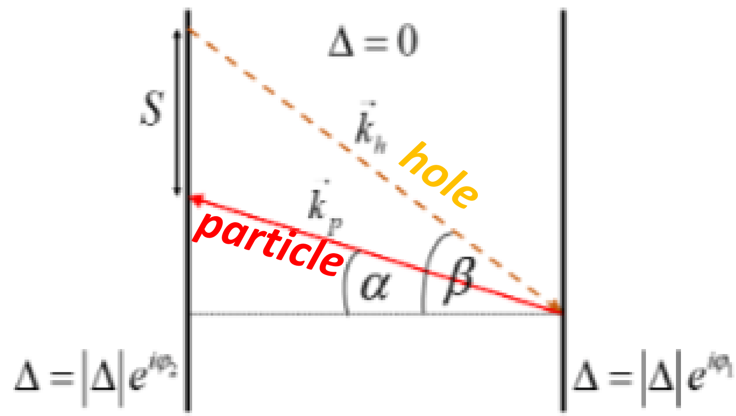
Velocity component along the vortex line

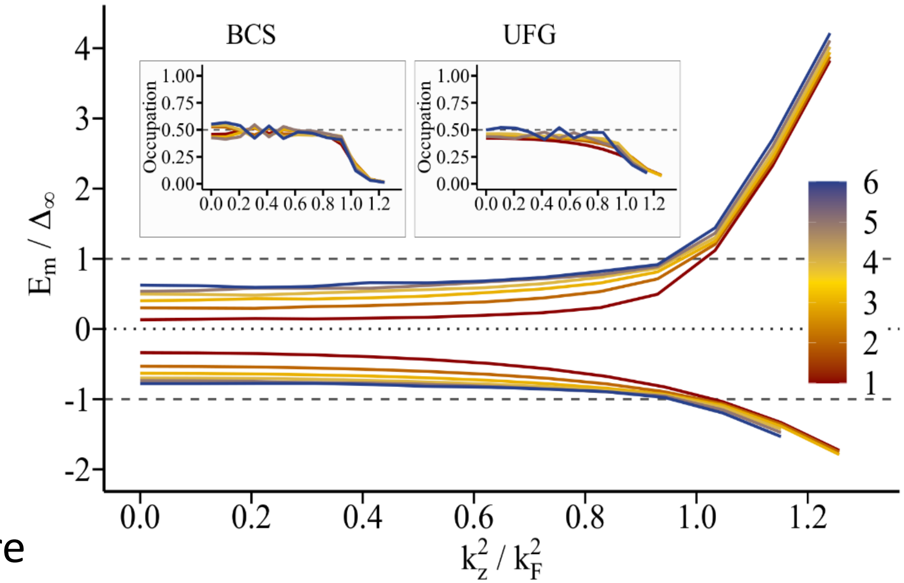
It gives the same dispersion relations as above up to the second order.

$$M_{eff}^{-1}(L_z) \approx \frac{2}{3}\left(\frac{|\Delta_\infty|}{\varepsilon_F}\right)^2 \frac{L_z}{\hbar}$$

Effective mass of quasiparticle in the core carrying ang. mom. Lz

Schematic picture of Andreev reflection of particle-hole moving along the vortex line

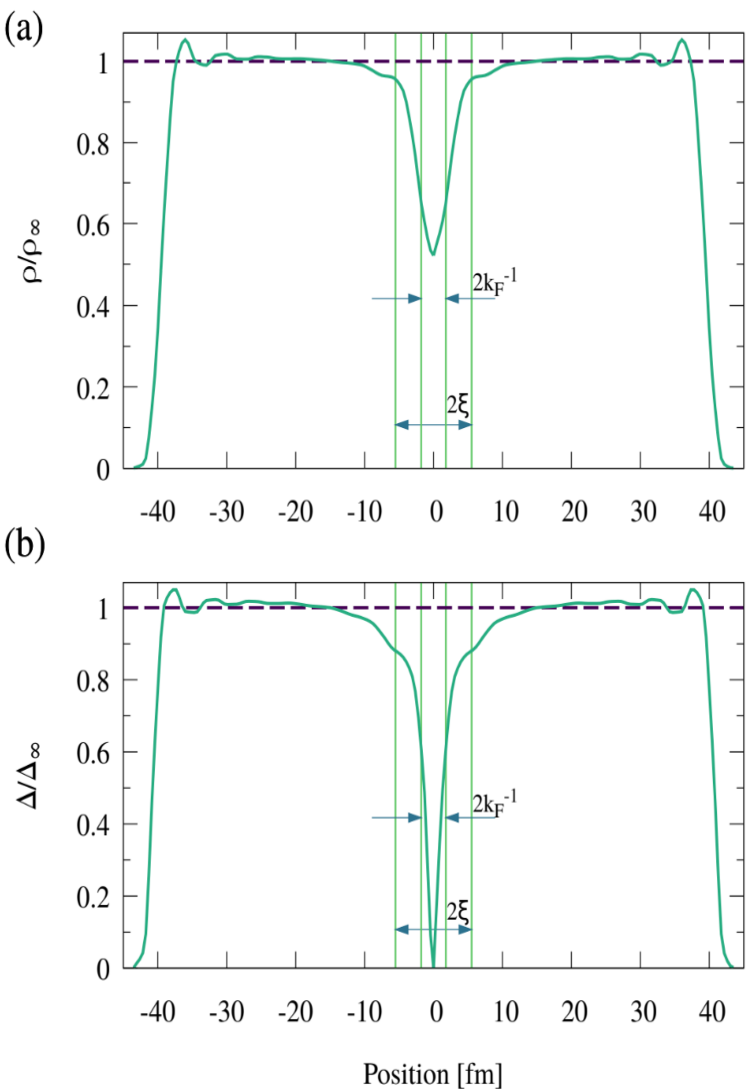


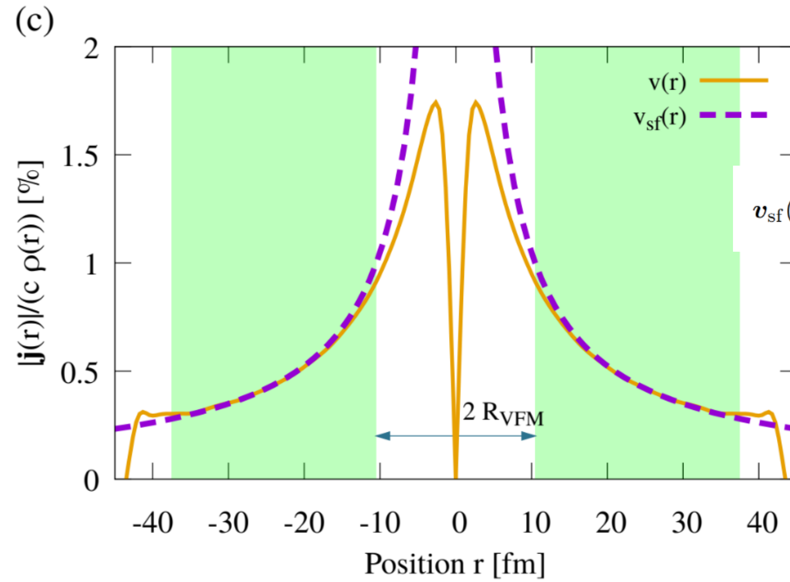


P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

Note that large value of effective mass along the vortex line originate from the fact that the occupations of hole and particle states below the gap are approximately equal.

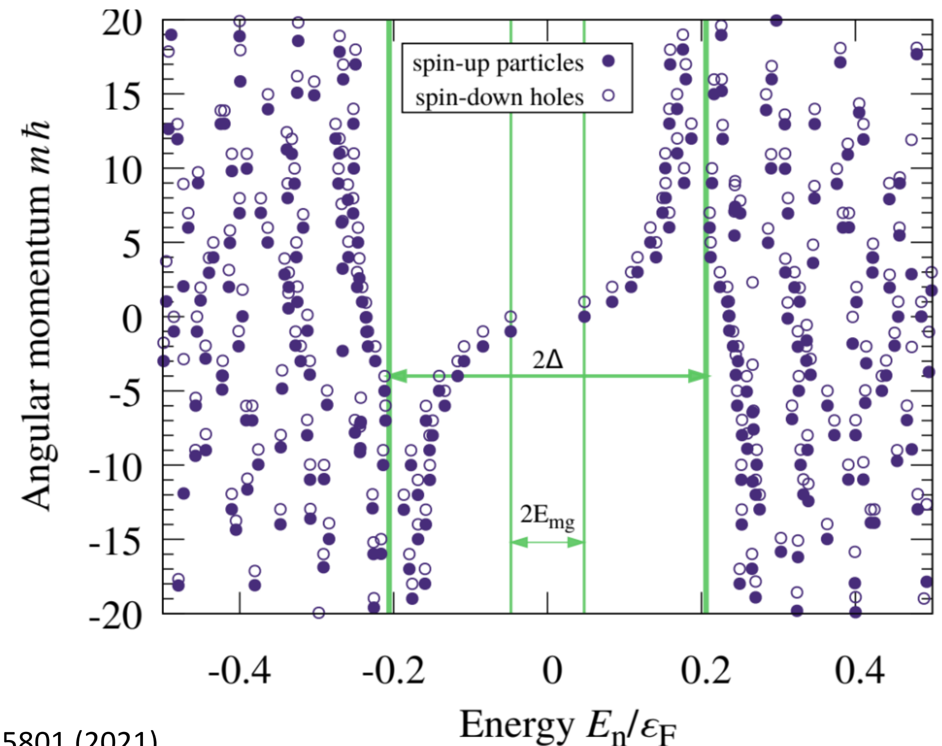
Example of HFB (BdG) calculations of the vortex structure.





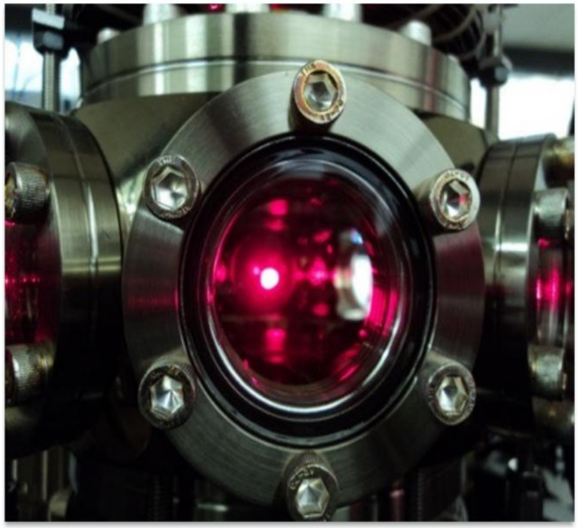
$$\boldsymbol{v}_{\rm sf}(\boldsymbol{r}) = \frac{\hbar}{2M}\boldsymbol{\nabla}\phi(\boldsymbol{r}) = \frac{\hbar}{2M}\frac{1}{r}\hat{\boldsymbol{e}}_{\boldsymbol{\phi}}.$$

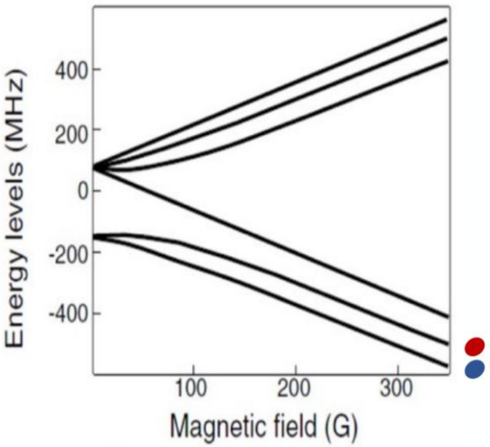
$$\boldsymbol{v}(\boldsymbol{r}) = \hbar\frac{\boldsymbol{j}(\boldsymbol{r})}{\rho(\boldsymbol{r})}.$$

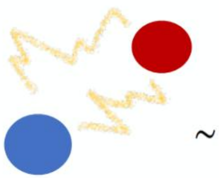


D. Pęcak N. Chamel, P.M., G. Wlazłowski. Phys. Rev. C 104, 055801 (2021)

LITHIUM-6 FERMI SUPERFLUIDS





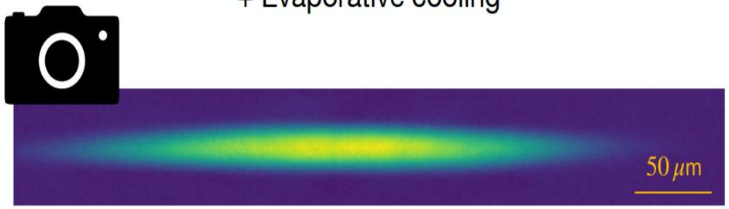


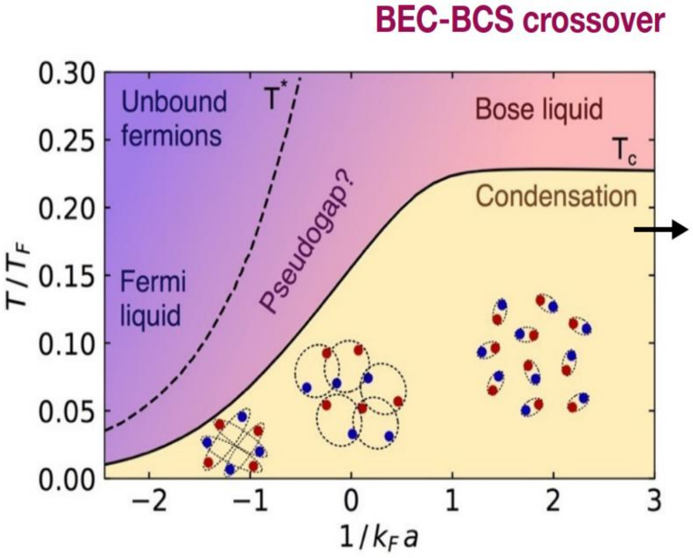
$$\sim \frac{4\pi\hbar^2}{m} a\,\delta(r)$$

a → Scattering length

N $\sim 10^5$ at $T \sim 30$ nK

Magneto optical trap + D_1 molasses + Evaporative cooling





→ 100% superfluid
<100% condensed

Courtesy of Giulia del Pace

Forces acting on a 2D vortex in an electrically neutral superfluid

Magnus Force

Dissipative Forces

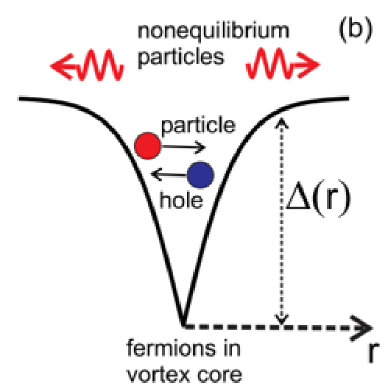
$$\boldsymbol{F} = A\boldsymbol{e}_z \times \left(\frac{d\boldsymbol{R}_V}{dt} - \boldsymbol{v}_s\right) + B\boldsymbol{e}_z \times \left(\frac{d\boldsymbol{R}_V}{dt} - \boldsymbol{v}_n\right) + C\left(\frac{d\boldsymbol{R}_V}{dt} - \boldsymbol{v}_n\right) + $$
$$+ \boldsymbol{F}_{\text{pinning}}\left(\boldsymbol{R}_V, \frac{d\boldsymbol{R}_V}{dt}, \boldsymbol{e}_z \times \frac{d\boldsymbol{R}_V}{dt}\right),$$

Pinning Force (in inohomogeneous superfluid)

How can we measure the influence of core states in ultracold gases?

Dissipative processes involving vortex dynamics.

Silaev, Phys. Rev. Lett. 108, 045303 (2012)
Kopnin, Rep. Prog. Phys. 65, 1633 (2002)
Stone, Phys. Rev. B54, 13222 (1996)
Kopnin, Volovik, Phys. Rev. B57, 8526 (1998)
....



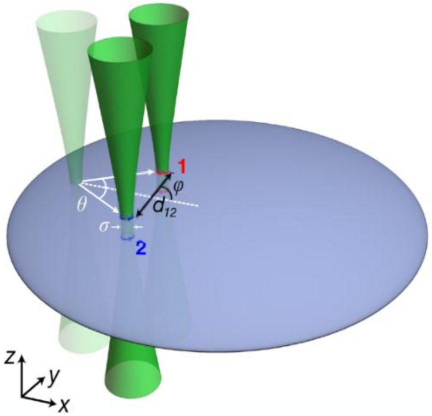
Classical treatment of states in the core (Boltzmann eq.).
More applicable in deep BCS limit unreachable in ultracold atoms.

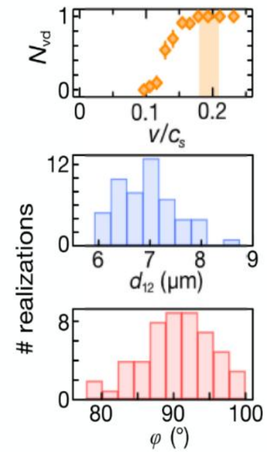
VORTICES IN ATOMIC FERMI SUPERFLUIDS

W. J. Kwon, et al. *Nature* **600**, 64–69 (2021).

Excitation of vortices

Chopstick method

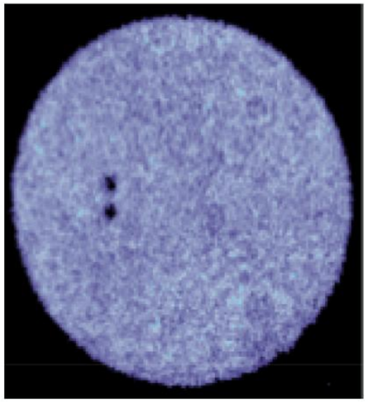




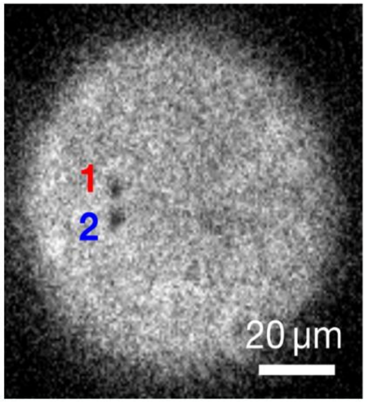
With Bose superfluids:
E. C. Samson, *Phys. Rev. A* **93**, 023603 (2016).
T. Neely et al., arXiv:2402.09920v2

Detection of vortices

Chopstick ON
In situ



Chopstick OFF
Time-of-fligth



- Create arbitrary configuration of **vortex dipole**
- Precisely track the vortex position

Courtesy of Giulia del Pace

Velocities of vortex-antivortex pair (vortex dipole)

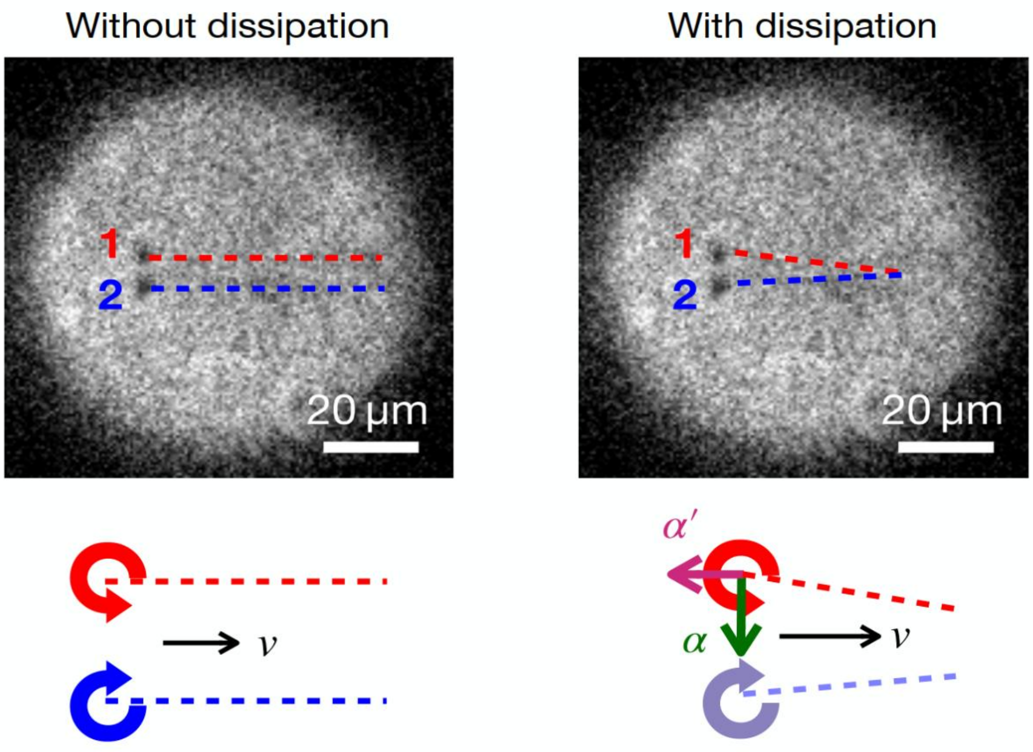
$$\boldsymbol{v}_1(t) = \frac{|\kappa|}{d(t)}\left[(1-\alpha')\hat{\boldsymbol{y}} - |\alpha|\hat{\boldsymbol{x}}\right],$$

$$\boldsymbol{v}_2(t) = \frac{|\kappa|}{d(t)}\left[(1-\alpha')\hat{\boldsymbol{y}} + |\alpha|\hat{\boldsymbol{x}}\right],$$

where $\alpha = \frac{\tilde{D}}{\tilde{D}^2+(1-\tilde{D}')^2}$, $1-\alpha' = \frac{1-\tilde{D}'}{\tilde{D}^2+(1-\tilde{D}')^2}$, $\tilde{D} = \frac{D}{\kappa\rho_s}$, $\tilde{D}' = \frac{D'}{\kappa\rho_s}$.

Dissipative Force

$$\boldsymbol{F}_N = D(\boldsymbol{v}_i - \boldsymbol{v}_n) + D'\hat{\boldsymbol{z}} \times (\boldsymbol{v}_i - \boldsymbol{v}_n)$$

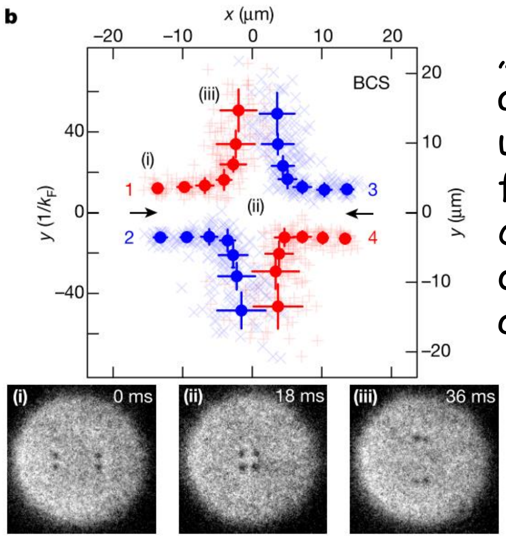


$E \propto \log d$

$v \propto 1/d$

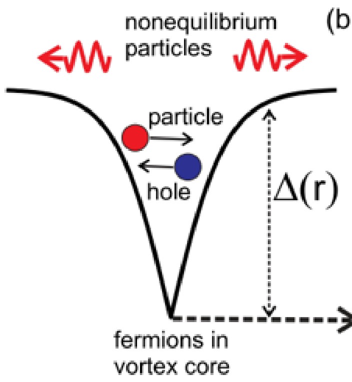
$E_i = E_f$ $\qquad$ $E_i > E_f$

Vortex-antivortex scattering in 2D



„Further, our few-vortex experiments extending across different superfluid regimes reveal non-universal dissipative dynamics, suggesting that fermionic quasiparticles localized inside the vortex core contribute significantly to dissipation, thereby opening the route to exploring new pathways for quantum turbulence decay, vortex by vortex."

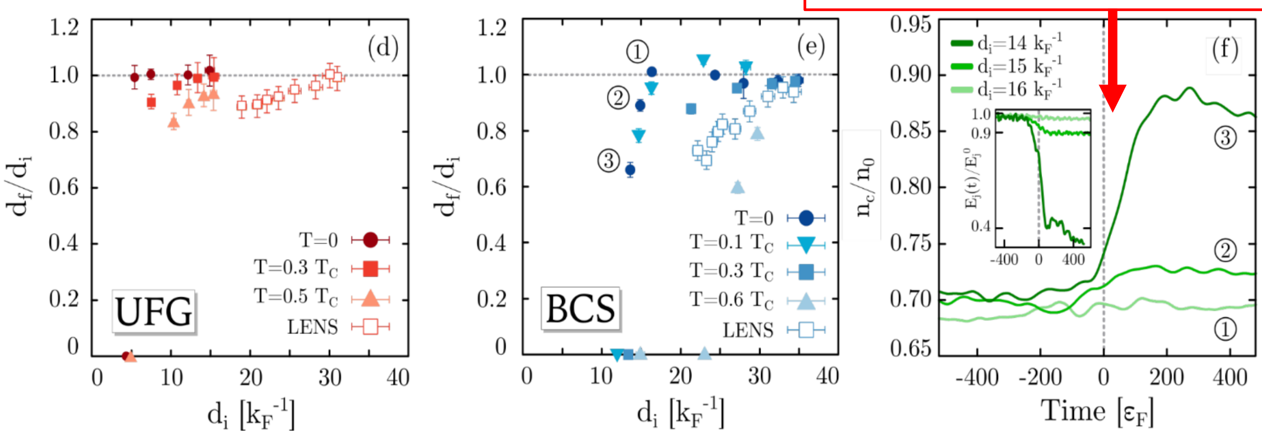
W.J. Kwon et al. Nature **600**, 64 (2021)



Exciting quasiparticles in the vortex core

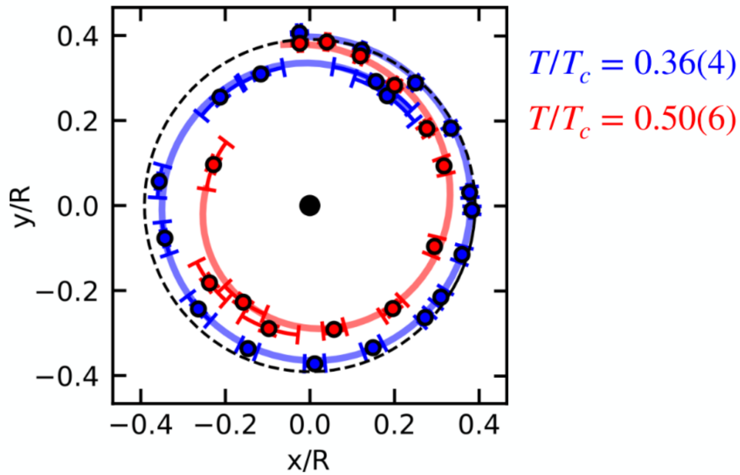
Indeed quasiparticles in the core are excited due to vortex acceleration but the effect is too weak to account for the total dissipation rate.

A. Barresi, A. Boulet, P.M., G. Wlazłowski, Phys. Rev. Lett. 130, 043001 (2023)



MUTUAL FRICTION IN A VORTEX DIPOLE

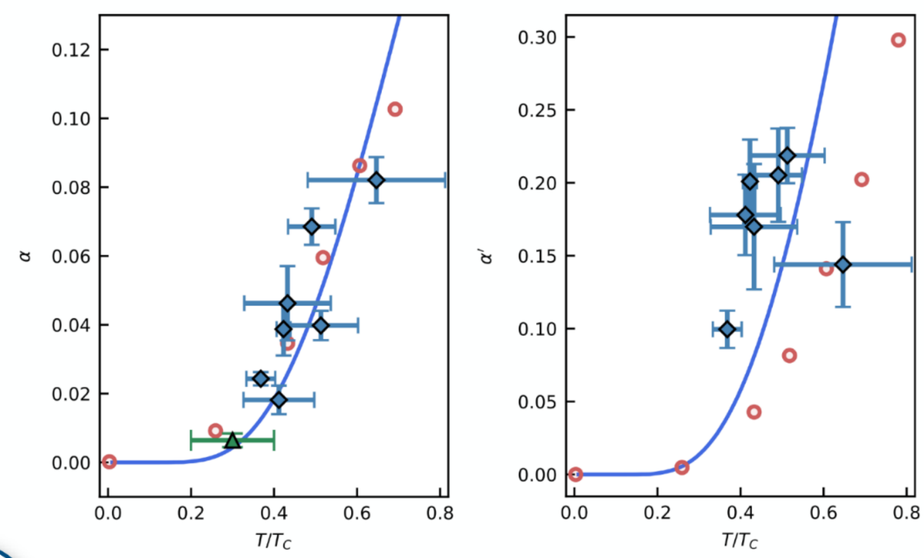
N. Grani, et al., arXiv:2503.21628 (2025)



Point Vortex Model (PVM)

- First experimental observation of **nonzero** α' in ultra cold atoms
- Connection with **microscopic mechanism** of dissipation

Mutual friction coefficients



- SLDA simulation
- Analytical model including localized QP

Kopnin, Rep. Prog. Phys. 65, 1633 (2002)

Courtesy of Giulia del Pace

Inhomogeneous systems: Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase

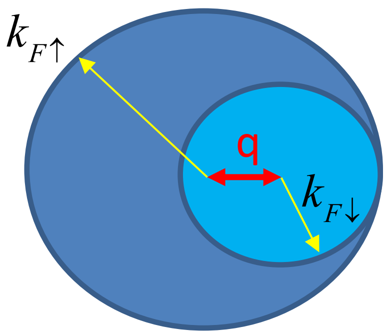
Larkin-Ovchinnikov (LO): $\Delta(r) \sim cos(\vec{q}\cdot\vec{r})$

Fulde-Ferrell (FF): $\Delta(r) \sim \exp(i\vec{q}\cdot\vec{r})$

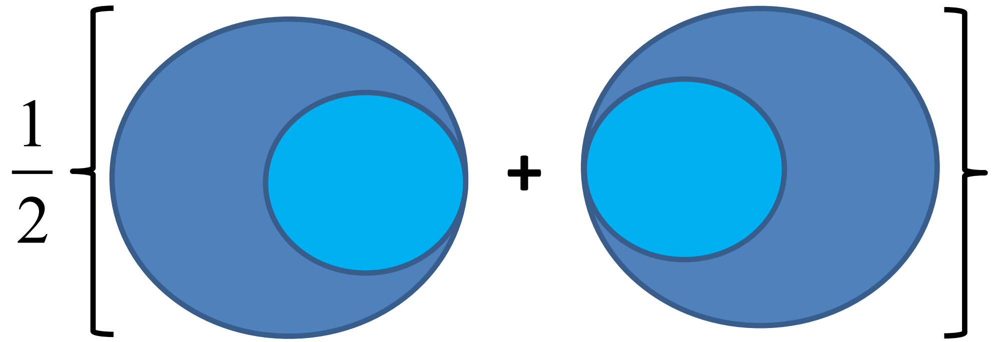
A.I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP 20, 762 (1965)
P. Fulde and R. A. Ferrell, Phys. Rev. 135, A550 (1964)

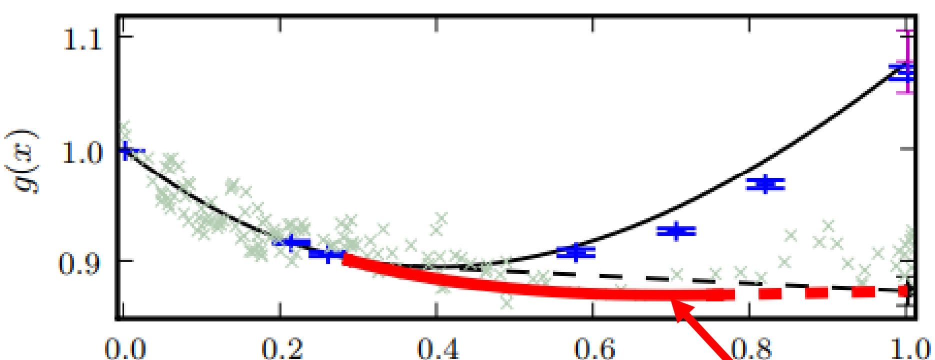
Spatial modulation of the pairing field cost energy proportional to q^2 but may be compensated by an increased pairing energy due to the mutual shift of Fermi spheres:

FF:



LO:





$E \sim [n_a g(x)]^{5/3}$

Bulgac & Forbes have shown, within DFT, that Larkin-Ovchinnikov (LO) phase may exist in the unitary Fermi gas (UFG) (realized experimentally in ultracold atomic clouds)

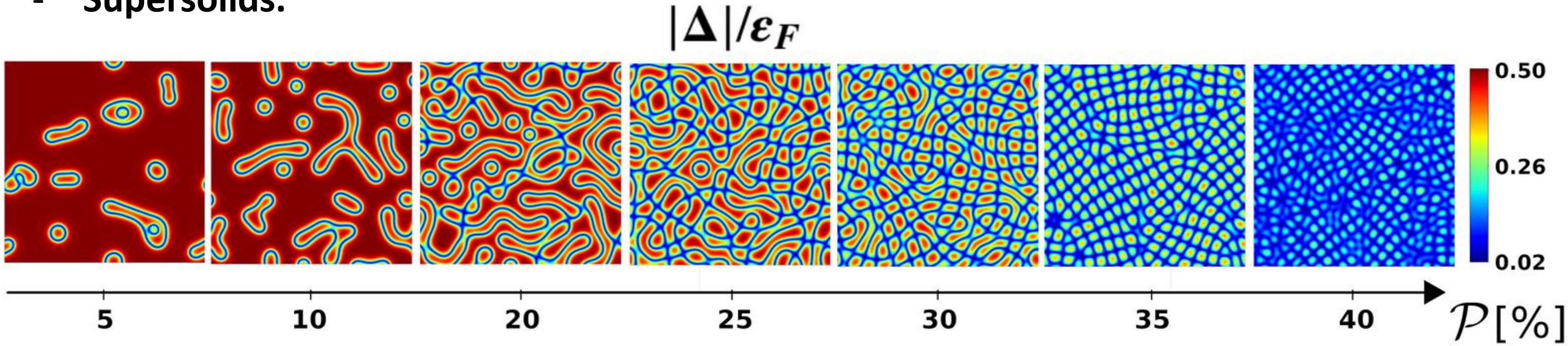
LO configuration – supersolid state

A. Bulgac, M.M.Forbes, Phys. Rev. Lett. 101,215301 (2008)
See also review of mean-field theories : Radzihovsky,Sheehy, Rep.Prog. Phys.73,076501(2010)

What is going to happen if we keep increasing spin imbalance?

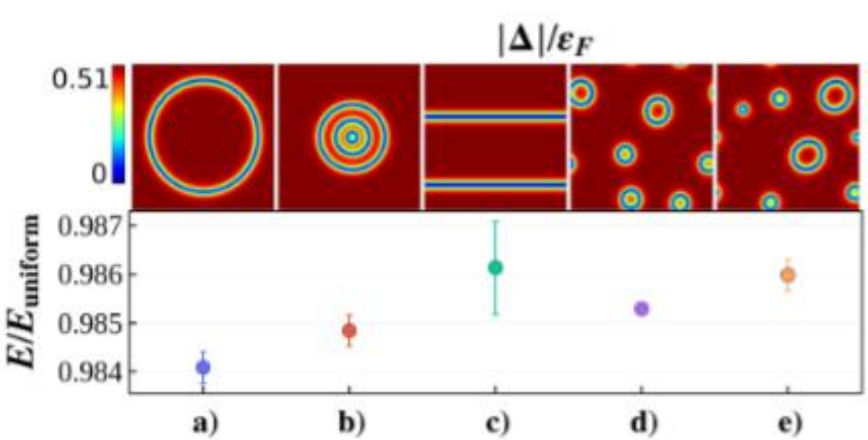
In general it will generate distortions of Fermi spheres locally and triggering the appearance of **pairing field inhomogeneity** leading to various patterns involving:

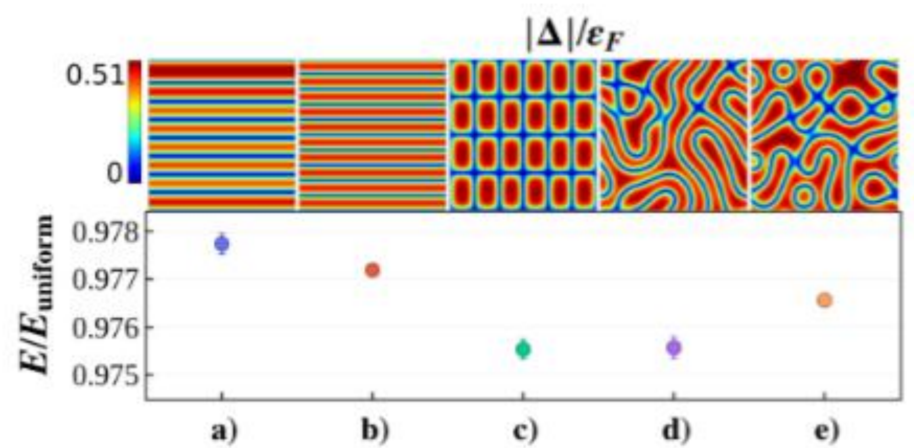
- **Separate impuritites (ferrons),**
- **Liquid crystal-like structure,**
- **Supersolids.**



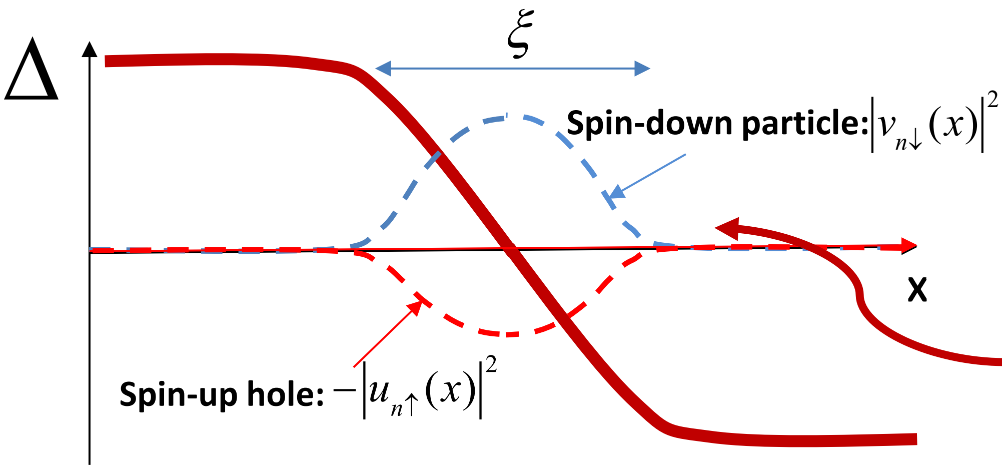
B. Tüzemen, T. Zawiślak, P.M., G. Wlazłowski, New J. Phys. 25, 033013 (2023)

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$





Andreev states and stability of pairing nodal points



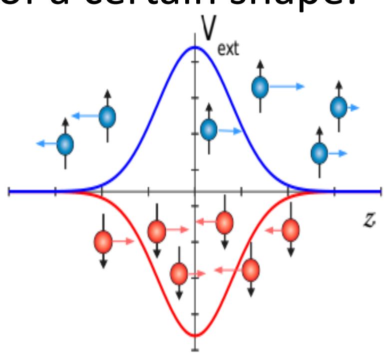
Due to quasiparticle scattering the localized Andreev states appear at the nodal point. These states induce local spin-polarization

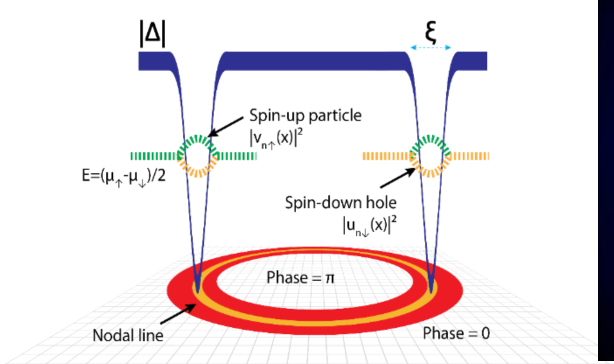
BdG in the Andreev approx. ($\Delta \ll k_F^2$)

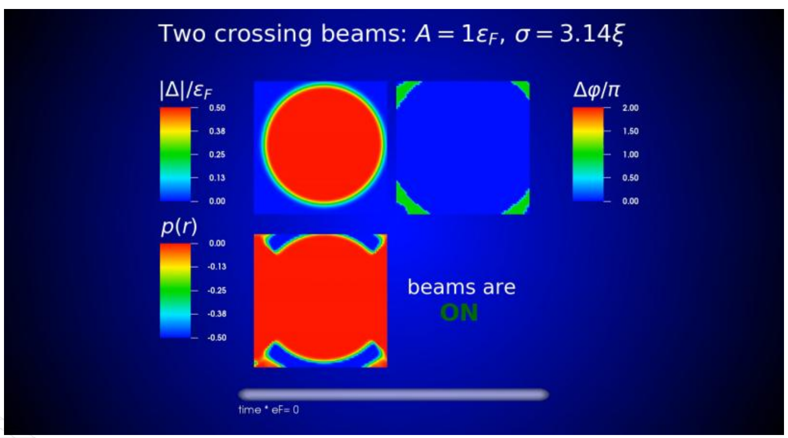
$$\begin{bmatrix} -2ik_F \dfrac{d}{dx} & \Delta(x) \\ \Delta^*(x) & 2ik_F \dfrac{d}{dx} \end{bmatrix} \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix} = E_n \begin{bmatrix} u_{n\uparrow}(x) \\ v_{n\downarrow}(x) \end{bmatrix}$$

Engineering the structure of nodal surfaces

Apply the spin-selective potential of a certain shape:



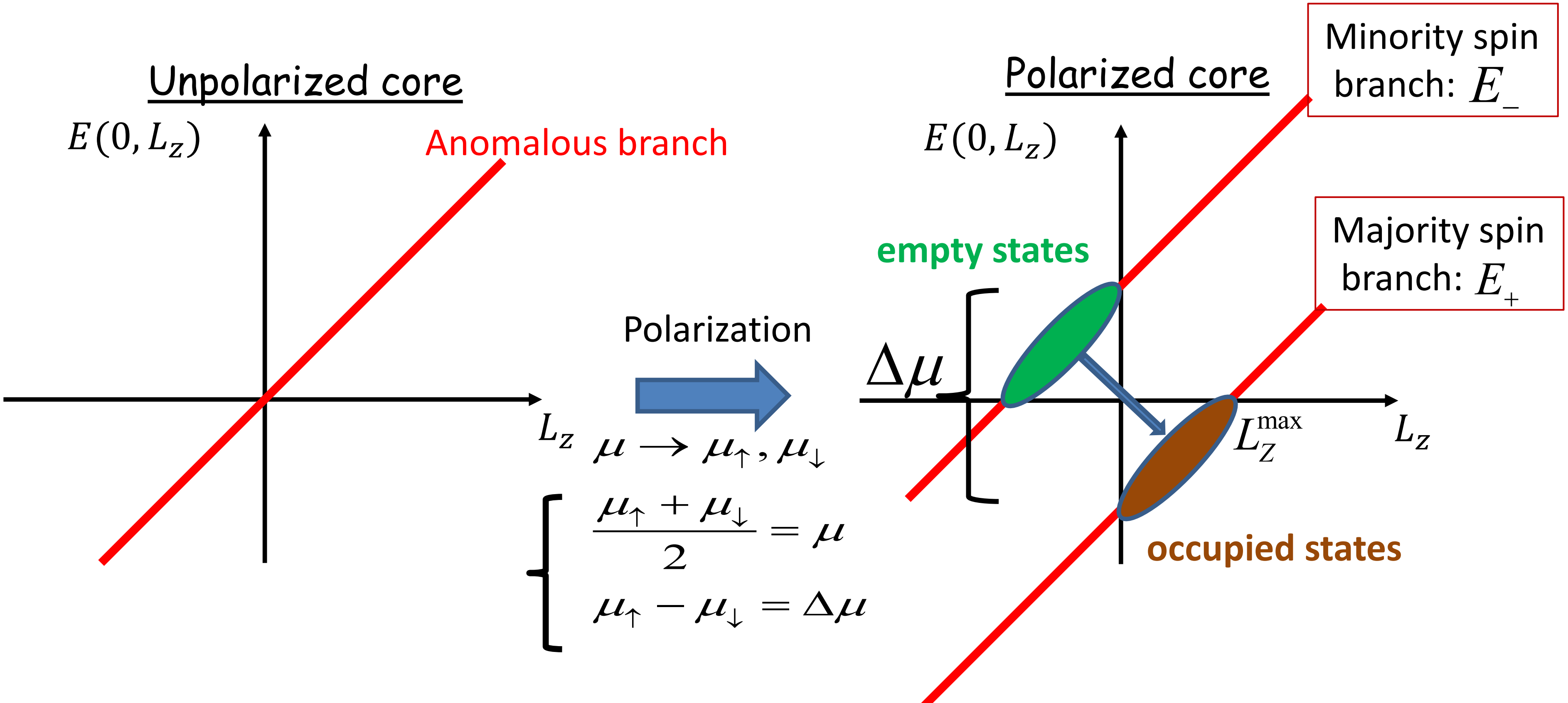




Wait until the proximity effects of the pairing field generate the nodal structure and remove the potential.

P. Magierski, B.Tüzemen, G.Wlazłowski, Phys. Rev. A 100, 033613 (2019); Phys. Rev. A 104, 033304 (2021)

Changes of the vortex core structure induced by spin polarization



$$\mu \rightarrow \mu_\uparrow, \mu_\downarrow$$

$$\begin{cases} \dfrac{\mu_\uparrow + \mu_\downarrow}{2} = \mu \\ \mu_\uparrow - \mu_\downarrow = \Delta\mu \end{cases}$$

Branches are split proportionally to polarization

$$E_\pm(0, L_Z) \approx \frac{|\Delta_\infty|^2}{\varepsilon_F \dfrac{r_V}{\xi}\left(\dfrac{r_V}{\xi}+1\right)} \frac{L_Z}{\hbar} \mp \frac{\Delta\mu}{2}$$

Certain fraction of majority spin particles rotate in the opposite direction!

$$L_Z^{\max} \approx \frac{1}{2} \frac{\varepsilon_F}{|\Delta_\infty|^2} \frac{r_V}{\xi}\left(\frac{r_V}{\xi}+1\right) \hbar\Delta\mu$$

Two consequences of vortex core polarization:

1) Minigap vanishes.

2) Direction of the current in the core reverses.

1) Since the polarization correspond to relative shift of anomalous branches therefore the quasiparticle spectrum of spin-up and spin-down components is asymmetric for $k_Z = 0$.

However the symmetry of the spectrum has to be restored in the limit of $k_Z \to \infty$. Since for a straight vortex one can decouple the degree of freedom along the vortex line:

$$H = \begin{pmatrix} h_{2D}(\mathbf{r}) + \frac{1}{2}k_z^2 - \mu_\uparrow & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h_{2D}^*(\mathbf{r}) - \frac{1}{2}k_z^2 + \mu_\downarrow \end{pmatrix}$$

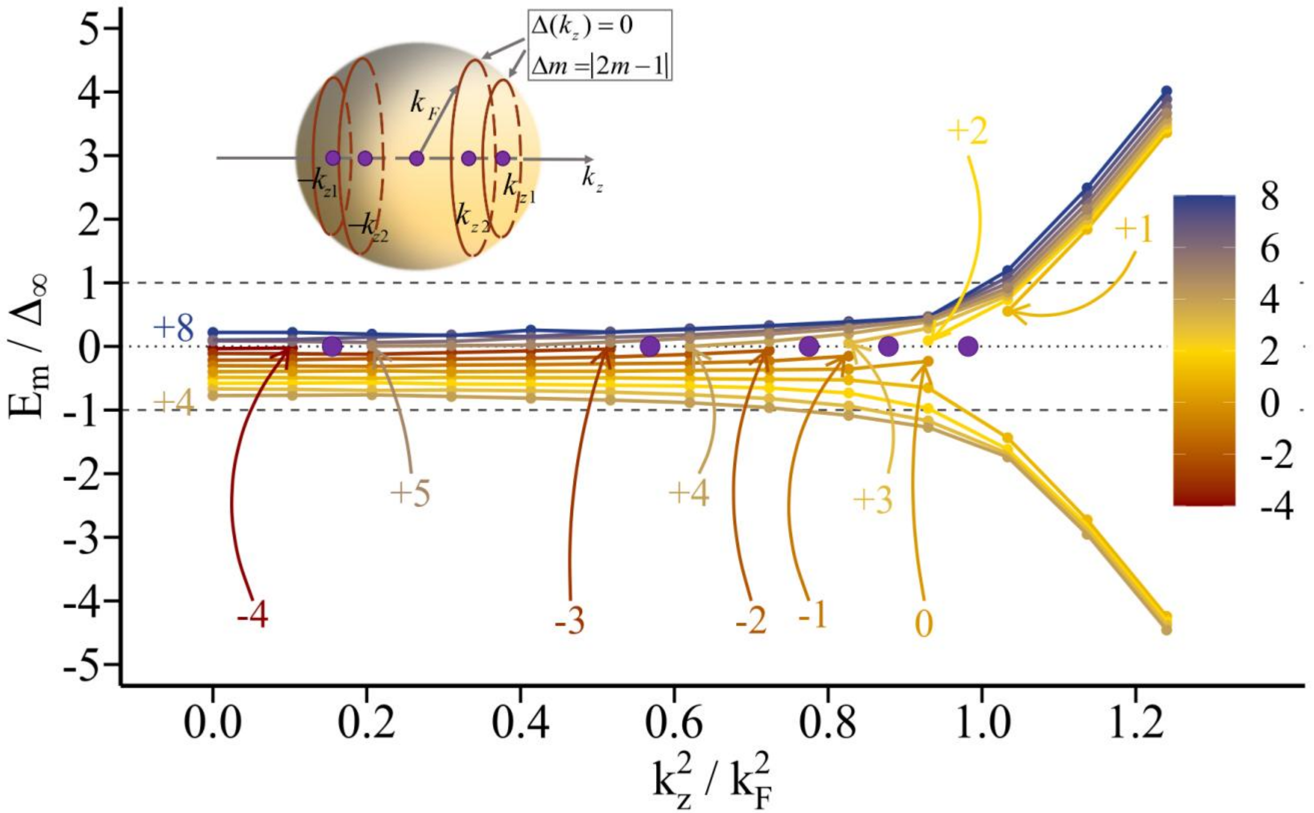
therefore $E(k_Z) \propto \pm k_Z^2$ when $k_Z \to \infty$

As a result there must exist a sequence of values: $k_Z = \pm k_{Z1}, \pm k_{Z2}, \ldots$ for which:

$$E(\pm k_{Zi}) = 0$$

Moreover the crossings occur between levels of particular projection of angular momentum on the vortex line.

Namely, the crossing occurs in such a way that the particle state: $v_{\uparrow}$ of ang. momentum ***m*** is converted into a hole $u_{\uparrow}$ of momentum ***–m+1***
Hence the configuration changes by $\Delta m = |2m-1|$

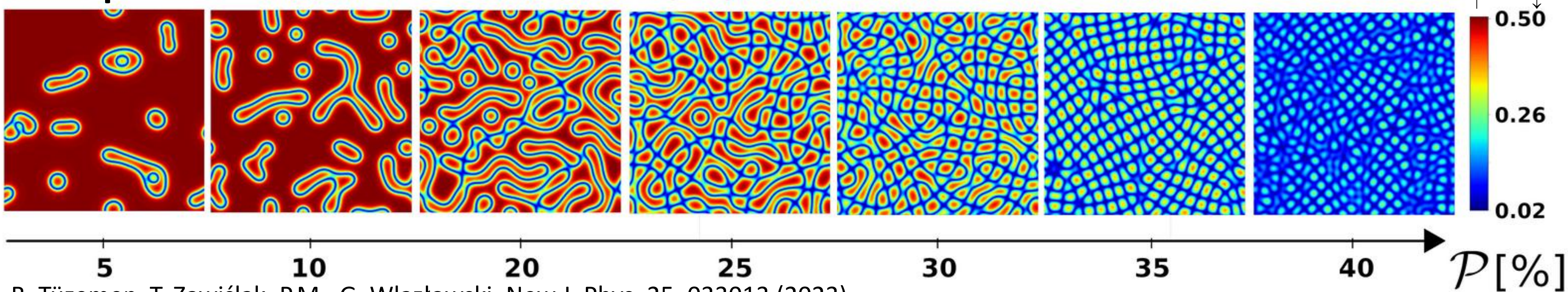


P.M. G. Wlazłowski, A. Makowski, K. Kobuszewski, Phys. Rev. A 106, 033322 (2022)

What is going to happen if we introduce spin imbalance?

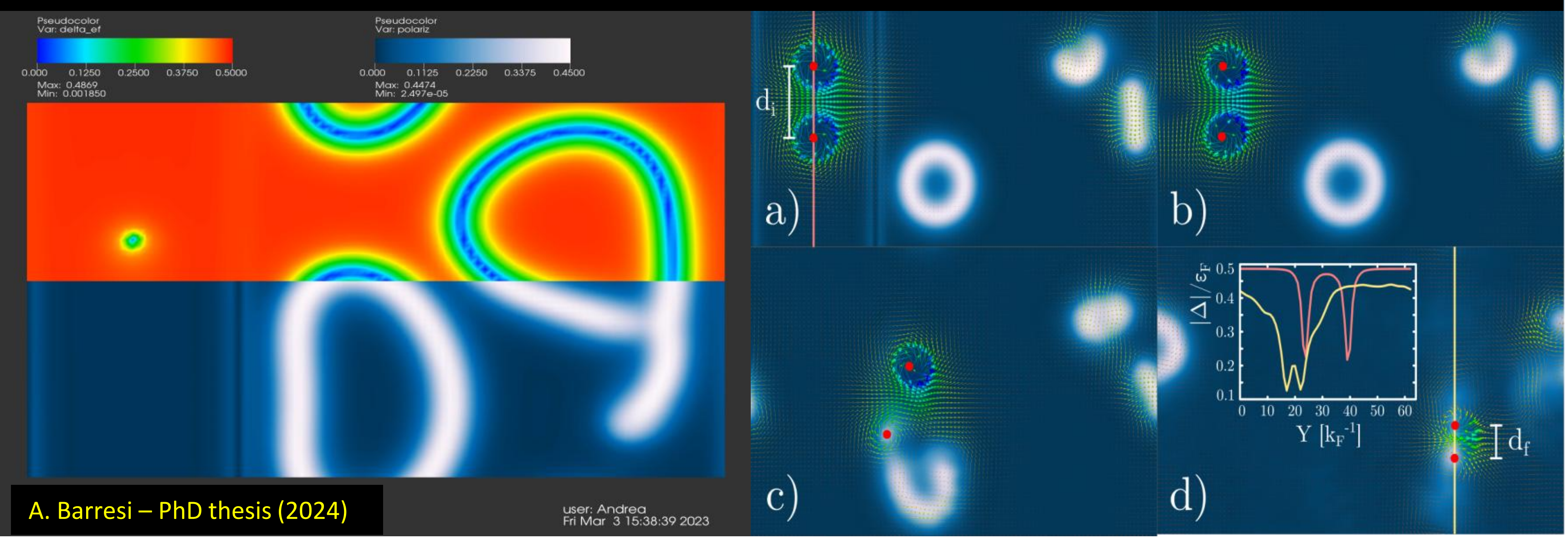
In general it will generate distortions of Fermi spheres locally and triggering the appearance of **pairing field inhomogeneity** leading to various patterns involving:

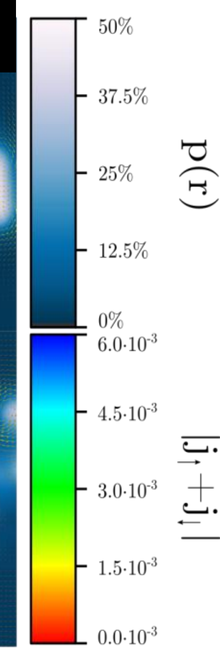
- **Separate impuritites (ferrons),**
- **Liquid crystal-like structure,**
- **Supersolids.**



B. Tüzemen, T. Zawiślak, P.M., G. Wlazłowski, New J. Phys. 25, 033013 (2023).

Dynamics of a vortex dipole in spin imbalanced Fermi superfluid.
Strong enhancement of vortex dipole energy dissipation.



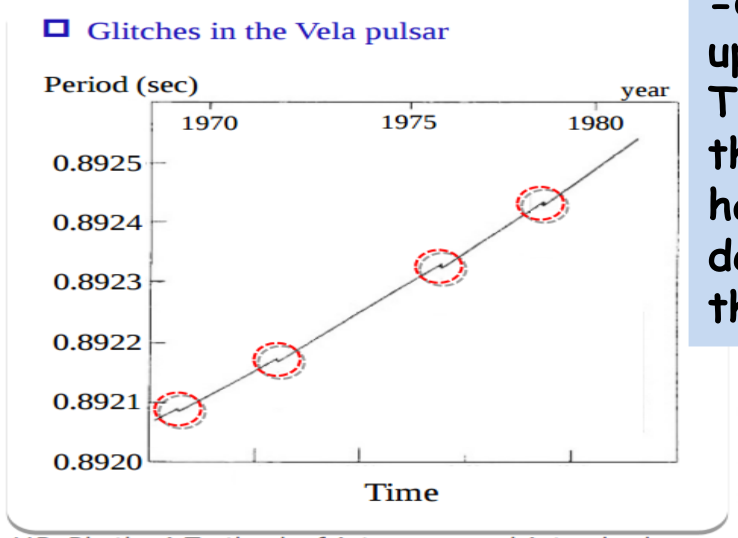


A. Barresi – PhD thesis (2024)

Modelling neutron star interior

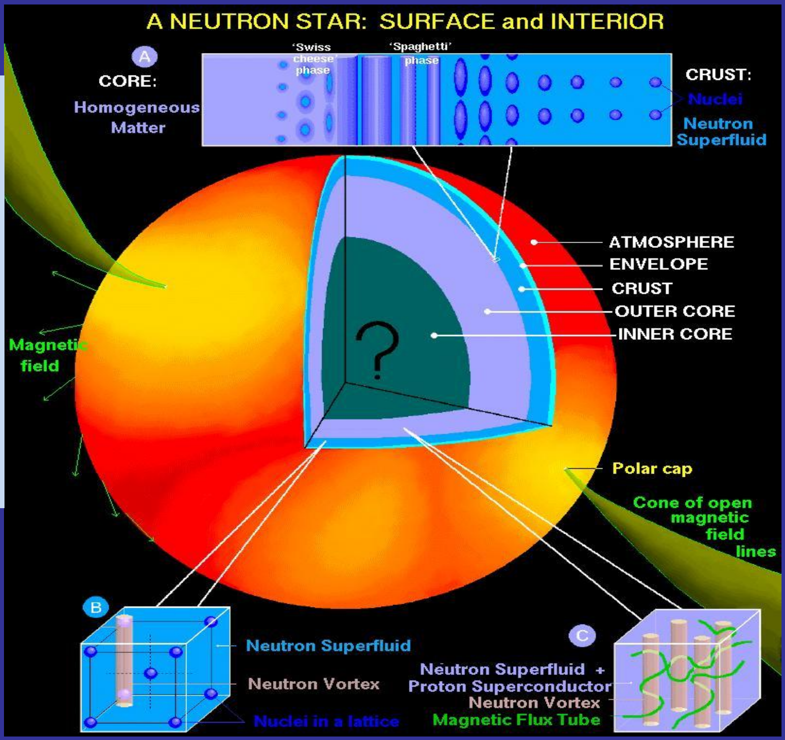
Neutron star is a huge superfluid

Glitch: a sudden increase of the rotational frequency



V.B. Bhatia, A Textbook of Astronomy and Astrophysics with Elements of Cosmology, Alpha Science, 2001.

glitch phenomenon =a sudden speed up of rotation.
To date more than 300 glitches have been detected in more than 100 pulsars



**Glitch phenomenon is commonly believed to be related to rearrangement of vortices in the interior of neutron stars (Anderson, Itoh, Nature 256, 25 (1975))
It would require however a correlated behavior of huge number of quantum vortices and the mechanism of such collective rearrangement is still a mystery.**

Large scale dynamical model of neutron star interior (in particular neutron star crust), based on microscopic input from nuclear theory, is required.
In particular: vortex-impurity interaction, deformation modes of nuclear lattice, effective masses of nuclear impurities and couplings between lattice vibrations and neutron superfluid medium, need to be determined.

Properties of a vortex across the neutron star crust

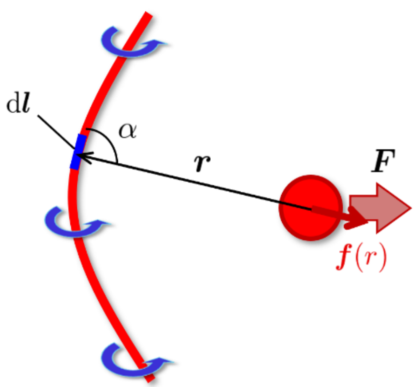
ρ_∞ (fm^{-3})	0.00036	0.0059	0.0112	0.0189	0.0231	0.0333
k_F^{-1} (fm)	4.52	1.79	1.45	1.21	1.14	1.01
ξ (fm)	8.44	5.53	5.97	7.00	7.78	10.28
R_{VFM} (fm)	15.0	10.5	10.5	12.0	13.5	16.5
Δ_∞ (MeV)	0.35	1.33	1.53	1.55	1.50	1.28
T_{crit} (MeV)	0.20	0.76	0.87	0.88	0.85	0.73
ε_{F} (MeV)	1.01	6.48	9.93	14.09	16.10	20.53
μ (MeV)	0.80	4.21	5.80	7.30	7.91	9.09
E_{mg} (MeV)	0.090	0.308	0.261	0.199	0.152	0.009
B_{crit} (10^{15} G)	7.76	26.5	22.5	17.2	13.1	0.82

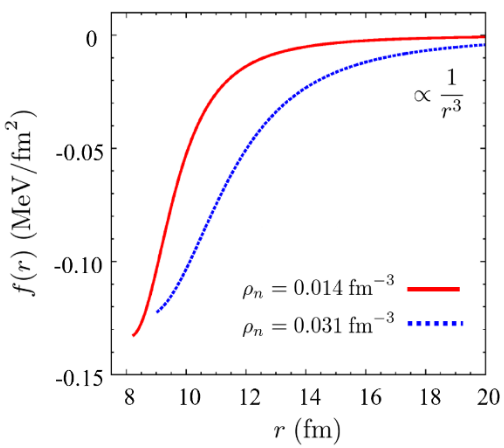
Minigap values

Magnetic field needed to polarize the core

D. Pęcak, N. Chamel, P.M., G. Wlazłowski, Phys. Rev. C**104**, 055801 (2021)

Vortex – impurity interaction (pinning force)



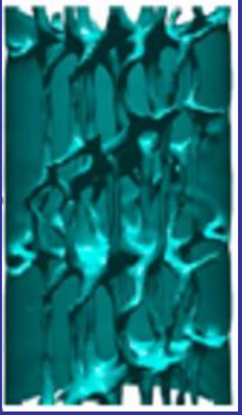


G. Wlazłowski, K. Sekizawa, P. Magierski, A. Bulgac, M.M. Forbes, Phys. Rev. Lett. 117, 232701(2016)

Is neutron star a turbulent system?

- What are differences and similarities of turbulence and its decay in Fermi and Bose superfluids?

A. Bulgac, A. Luo, P. Magierski, K.Roche, Y. Yu, Science 332, 1288 (2011).
M. Tylutki, G. Wlazłowski, Phys. Rev. A103, 051302 (2021).
K.Hossain, K.Kobuszewski, M.M.Forbes, P. Magierski, K.Sekizawa, G.Wlazłowski Phys. Rev. A 105, 013304 (2022).
G. Wlazłowski, M.M. Forbes, S.R. Sarkar, A. Marek, M. Szpindler, PNAS Nexus 3, 160 (2024).

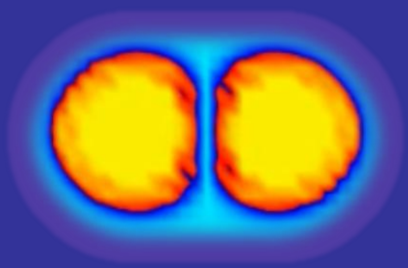


Quantum turbulence

K. Hossain (WSU)
M.M. Forbes (WSU)
K. Kobuszewski (WUT)
S. Sarkar (WSU)
G. Wlazłowski (WUT)

Vortex dynamics in neutron star crust

N. Chamel (ULB)
D. Pęcak (WUT)
G. Wlazłowski (WUT)



Nuclear collisions

M. Barton (WUT)
A. Boulet (WUT)
A. Makowski (WUT)
K. Sekizawa (Tokyo I.)
G. Wlazłowski (WUT)

Josephson junction in atomic Fermi gases - dissipative effects

N. Proukakis (NU)
M. Tylutki (WUT)
G. Wlazłowski (WUT)
K. Xhani (Univ. Of Torino)
and LENS exp. Group,

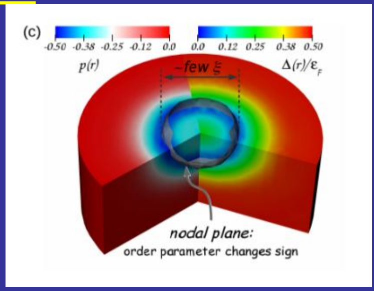
Nonequilibrium superfluidity in Fermi systems

Collisions of vortex-antivortex pairs

A. Barresi (WUT)
A. Boulet (WUT)
G. Wlazłowski (WUT)
and LENS exp. Group

Spin-imbalanced Fermi gases

B. Tuzemen (WUT)
G. Wlazłowski (WUT)
T. Zawiślak (WUT,
Univ. Of Trento)



Solving time-dependent problem for superfluids...

The real-time dynamics is given by equations, which are formally equivalent to the Time-Dependent HFB (TDHFB) or Time-Dependent Bogolubov-de Gennes (TDBdG) equations

$$h \sim f_1(n,\nu,\ldots)\nabla^2 + \boldsymbol{f}_2(n,\nu,\ldots)\cdot\nabla + f_3(n,\nu,\ldots)$$

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t)\\ u_{n,b}(\boldsymbol{r},t)\\ v_{n,a}(\boldsymbol{r},t)\\ v_{n,b}(\boldsymbol{r},t)\end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) & 0 & 0 & \Delta(\boldsymbol{r},t)\\ 0 & h_b(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0\\ 0 & -\Delta^*(\boldsymbol{r},t) & -h_a^*(\boldsymbol{r},t) & 0\\ \Delta^*(\boldsymbol{r},t) & 0 & 0 & -h_b^*(\boldsymbol{r},t)\end{pmatrix}\begin{pmatrix} u_{n,a}(\boldsymbol{r},t)\\ u_{n,b}(\boldsymbol{r},t)\\ v_{n,a}(\boldsymbol{r},t)\\ v_{n,b}(\boldsymbol{r},t)\end{pmatrix}$$

We explicitly track fermionic degrees of freedom!

where h and Δ depends on "densities":

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c}|v_{n,\sigma}(\boldsymbol{r},t)|^2, \qquad \tau_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c}|\nabla v_{n,\sigma}(\boldsymbol{r},t)|^2,$$

$$\chi_c(r,t) = \sum_{E_n<E_c} u_{n,\uparrow}(\boldsymbol{r},t)v^*_{n,\downarrow}(\boldsymbol{r},t), \qquad \boldsymbol{j}_\sigma(\boldsymbol{r},t) = \sum_{E_n<E_c}\mathrm{Im}[v^*_{n,\sigma}(\boldsymbol{r},t)\nabla v_{n,\sigma}(\boldsymbol{r},t)],$$

$$\Delta(\mathbf{r}) = g_{eff}(\mathbf{r})\chi_c(\mathbf{r})$$

$$\frac{1}{g_{eff}(\mathbf{r})} = \frac{1}{g(\mathbf{r})} - \frac{mk_c(\mathbf{r})}{2\pi^2\hbar^2}\left(1 - \frac{k_F(\mathbf{r})}{2k_c(\mathbf{r})}\ln\frac{k_c(\mathbf{r})+k_F(\mathbf{r})}{k_c(\mathbf{r})-k_F(\mathbf{r})}\right)$$

A. Bulgac, Y. Yu, Phys. Rev. Lett. 88 (2002) 042504
A. Bulgac, Phys. Rev. C65 (2002) 051305

huge number of nonlinear coupled 3D Partial Differential Equations
(in practice n=1,2,…, 10^5 - 10^6)

Present computing capabilities:

- full 3D (unconstrained) superfluid dynamics
- spatial mesh up to 100^3
- max. number of particles of the order of 10^4
- up to 10^6 time steps

(for cold atomic systems – time scale: a few ms
for nuclei – time scale: 100 zs)

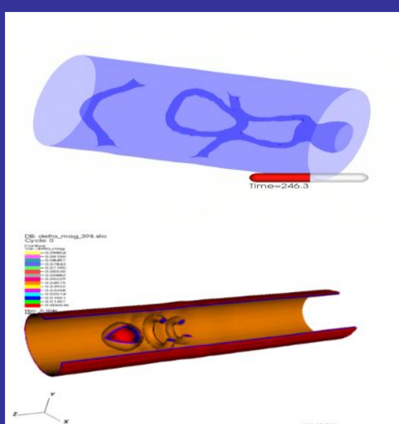
- P. Magierski, *Nuclear Reactions and Superfluid Time Dependent Density Functional Theory*, Frontiers in Nuclear and Particle Physics, vol. 2, 57 (2019)
- A. Bulgac, *Time-Dependent Density Functional Theory and Real-Time Dynamics of Fermi Superfluids*, Ann. Rev. Nucl. Part. Sci. 63, 97 (2013)
- A. Bulgac, M.M. Forbes, P. Magierski, Lecture Notes in Physics, Vol. 836, Chap. 9, p.305-373 (2012)

Superconducting systems of interest

$$\frac{\Delta}{\varepsilon_F} \leq 0.5$$

Ultracold atomic (fermionic) gases. Unitary regime.

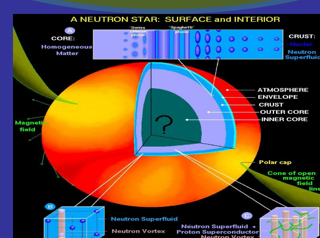
Dynamics of quantum vortices, solitonic excitations, quantum turbulence



$$\frac{\Delta}{\varepsilon_F} \leq 0.1 - 0.2$$

Astrophysical applications.

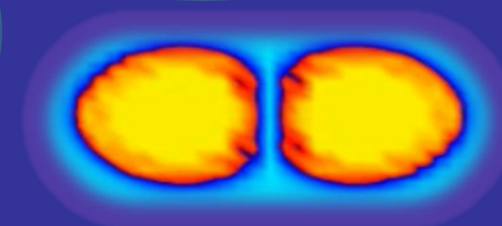
Modelling of neutron star interior (glitches): vortex dynamics, dynamics of inhomogeneous nuclear matter.

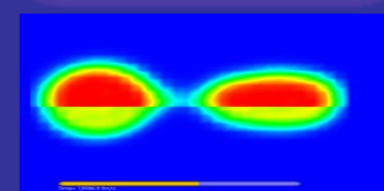


$$\frac{\Delta}{\varepsilon_F} \leq 0.03$$

Nuclear physics.

Induced nuclear fission, fusion, collisions.





$\frac{\Delta}{\varepsilon_F}$ **- Pairing gap to Fermi energy ratio**